Instabilities and turbulence in fusion devices

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Outline

- Introduction
- MHD instabilities
- Drift waves
- Turbulence, transport and fluctuation measurements

Instabilities and turbulence : why?

Lawson criterion $P_{fusion} > P_{loss}$

$$\beta B^2 \tau_E \geq 4T^2 s$$

Plasma «beta»

$$\beta = \frac{P}{B^2/2\mu_0}$$

MHD Stability

Magnetic field

Cost

Confinement time

$$\tau_E = \frac{W}{P_{add}}$$

Turbulence

MHD instabilities

- In single fluid MHD, two main destabilizing terms:

1) Pressure gradient → interchange
2) Current density gradient → kink

- In fusion devices, radiative instabilities → density limit

Single fluid ideal MHD

- MHD equations

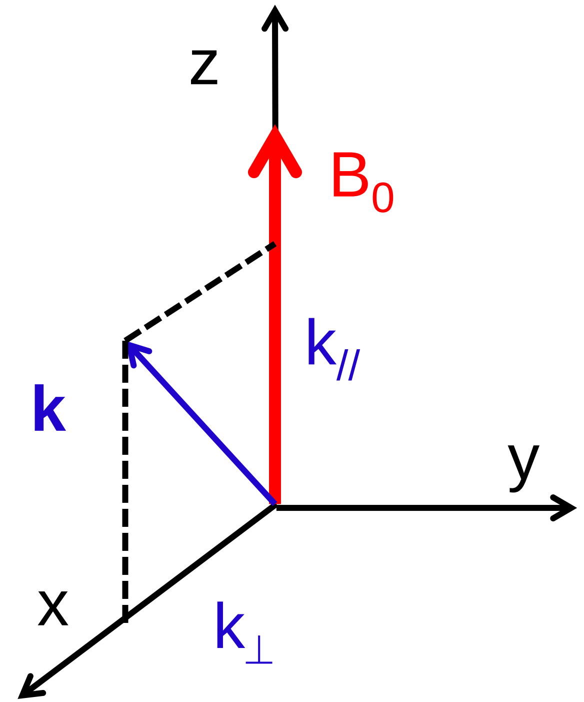
Velocity — Magnetic field

$$\rho d_t \mathbf{V} = -\nabla P + \mathbf{J} \times \mathbf{B}$$

$$d_t \mathbf{B} = (\mathbf{B} \cdot \nabla)\mathbf{V} - \mathbf{B}(\nabla \cdot \mathbf{V})$$

$$d_t P = -\Gamma P (\nabla \cdot \mathbf{V})$$

Pressure — Adiabatic index



- Lagrangian derivative $d_t = \partial_t + \mathbf{V} \cdot \nabla$

- MHD displacement $$\frac{d\boldsymbol{\xi}(\mathbf{x},t)}{dt} = \mathbf{V}(\mathbf{x},t)$$

Alfvén waves

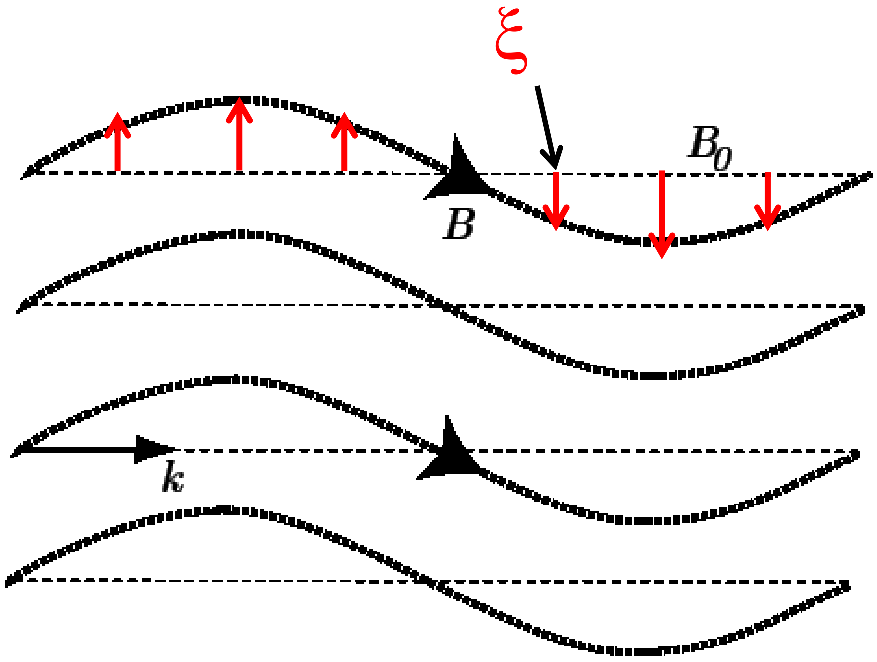
- 3 solutions: shear Alfvén wave, fast and slow magneto-acoustic waves

- Shear Alfvén wave

$$\omega = k_{\parallel} v_A$$

- Alfvén velocity

$$v_A = \sqrt{\frac{B^2}{\mu_0 \rho}}$$



MHD instabilities : kink

- A flexible wire plunged in a magnetic field is unstable

- Displacement

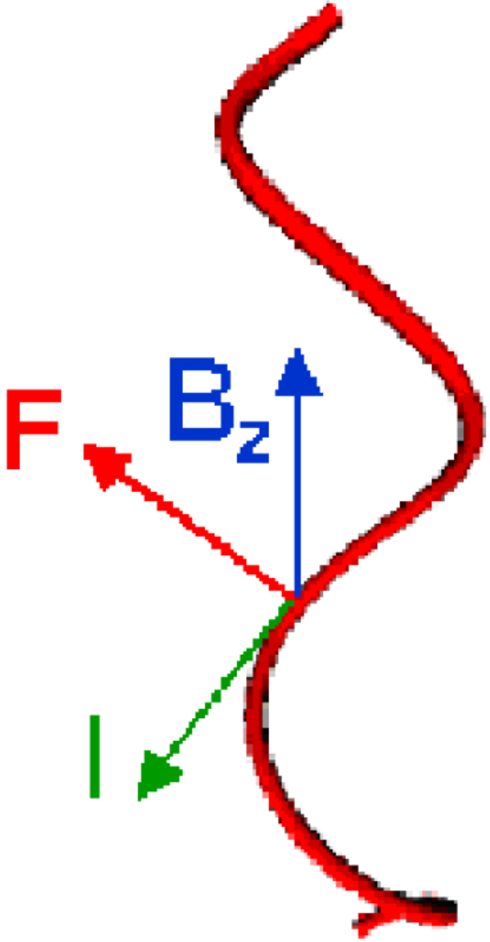
$$\boldsymbol{\xi} = \xi_r cos(kz)\mathbf{e_x} + \xi_r sin(kz)\mathbf{e_y}$$

- Perturbed current

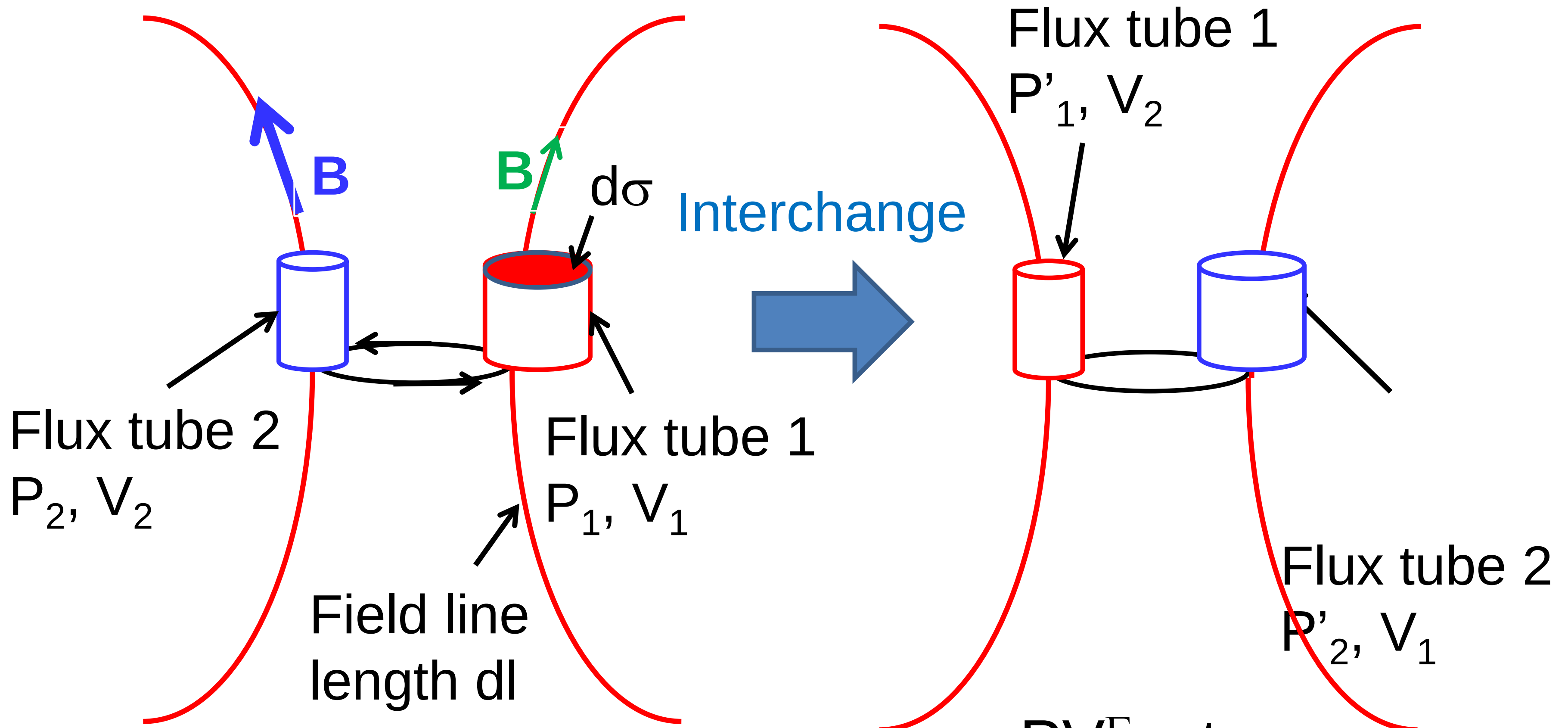
$$\delta\mathbf{I} = I_0 \frac{\partial \boldsymbol{\xi}}{\partial z}$$

- Force balance equation

$$\mu_\rho \frac{\partial^2 \xi_r}{\partial t^2} = I_0 B_0 k \xi_r$$



MHD instabilities : interchange



PV^{Γ}=cte

$$\delta W = \delta P \delta V \simeq \delta P \delta \int \frac{d\ell}{B} \propto -\delta P \delta B$$

Not always true

Interchange in a tokamak - ballooning modes

- Low field side is unstable $\rightarrow$ "ballooning"

