



Reconnection in Magnetically Confined Fusion Plasmas

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Theory of reconnection in magnetically confined fusion plasmas

- recap of MHD equations and the role of resistivity
- formation of magnetic islands in tokamaks
- linear and nonlinear growth

Experimental examples of reconnection in tokamaks

- classical and neoclassical tearing modes
- rapid reconnection events: sawteeth
- others: MHD 'dynamo' through tearing modes



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The one fluid MHD equations



$$\frac{\partial \rho}{\partial t} = -\nabla \cdot (n\vec{v})$$

equation of continuity

$$\rho \left(\frac{\partial \vec{v}}{\partial t} + (\vec{v} \cdot \nabla) \vec{v} \right) = -\nabla p + \vec{j} \times \vec{B}$$

force equation

$$\vec{E} = -\vec{v} \times \vec{B} + \frac{1}{\sigma} \vec{j}$$

Ohm's law

$$\frac{d}{dt} \left(\frac{p}{\rho^\gamma} \right) = 0$$

equation of state

plus Maxwell's equations for E und B



Consider equilibrium (i.e. $dv/dt = 0$)

$$\nabla p = \vec{j} \times \vec{B} = \frac{1}{\mu_0} (\nabla \times \vec{B}) \times \vec{B}$$

So that two contributions to force balance can be identified:

$$\nabla_{\perp} \left(p + \frac{B^2}{2\mu_0} \right) + \frac{B^2}{\mu_0 R_c} \vec{e}_{R_c} = 0$$

Magnetic pressure

Field line tension



Consider equilibrium (i.e. $dv/dt = 0$)

$$\nabla p = \vec{j} \times \vec{B} = \frac{1}{\mu_0} (\nabla \times \vec{B}) \times \vec{B}$$

Use magnetic pressure to determine 'speed of sound':

$$c_s = \sqrt{\gamma p / \rho} \approx \frac{B}{\sqrt{\mu_0 \rho}} \quad (\text{for } \beta = p_{kin}/p_{mag} \ll 1)$$

Alfven time scale:

$$\tau_A = \frac{L}{v_A} = \frac{L}{\frac{B}{\sqrt{\mu_0 \rho}}}$$

Fast ($\sim \mu\text{s}$ for 0.5 m) since mass is small in fusion experiments



Consider equilibrium Ohm's law... $\vec{E} = -\vec{v} \times \vec{B} + \frac{1}{\sigma} \vec{j}$

...and analyse how magnetic field can change:

$$\frac{\partial \vec{B}}{\partial t} = -\nabla \times \vec{E} = \nabla \times (\vec{v} \times \vec{B}) - \frac{1}{\mu_0 \sigma} \nabla \times (\nabla \times \vec{B})$$

$$\Rightarrow \frac{\partial \vec{B}}{\partial t} = \nabla \times (\vec{v} \times \vec{B}) + \frac{1}{\mu_0 \sigma} \Delta \vec{B}$$

Flux conserving
plasma motion (ideal MHD, τ_A)

Diffusive change of flux
(resistive MHD, τ_R)



Consider equilibrium Ohm's law... $\vec{E} = -\vec{v} \times \vec{B} + \frac{1}{\sigma} \vec{j}$

...and analyse how magnetic field can change:

$$\begin{aligned}\frac{\partial \vec{B}}{\partial t} &= -\nabla \times \vec{E} = \nabla \times (\vec{v} \times \vec{B}) - \frac{1}{\mu_0 \sigma} \nabla \times (\nabla \times \vec{B}) \\ \Rightarrow \frac{\partial \vec{B}}{\partial t} &= \nabla \times (\vec{v} \times \vec{B}) + \frac{1}{\mu_0 \sigma} \Delta \vec{B}\end{aligned}$$