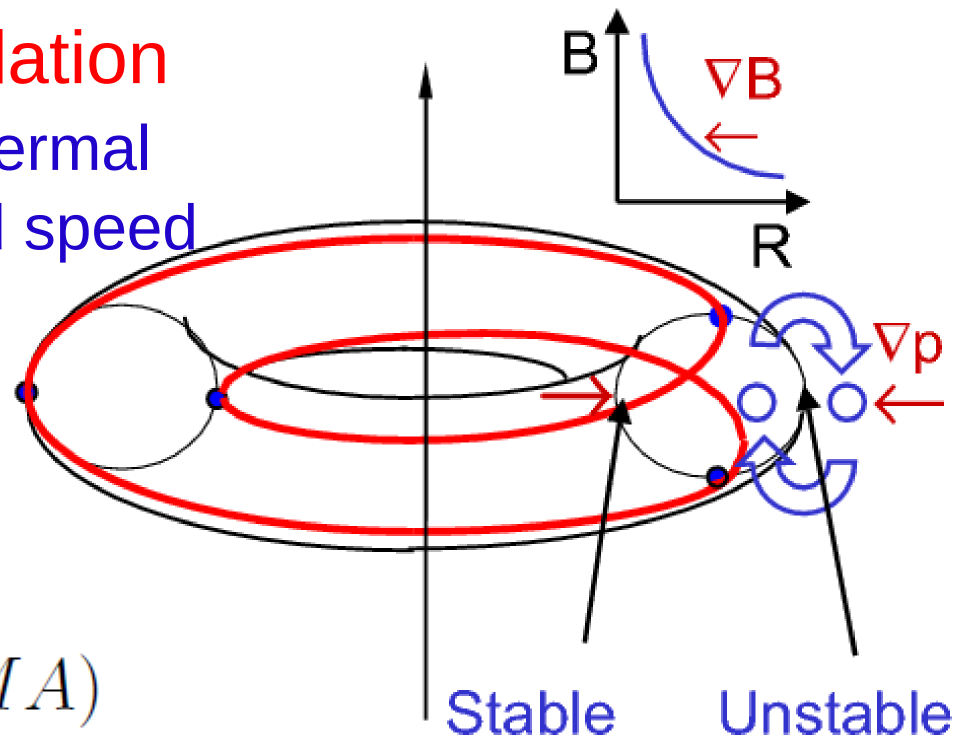
- Schematic dispersion relation

Field curvature

Isothermal sound speed

$$\omega^2 = k_\parallel^2 v_A^2 + \kappa \frac{c_S^2}{L_p}$$

Pressure gradient length



- Threshold $\beta(\%) > 3\frac{I(MA)}{aB}$

Radiative instability

- $P_{rad} > P_{heat}$ not sustainable

density → radiative power

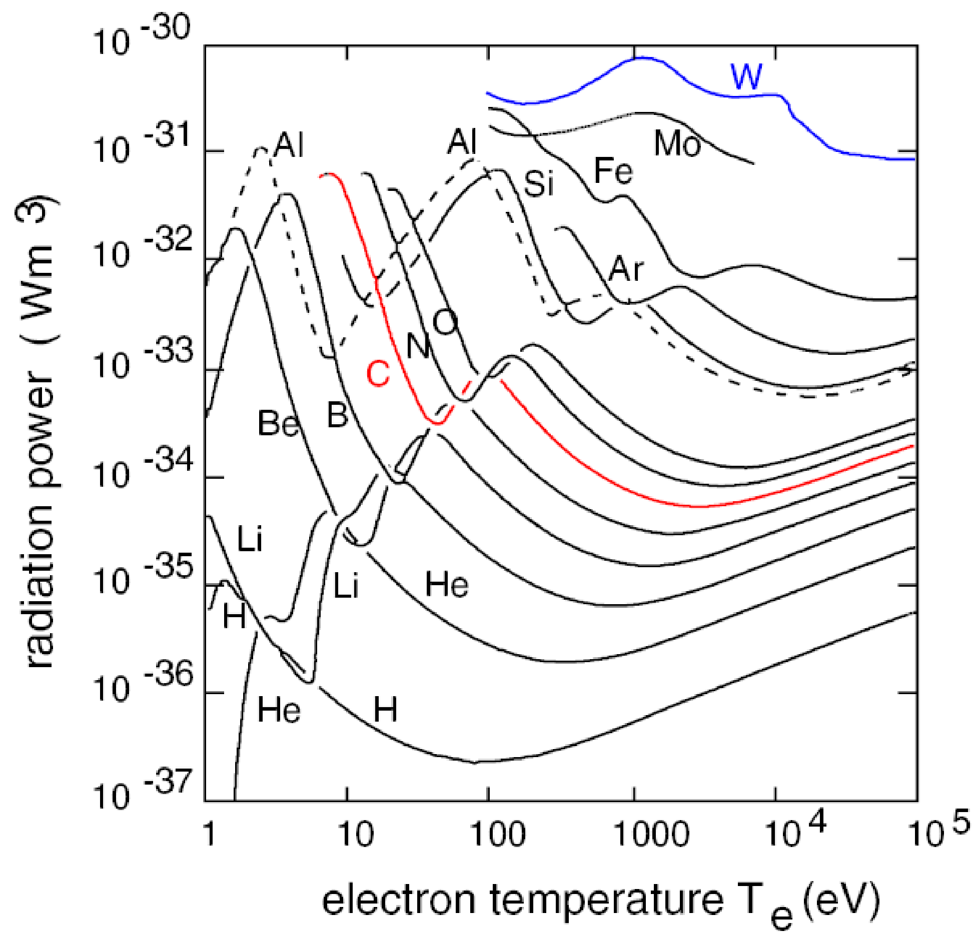
$$P_{rad} \simeq C_{imp} n_e^2 L(T)$$

resistivity → Current density

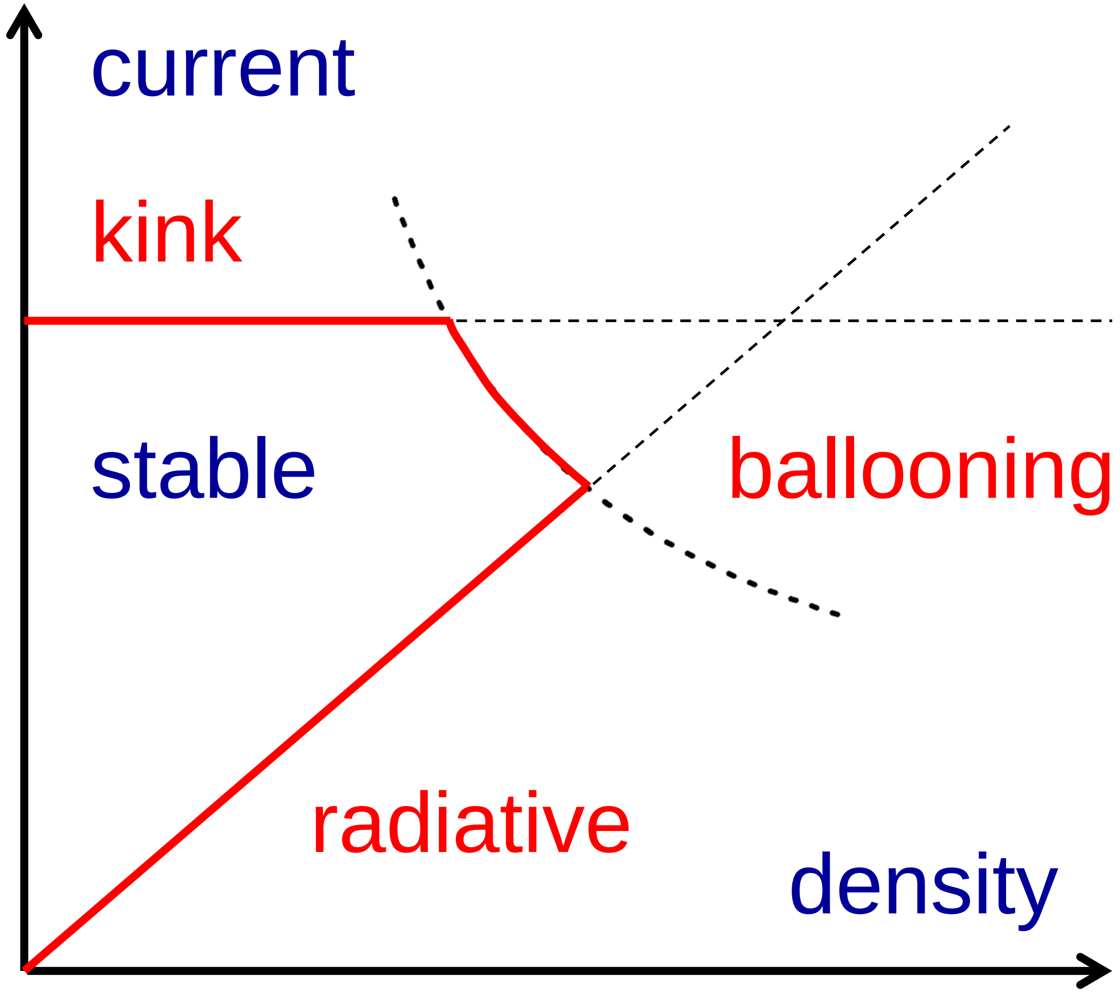
$$P_{heat} \simeq \eta j^2$$

- Instability criteria

$$n_e(10^{20} m^{-3}) \geq \frac{I(MA)}{\pi a^2}$$



Stability domain of a fusion device



Drift waves and turbulence

- MHD validity conditions

$$k_\perp \delta_{orbit} < 1 \quad \text{and} \quad \omega \gg \mathbf{k}\cdot\mathbf{v}$$

- Not good enough to describe low frequency, high wavenumbers instabilities

- However basic drive mechanisms still hold

- Transition to turbulence $\rightarrow$ rules confinement time

Low frequency limit

- Momentum equation for each species

$$nmd_t\mathbf{V} = -\nabla P + ne\left(\mathbf{E} + \mathbf{V}\times\mathbf{B}\right) + \ \ldots$$

- Strong guide field B

Polarization drift

Parallel flow

$$\mathbf{V} = V_{\parallel}\mathbf{b} + \mathbf{V}_E + \mathbf{V}^* + \mathbf{V}_{pol} + \ \ldots$$

EXB velocity

Diamagnetic velocity

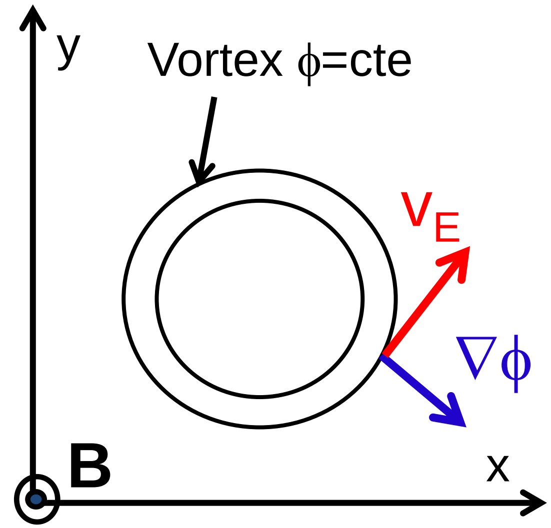
$$\mathbf{V}_E = \frac{\mathbf{E}\times\mathbf{B}}{B^2}$$

$$\mathbf{V}^* = \frac{\mathbf{B}}{B^2}\times\frac{\nabla P}{ne}$$

Drift wave – ion response

- Simple slab geometry, electrostatic, fluid ions

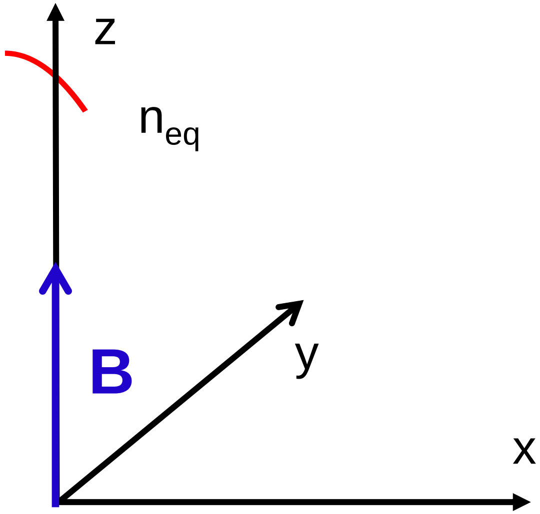
$$\frac{\partial n_i}{\partial t} + \nabla \cdot (n_i \mathbf{V}_E) = 0$$



- Linear ion density response

$$\delta n_i(\mathbf{x}, t) = n_{i,\mathbf{k}\omega} e^{i(\mathbf{k}\cdot\mathbf{x} - \omega t)} + c.c.$$

$$n_{i,\mathbf{k}\omega} = -\frac{k_y}{B}\frac{\partial n_{eq}}{\partial x}\phi_{\mathbf{k}\omega}$$



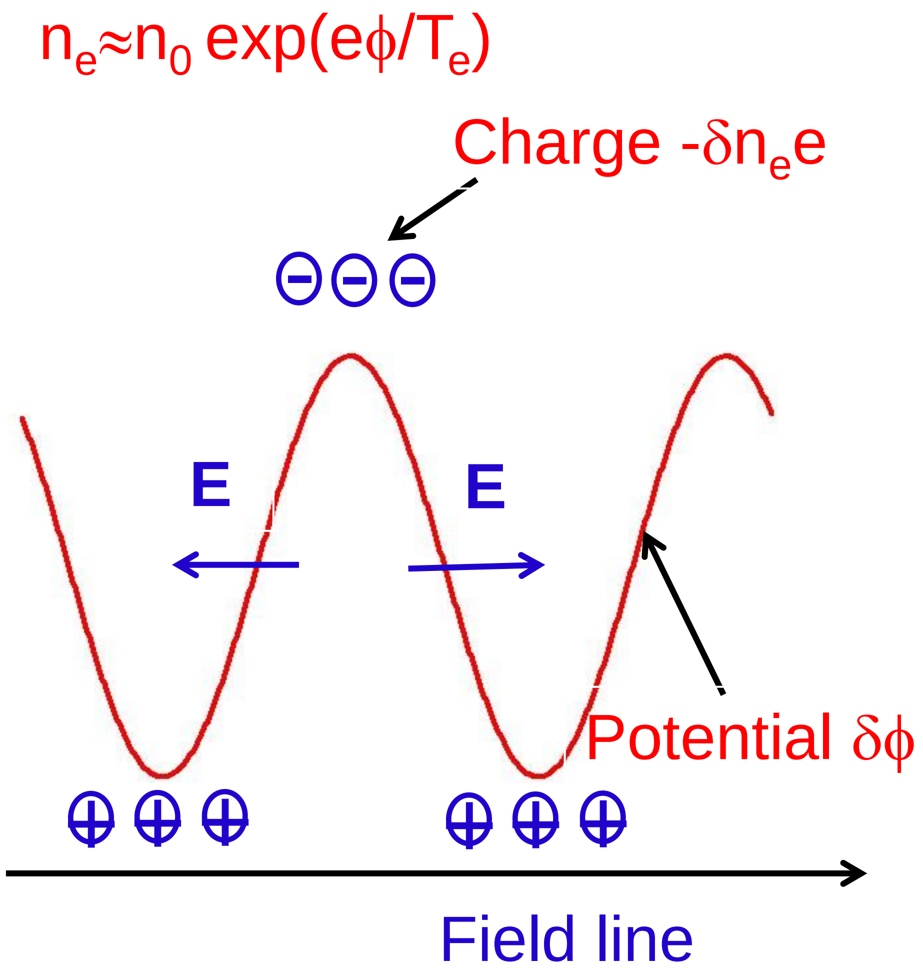
Drift wave – electron response

- fast motion along field lines → adiabatic electrons

$$\frac{\delta n_e}{n_{eq}} = \frac{e\delta\phi}{T_e}$$

- electro-neutrality ($k\lambda_D \ll 1$)

$$n_e = n_i$$



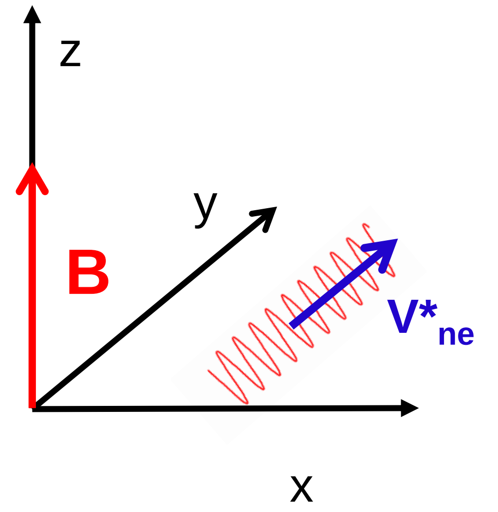
Drift wave (cont.)

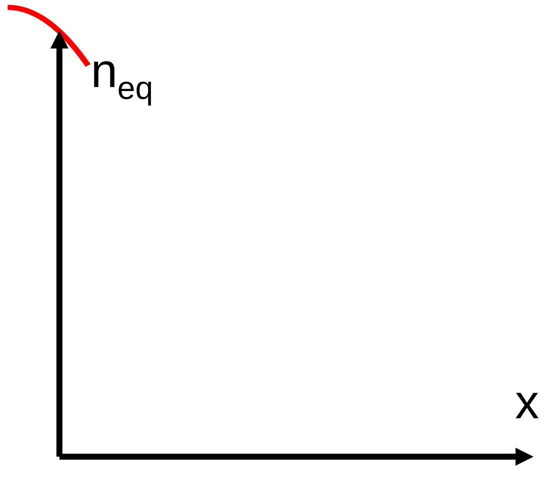
- Density gradient in the x direction, uniform B, y,z periodic

- Phase velocity = electron diamagnetic velocity

$$v_{ph} = \frac{\omega}{k_y} = V^*_{ne}$$

$$\mathbf{V}^*_{ne} = -\frac{T_e}{eB}\frac{\mathbf{B}}{B^2} \times \frac{\nabla n_e}{n_e}$$





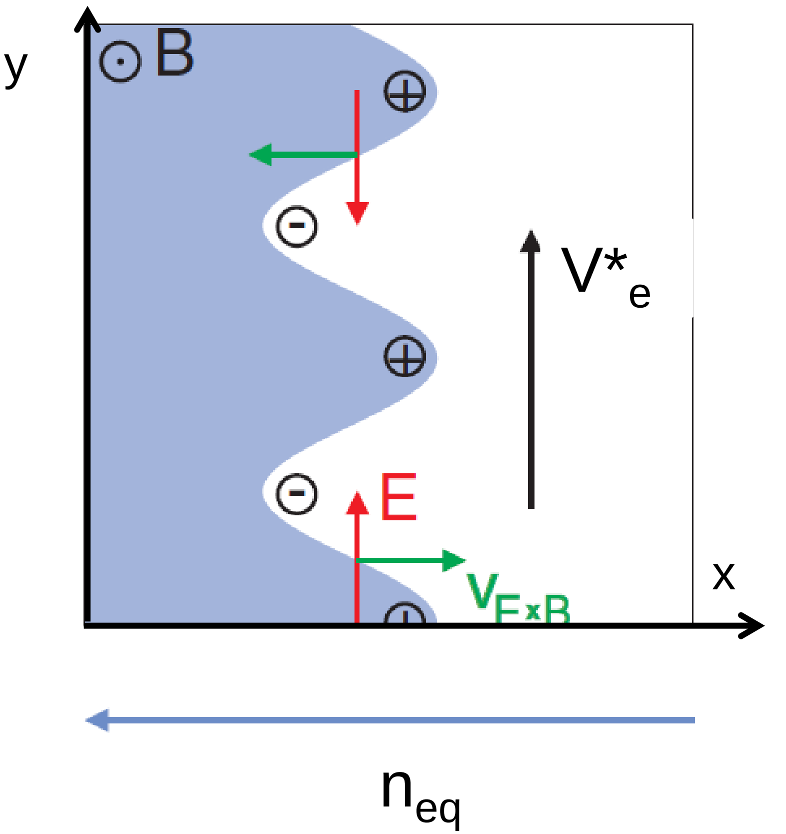
Drift wave (cont.) – schematic picture

- Start with a density n_i=n_e corrugation

- Fast electron response along field lines → potential adjusts → electric field E