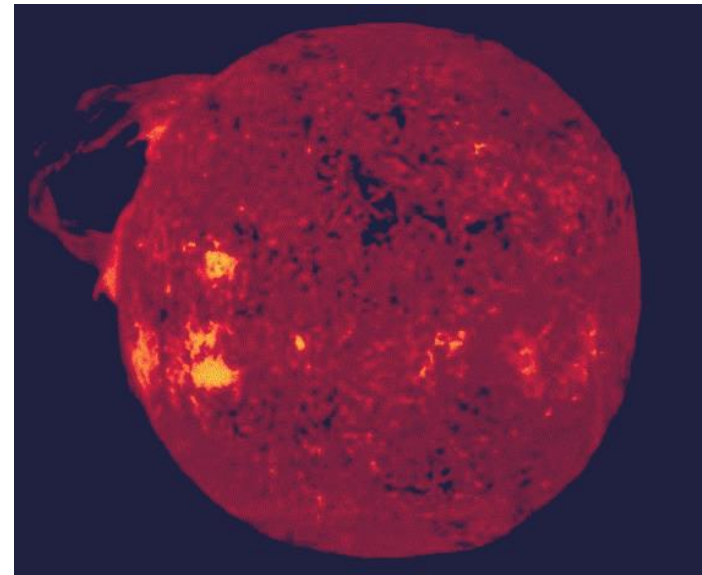
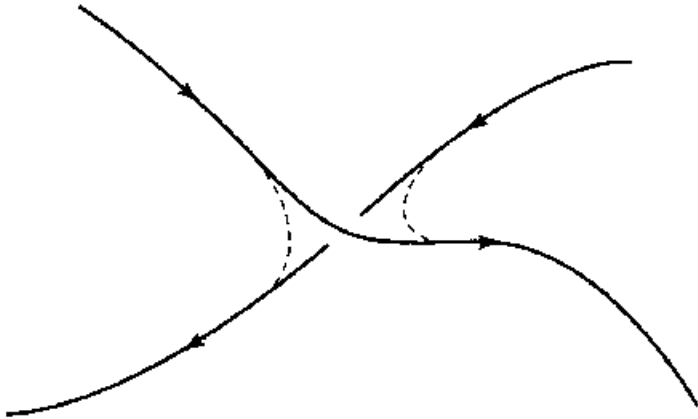
Typical time scale of resistive MHD:

$$\tau_R = \mu_0 \sigma L^2$$

Since σ is large for a hot plasma, τ_R is slow (\sim sec for 0.5 m) – irrelevant?



Due to high electrical conductivity, magnetic flux is frozen into plasma
⇒ magnetic field lines and plasma move together



A change of magnetic topology is only possible through reconnection

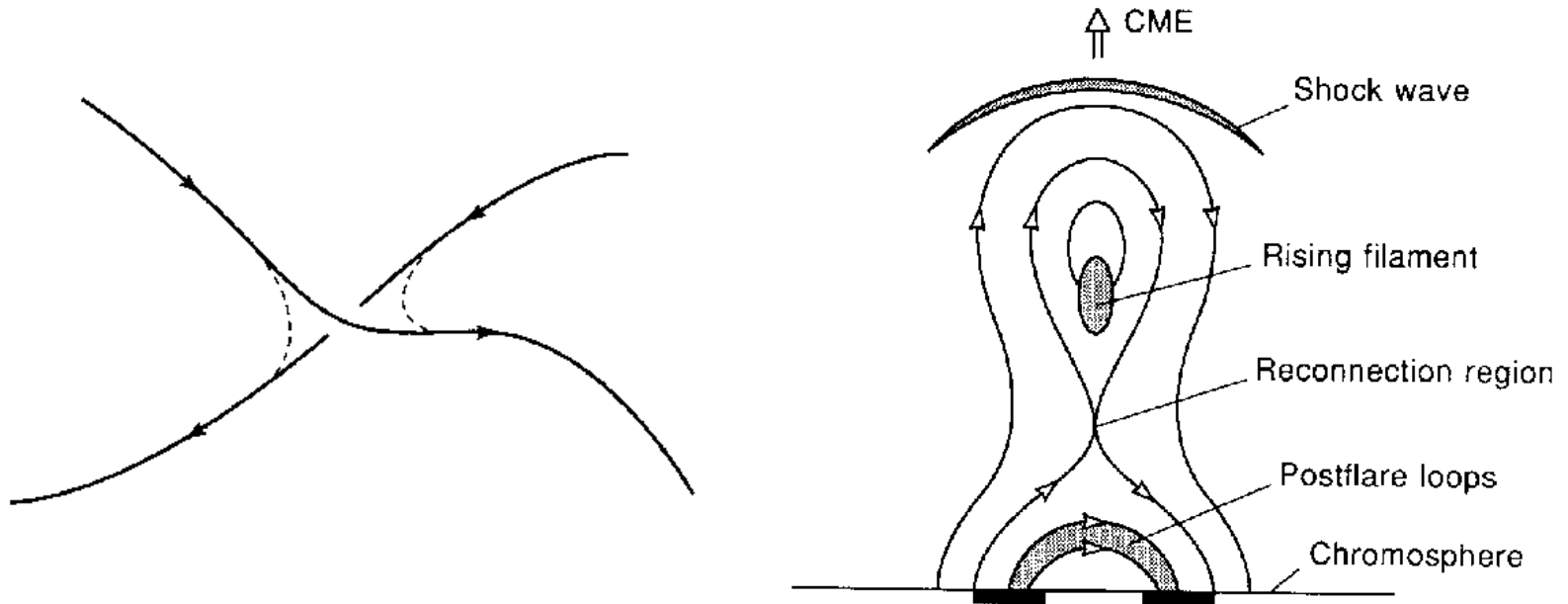
- opposing field lines reconnect and form new topological objects
- requires finite resistivity in the reconnection region



Reconnection in a hot fusion plasma



Due to high electrical conductivity, magnetic flux is frozen into plasma
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Example: Coronal Mass Ejection (CME) from the sun



Theory of reconnection in magnetically confined fusion plasmas

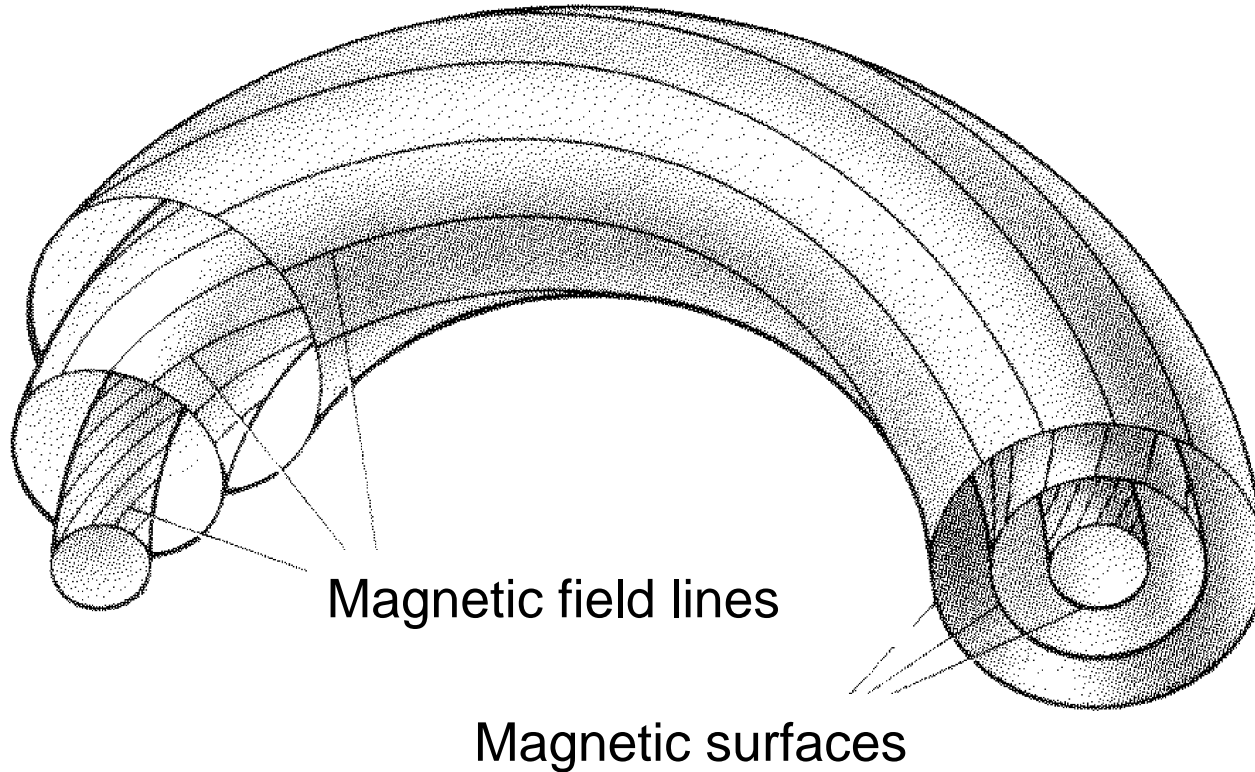
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Plasma can be confined in a magnetic field