- ExB drifts shifts the perturbation along V^*_e

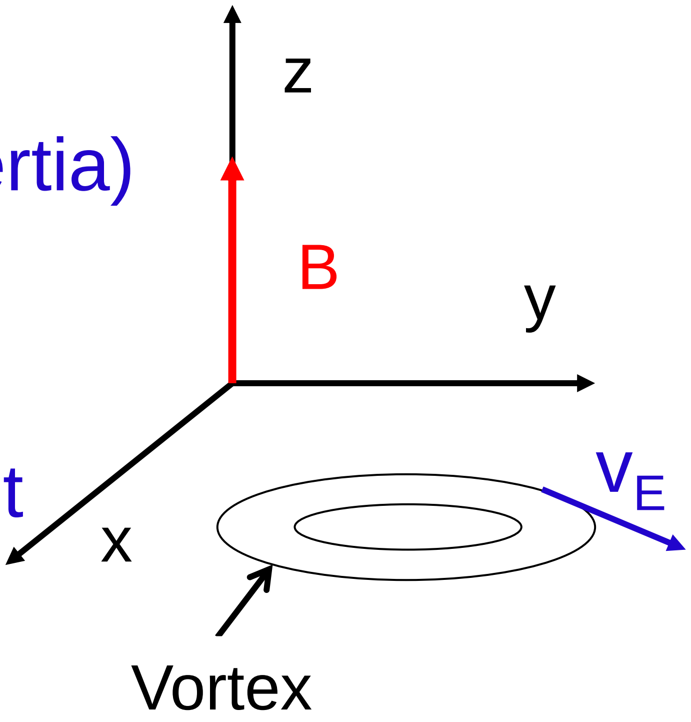
Grulke & Klinger 02



The ion inertia plays a crucial for non linear saturation

- Basic model: no instability, infinite number of non linear solutions
- Add the polarization drift (ion inertia)

$$\mathbf{V} = \mathbf{V}_E + \mathbf{V}^* + \mathbf{V}_{pol} + \ \dots$$



Divergence of polarization current

$$\nabla \cdot (ne\mathbf{V}_{pol}) \simeq -\frac{n_{eq}m_i}{B^2} d_t \nabla_\perp^2 \phi$$

$$\mathbf{v_E} = \frac{\mathbf{B}}{\mathrm{B}^2} \times \nabla\phi$$

Lagrangian derivative

$$d_t = \partial_t + \mathbf{V}_E \cdot \nabla$$

A paradigm : the CHM equation

- Same assumptions+ polarization drift

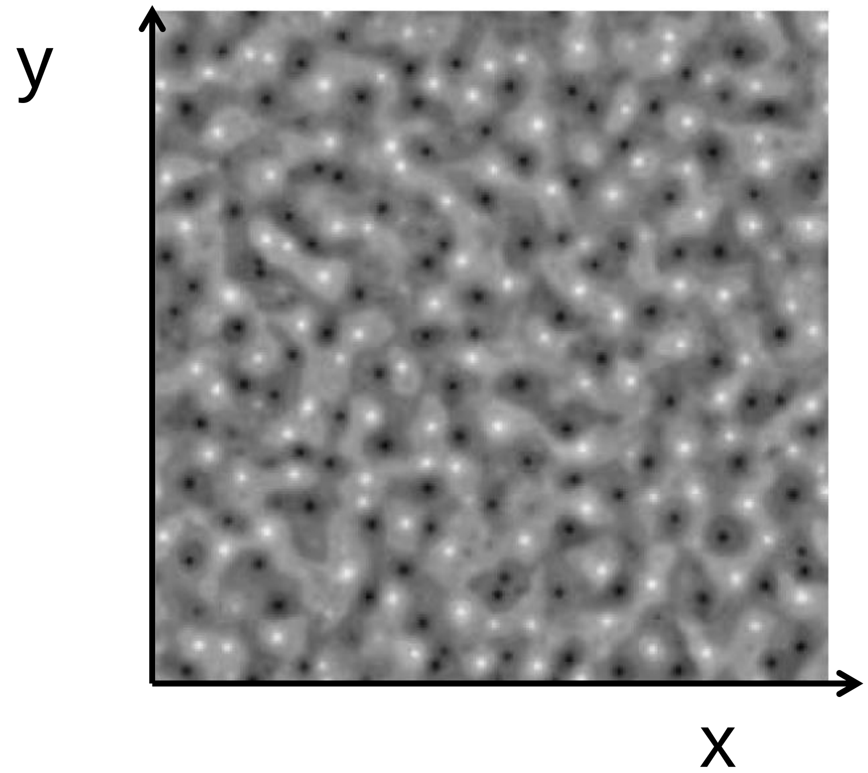
- Charney-Hasegawa-Mima (CHM) equation

$$d_t\left(\phi - \rho_s^2\nabla_\perp^2\phi\right) + \mathbf{V}_{ne}^* \cdot \nabla\phi = 0$$

Ion gyroradius $\rho_s = \frac{\sqrt{m_s T_e}}{eB}$

- Dispersion relation

$$\omega = \frac{k_y V_{ne}^*}{1 + k_\perp^2 \rho_s^2}$$



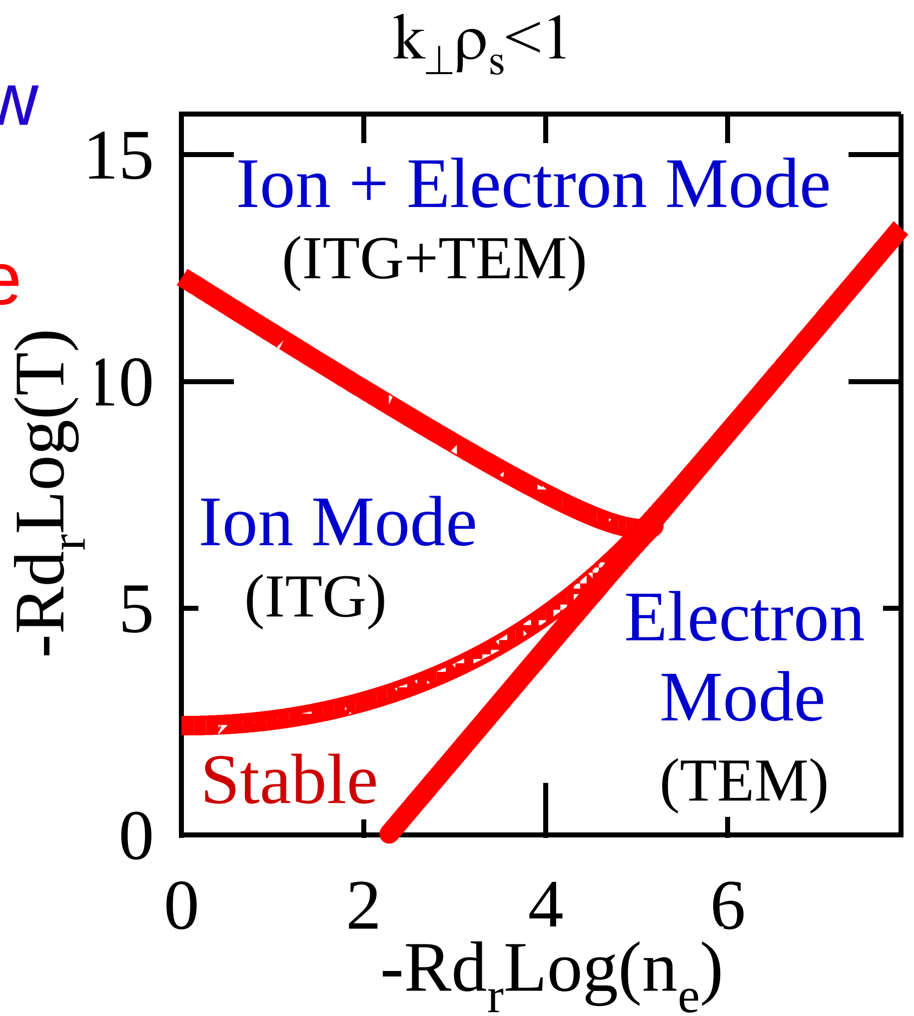
Bofetta 02

Example of a tokamak: electrostatic modes

- Dominant instabilities at low frequency : drift waves mainly driven by interchange

- Kinetic type: driven via resonances by electrons or ions.

- Threshold in temperature and density gradients



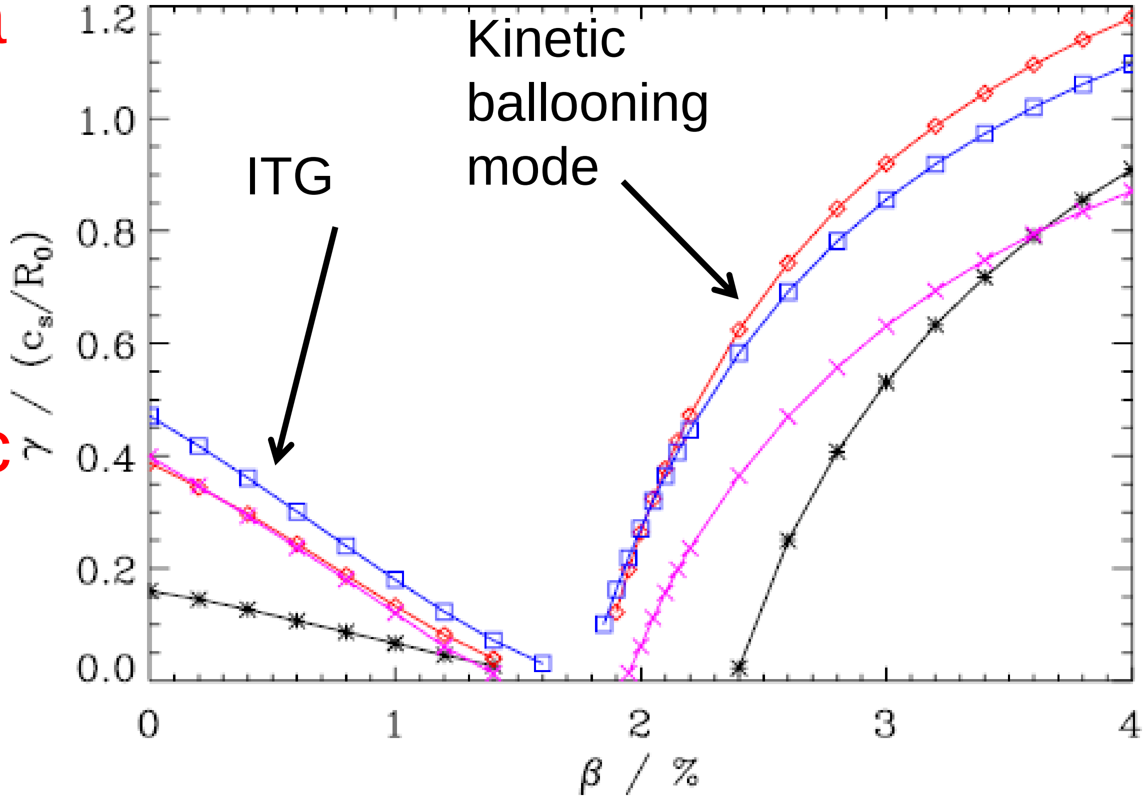
Connection with MHD

Pueschel 2010

- Drift waves dominate at low beta

$$\beta = \frac{2P}{B^2/2\mu_0}$$

- At high beta, kinetic ballooning modes become unstable



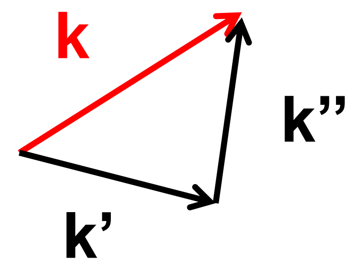
Non linear saturation

- Determines the confinement time
- Variety of non linear dynamics:

1) Few modes : steady saturated state, relaxation oscillations, explosive behavior, …
2) Many coupled modes : usually evolve towards turbulence. Turbulent state is different if waves are involved.