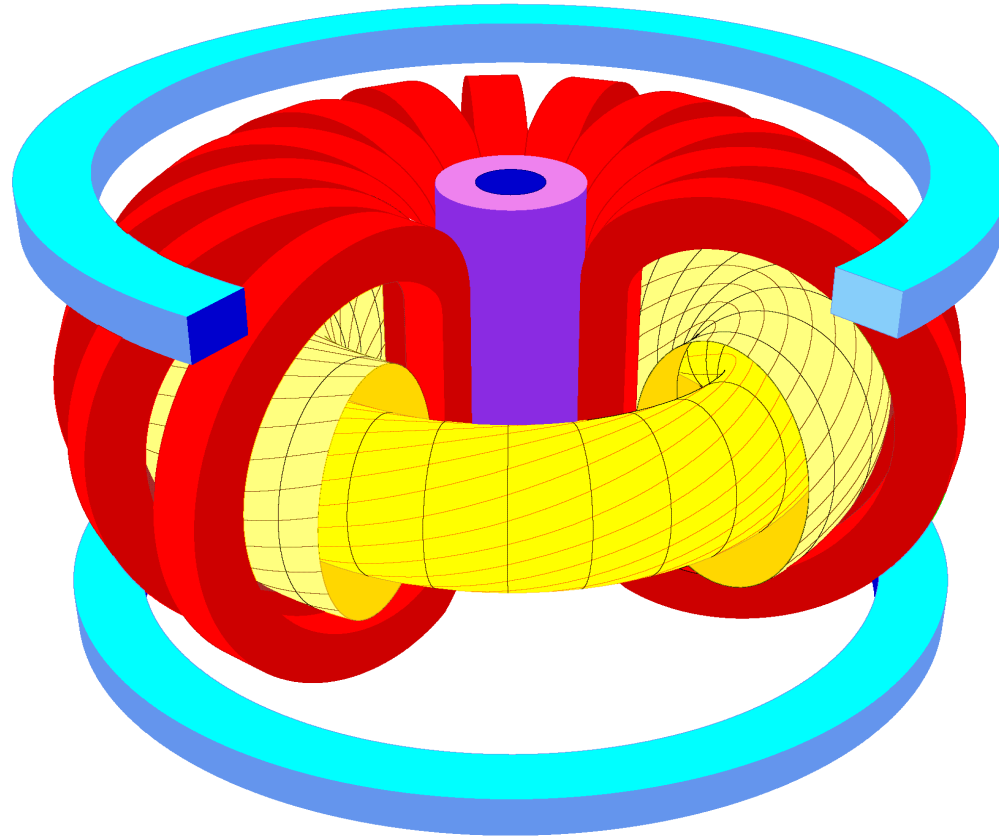
Toroidal systems avoid end losses along magnetic field

- Need to twist field lines helically to compensate particle drifts
- Safety factor q : number of toroidal turns a field line completes for one poloidal turn