Mode coupling is one route to get saturation

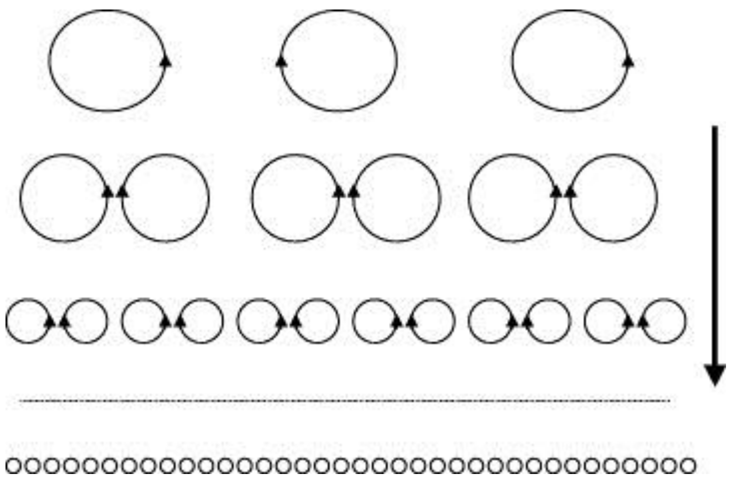
- Generic form of a non linear equation in Fourier space



$$\phi(\mathbf{x},t)=\sum_{\mathbf{k}}\phi_{\mathbf{k}}(t)\exp\{i\mathbf{k}\cdot\mathbf{x}\}$$

$$\partial_t\phi_{\mathbf{k}}(t)=-i\omega(\mathbf{k})\phi_{\mathbf{k}}(t)+\sum_{\mathbf{k}'\mathbf{k}''}\Lambda_{\mathbf{k}'\mathbf{k}''}\phi_{\mathbf{k}'}(t)\phi_{\mathbf{k}''}(t)$$

- Triad $\mathbf{k}'+\mathbf{k}''=\mathbf{k}$



- Coupling leads to energy transfer between waves – if coupling is ''local'' $\rightarrow$ cascade

CHM equation conserves two quadratic invariants

- In absence of dissipation, two quadratic invariants

« energy »

$$E = \frac{1}{2}\int dxdy\,\left[|\phi|^2 + \rho_s^2\,|\nabla_\perp \phi|^2\right]$$

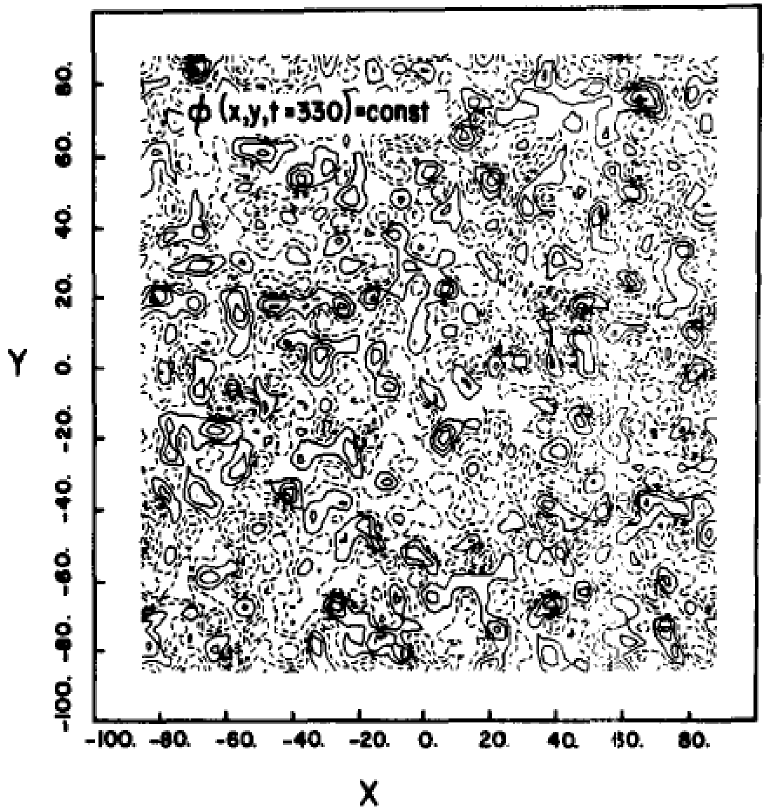
and « enstrophy »

$$\Omega = \frac{1}{2}\int dxdy\,\left[|\nabla_\perp \phi|^2 + \rho_s^2\,|\nabla_\perp^2 \phi|^2\right]$$

Many unstable modes: turbulence

- If many modes are unstable : the system evolves towards a turbulent state Waltz 83, Horton 86
- Behaves as a stochastic dynamical system

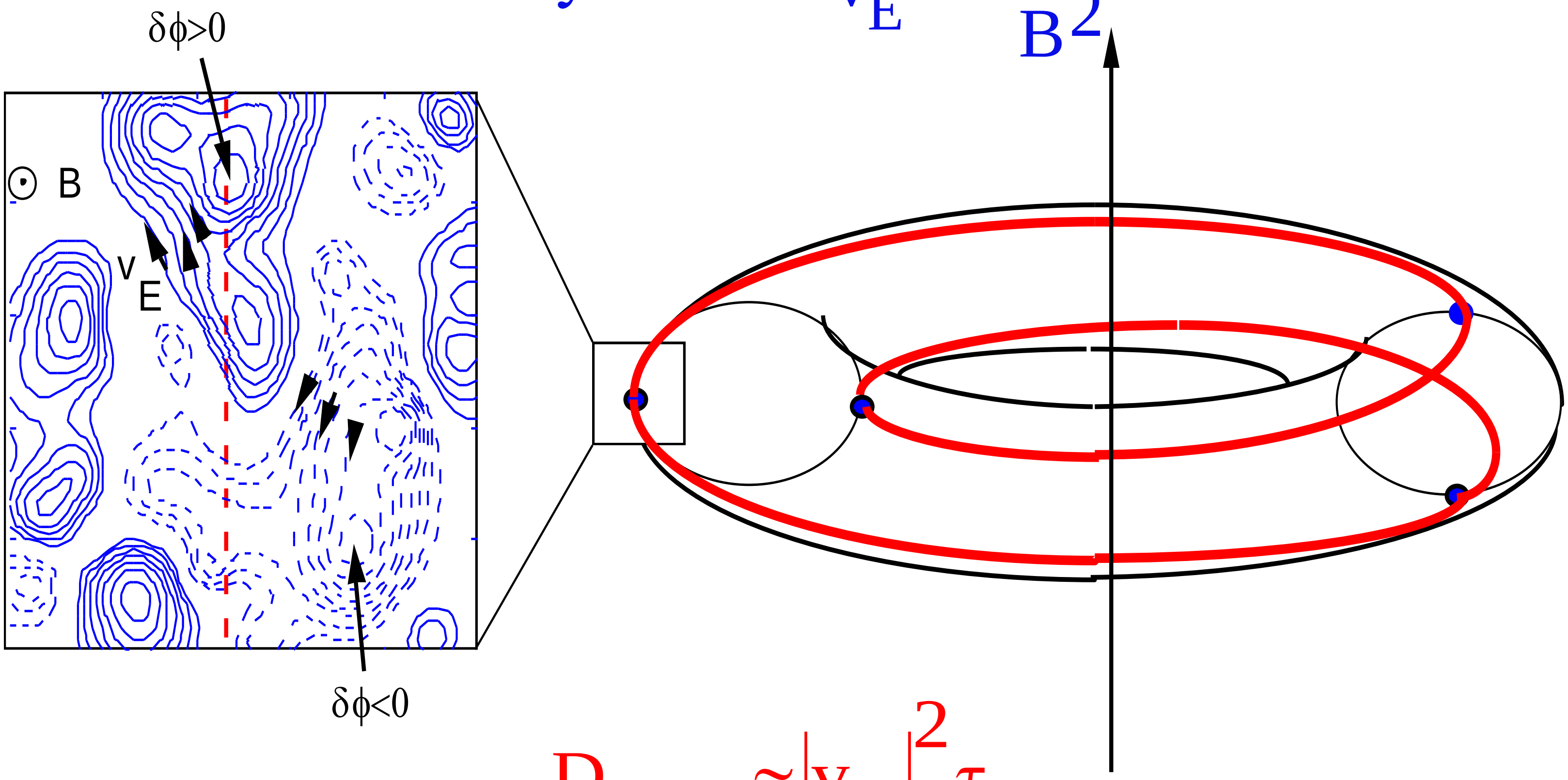
Horton 86



Turbulent transport: random walk

E×B drift velocity

$$v_E = \frac{B \times \nabla \phi}{B^2}$$



$$D_{turb.} \approx |v_E|^2 \tau_c$$

Transport equation

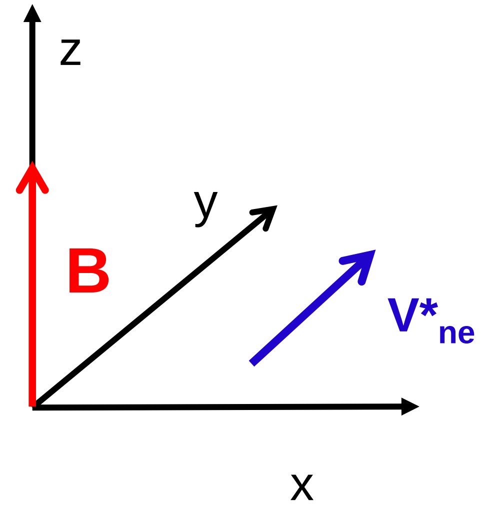
- Density equation averaged over periodic direction

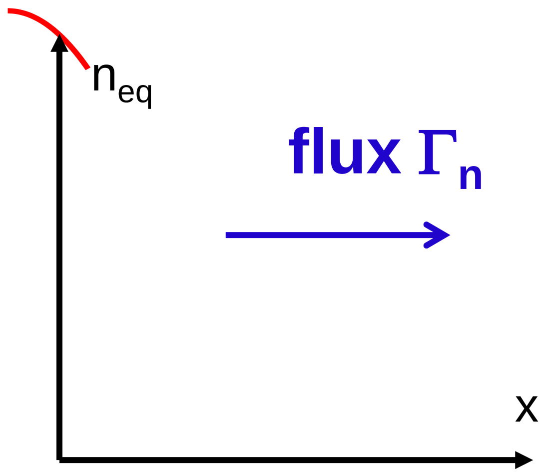
$$\frac{\partial n_{eq}}{\partial t} + \frac{\partial \Gamma_n}{\partial x} = 0$$

$$n_{eq}(x,t) = \int \frac{dy}{2\pi} n_i(x,y,t)$$

- Particle flux

$$\Gamma_n = \sum_{\mathbf{k},\omega} \mathbf{V}^*_{E,\mathbf{k}\omega} n_{i,\mathbf{k}\omega}$$





Quasi-linear theory

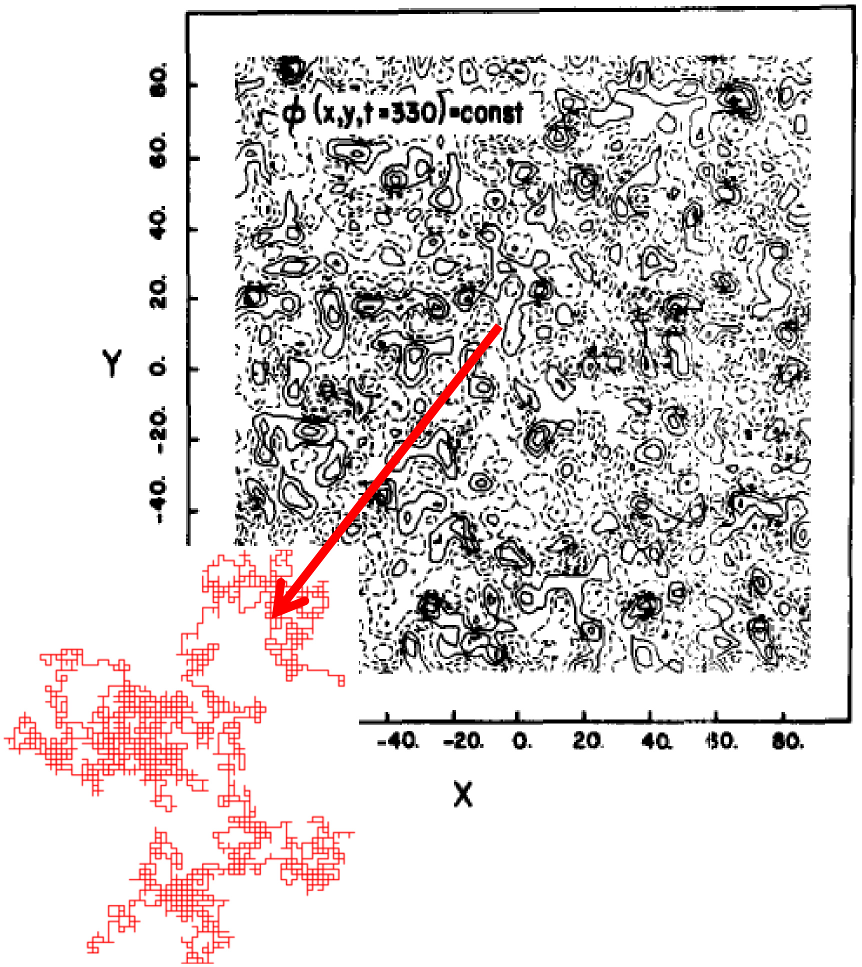
- Plug linear response into flux expression

$$\Gamma = -D_{QL} \frac{\partial n_{eq}}{\partial x}$$

$$D_{QL} = \pi \sum_{\mathbf{k},\omega} \left| \mathbf{V}_{E,\mathbf{k}\omega} \right|^2 \delta(\omega)$$

- Agrees with random walk estimate
- Oversimplified picture

Horton 86

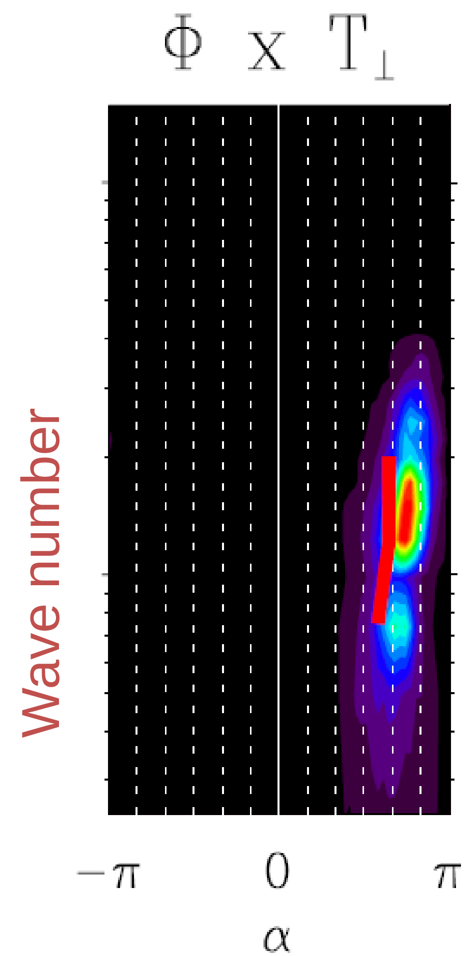


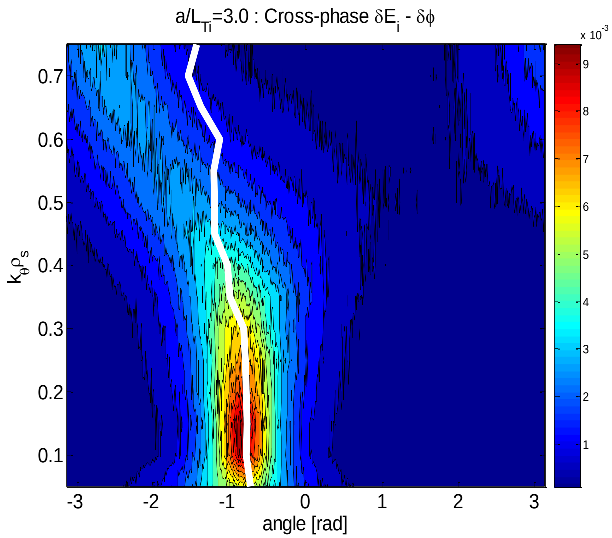
Quasi-linear theory is incomplete

- Key point: cross-phase between density and potential given by linear theory. Works quite well in many cases

- Theory is incomplete: amplitude and spectra unknown at this stage

Dannert 05





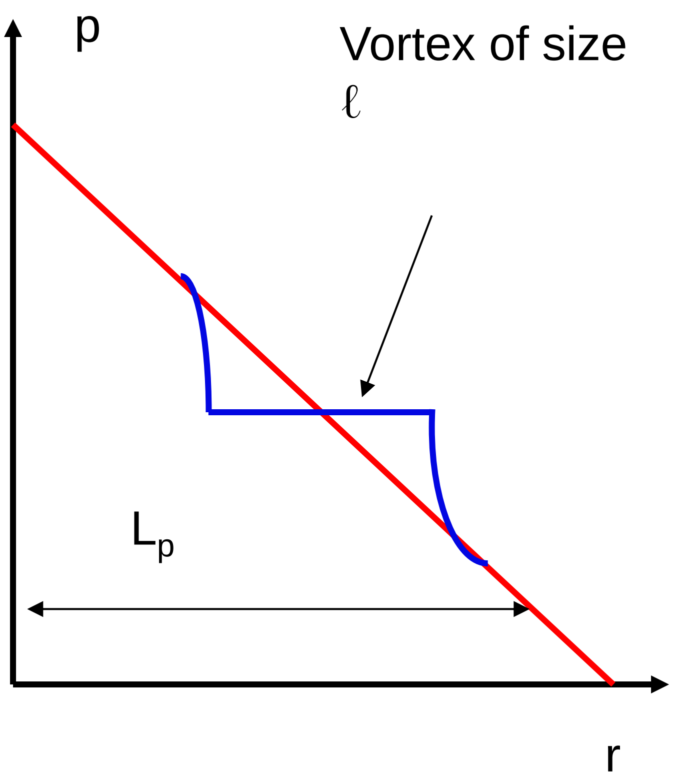
Casati 08

Analogy with vortex mixing : mixing-length estimate

Mixing of the pressure profile by vortex of size ℓ

→ "mixing length estimate" of the fluctuation level

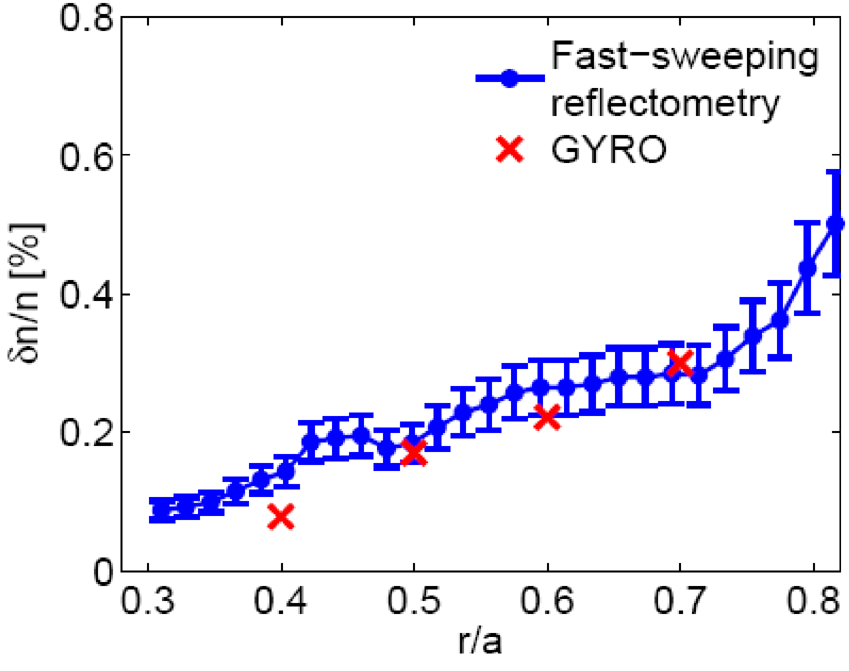
$$\frac{\delta n}{n_{eq}} \approx \frac{\ell}{L_p}$$



More on turbulence intensity

- Simplest mixing length estimate usually not good enough
- Improved expressions via fits of turbulence simulations
- Validation via comparison with measurements

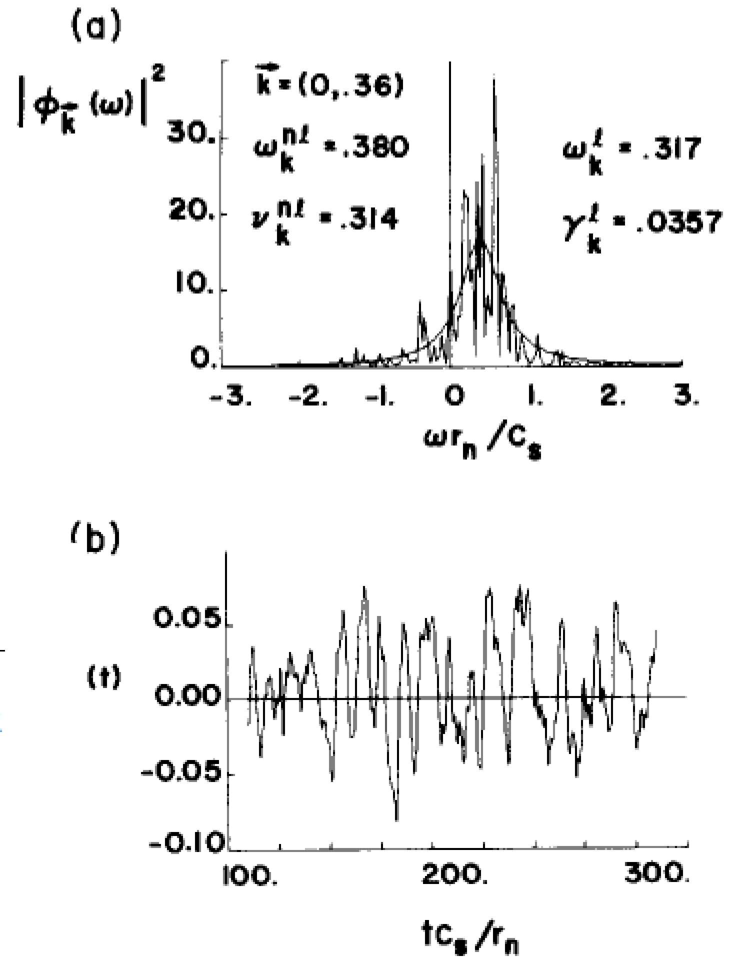
Casati 09



Frequency spectra

Horton 86

- Frequency spectra are broad.
- Prediction still an open issue
- Lorentzian often used

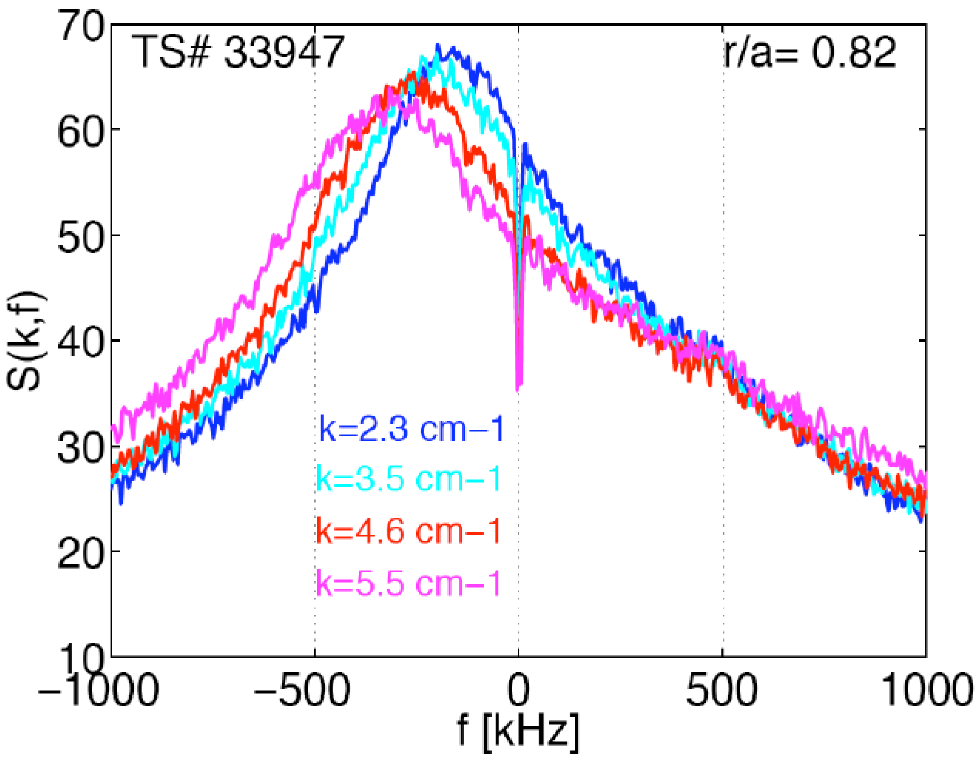


$$|\mathbf{V}_{E,\mathbf{k}\omega}|^2 \sim |\mathbf{V}_{E,\mathbf{k}}|^2 \frac{1}{\pi} \frac{\Delta\omega_{\mathbf{k}}}{(\omega-\omega_{\mathbf{k}})^2+\Delta\omega_{\mathbf{k}}^2}$$

with $\Delta\omega_k \sim \gamma_k$

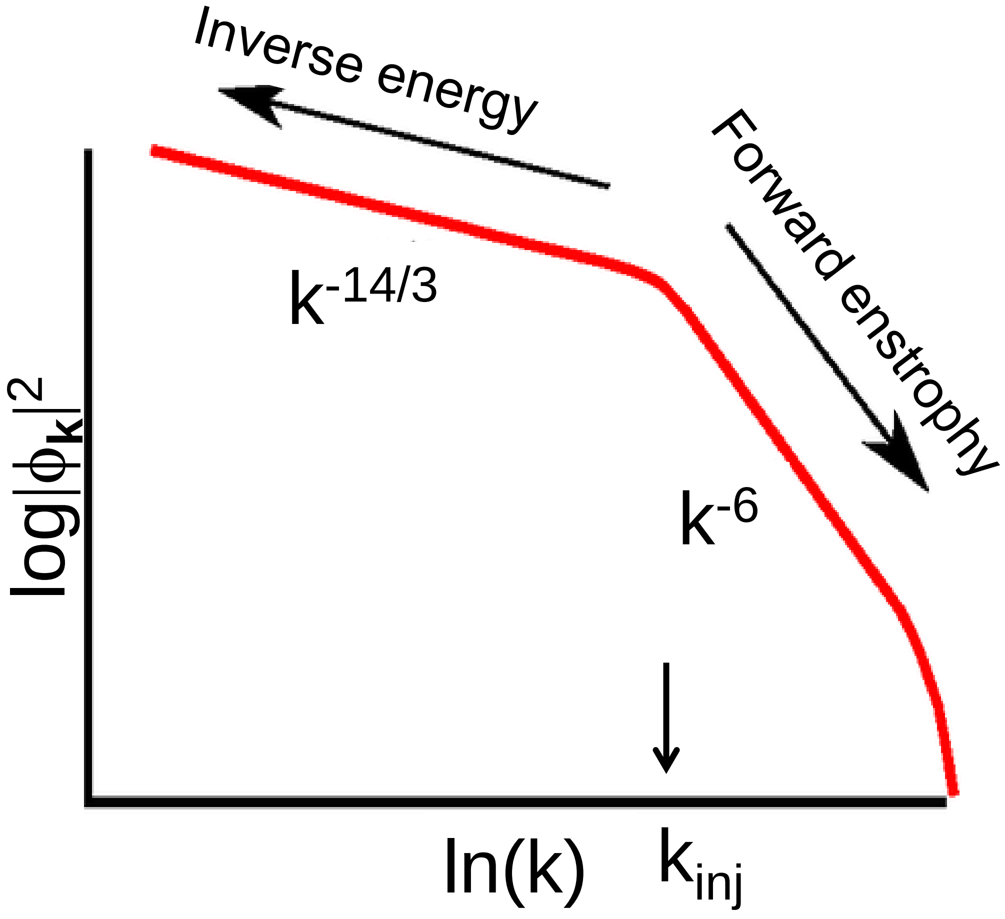
Frequency spectra –comparison to experiment

- Measured with Doppler reflectometry Hennequin 06
- Lorentzian fails – also $\Delta\omega_k \neq \gamma_k$
- Rather agrees with convective /diffusive model Hennequin 99



Wave-number spectra

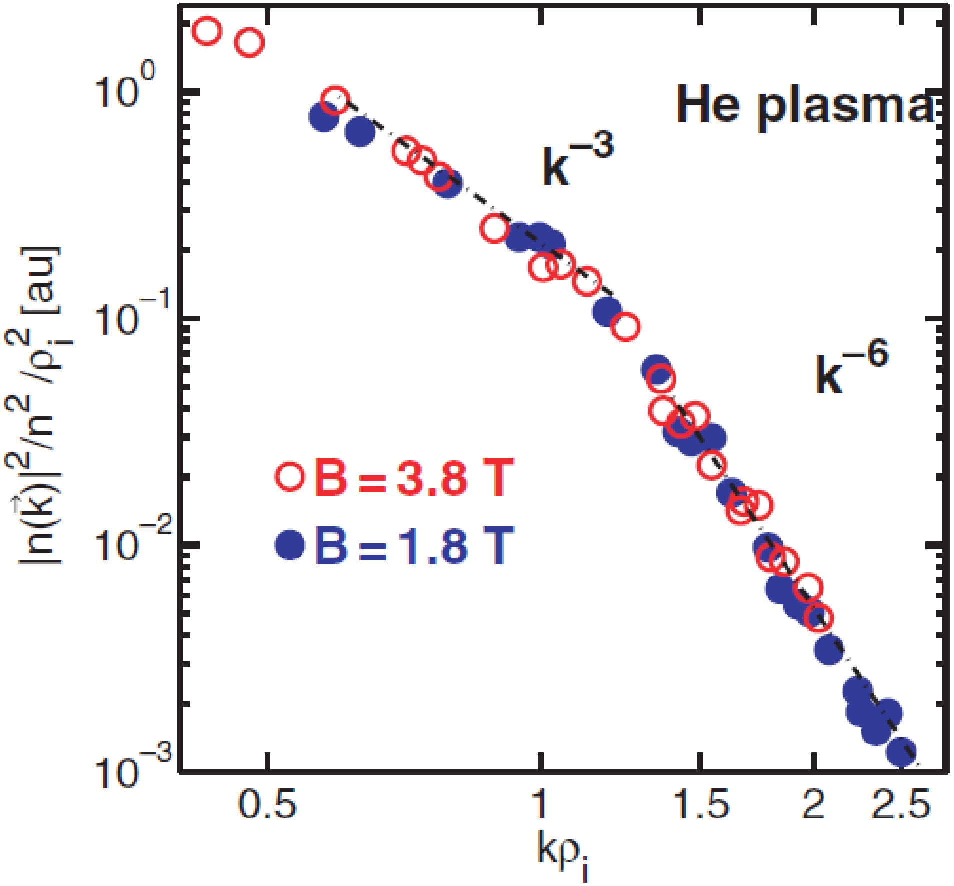
- A dual cascade is expected CHM
- Does not fit observation in tokamaks – many reasons:
- no inertial range
- coupling to large scale flows
- kinetic effects (Landau resonance, orbit width effects)



More on wave-number spectra

- Slope always the same
$\rightarrow$ NL effects dominant.
- Change of slope is usually observed at $k_\perp \rho_i \sim 1$ Hennequin 04.
- Possible explanations: effect of zonal flows Gurcan 09 , entropy cascade and critical balance Tatsuno 09, Barnes 11

Hennequin 04



Other channels for non linear saturation

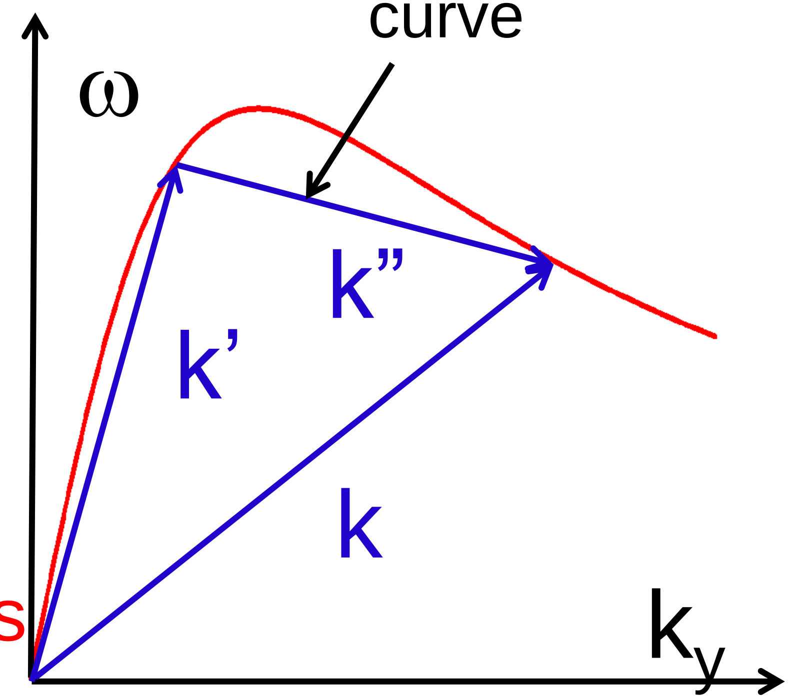
- Bragg's law **k**=**k'**+**k''** difficult to satisfy when all modes on the dispersion relation

- Coupling via radial profiles is an efficient mechanism **k**=(k_x,k_y=0)

- Two ways:

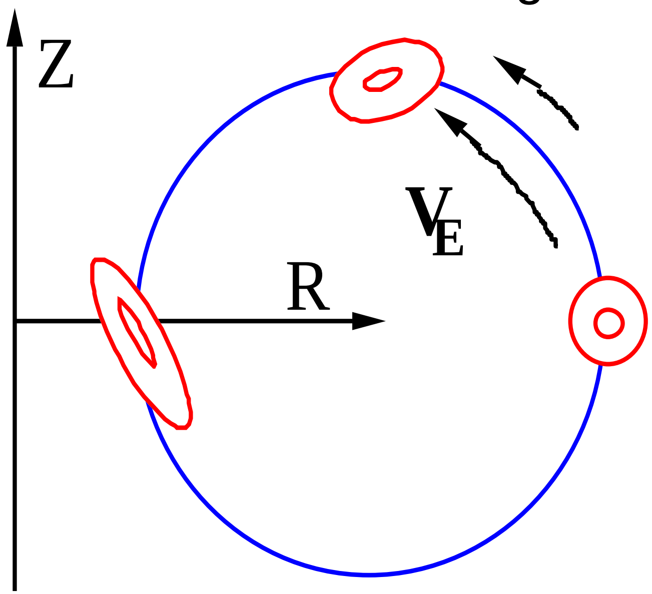
- Electric potential : zonal flows

- Density & temperature profile: streamers, avalanches

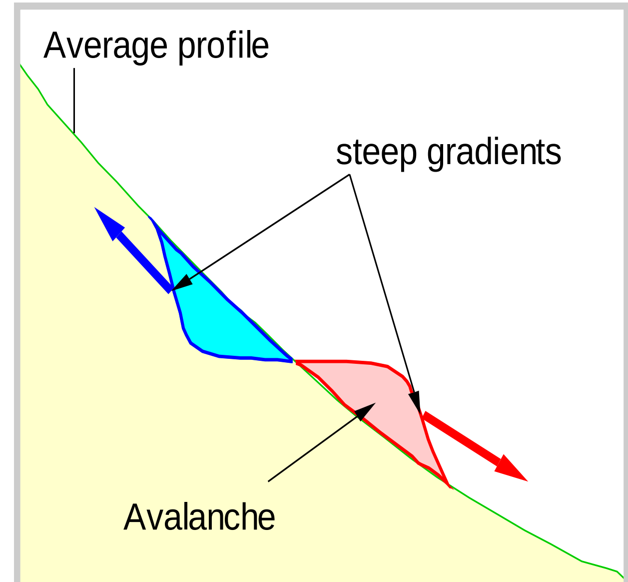


Structure dynamics

Zonal Flows Sagdeev 78, Hasegawa 79



Avalanches
Diamond & Hahm 95

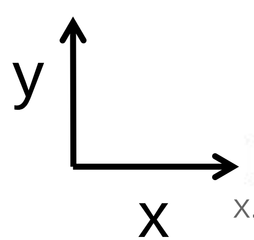


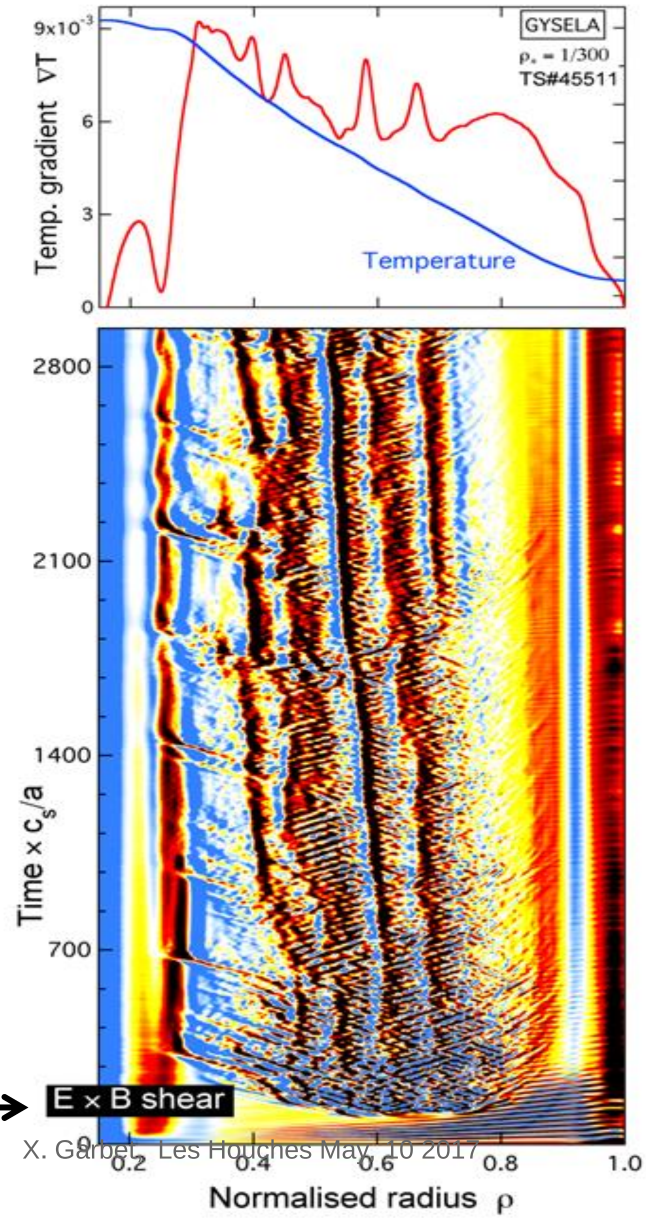
Streamers Jenko & Dorland 00

Turbulence self-organisation

Dif-Pradalier 15

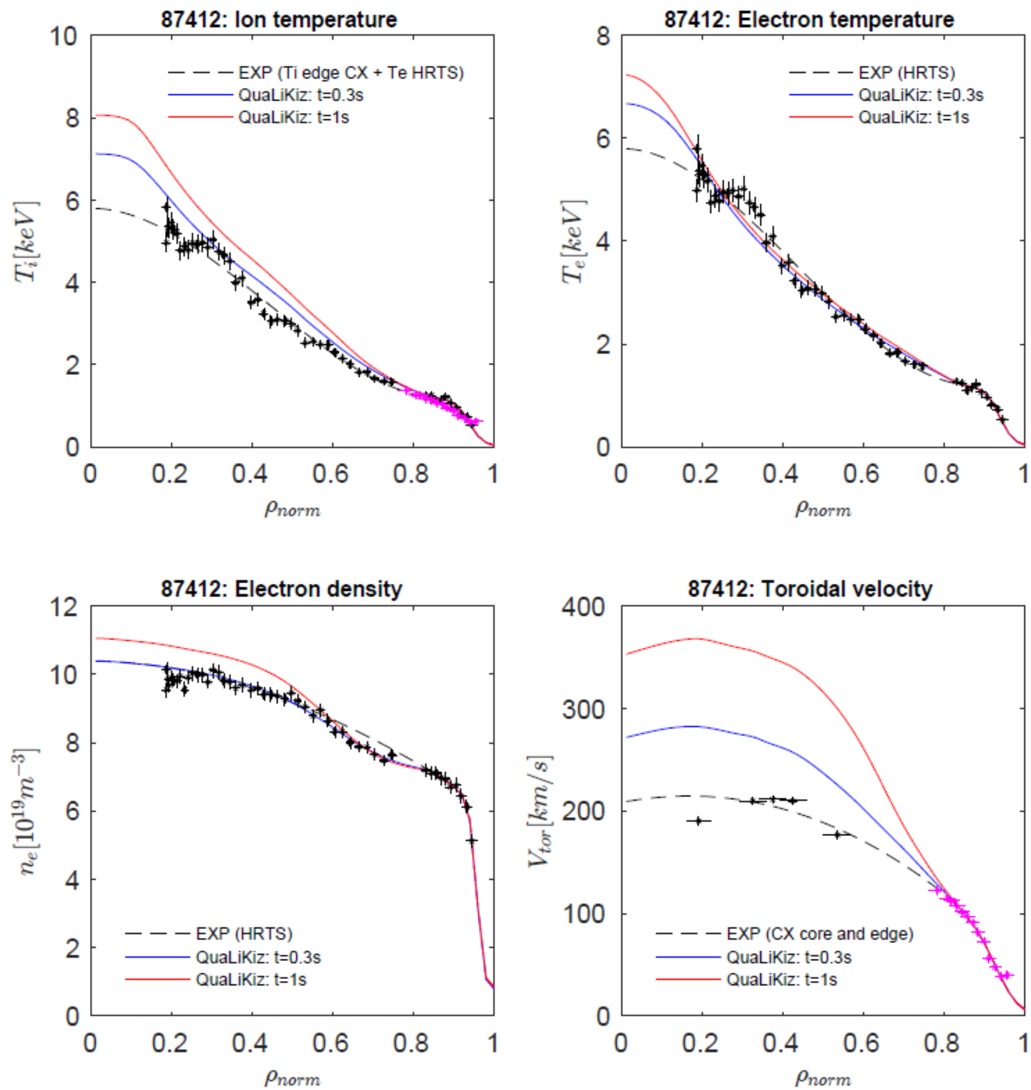
- Structures coexist → self-organisation
- Complex dynamics:
- avalanches/streamers enhance transport
- zonal flows reduce transport via vortex shearing





Reduced models

- Transport models based on quasi-linear theory + mixing length work beyond expectation.
- Hard to get a model that works in all conditions.



Citrin 17

Conclusions

- Linear stability is well documented – does not mean it is a simple exercise though…
- Non linear dynamics much more complex – No simple recipe !
- For turbulent states and wave/particle interaction via Landau resonances: quasi-linear theory often works well
- Predicting a level of fluctuations remains tricky.

Some useful textbooks

- D. B. Melrose “Instabilities in Space and Laboratory Plasmas” Cambridge UP 1986

- W. Baumjohann and R.A. Treumann “Basic Space Plasma Physics” Imperial UP 2012 vol I and II

- P. H. Diamond , S.-I. Itoh, K. Itoh, “Physical Kinetics of Turbulent Plasmas” Cambridge UP 2010

- D. Biskamp “Nonlinear Magnetohydrodynamics », 1997 Cambridge UP

- A.J. Lichtenberg and M.A. Liberman, “Regular and stochastic motion” Springer 1983

- Y. Elskens and D. Escande “Microscopic dynamics of plasmas and chaos”, IOP 2003