Plasma can be confined in a magnetic field

'Tokamak': poloidal field component from strong toroidal plasma current

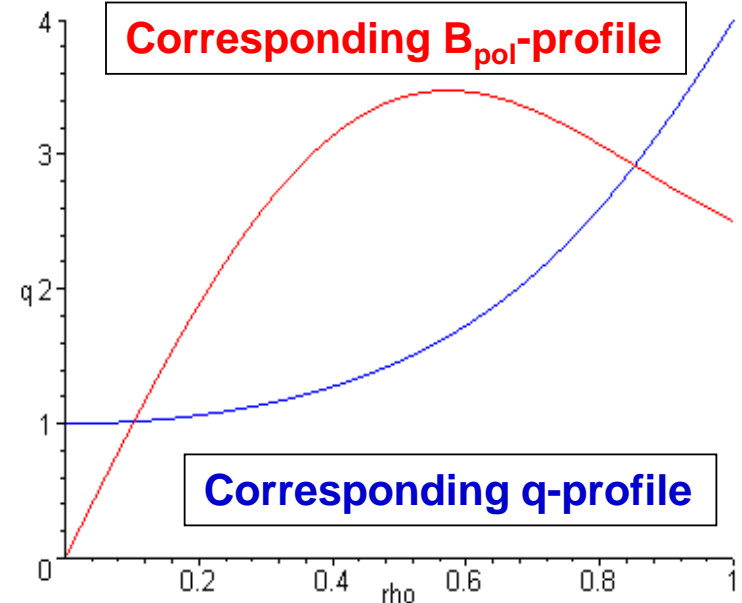
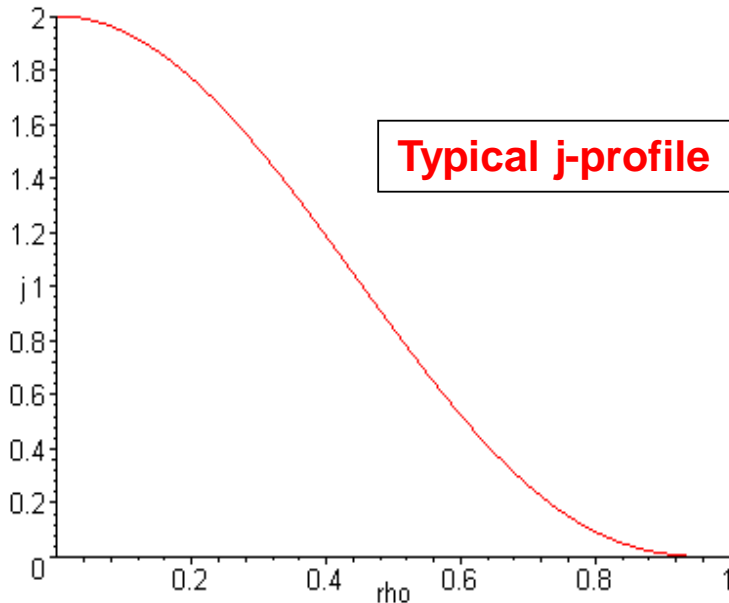


Due to radial variation of $T_e(r)$, there is also a radial variation $j(r)$

Free energy in the current density: the tokamak is prone to tearing modes



Reconnection in a tokamak



Safety factor q : number of toroidal turns a field line completes for one poloidal turn

$$q = \frac{r B_{pol}}{R B_{tor}} \quad \text{in cylindrical approximation}$$

For centrally peaked $j(r)$, B_{pol} increases weaker than linearly and hence, q increases monotonically from the centre to the edge

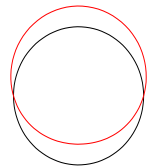


Reconnection on 'rational' magnetic surfaces

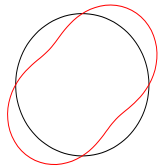


Torus has double periodicity

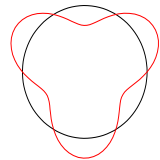
- instabilities with poloidal and toroidal 'quantum numbers'



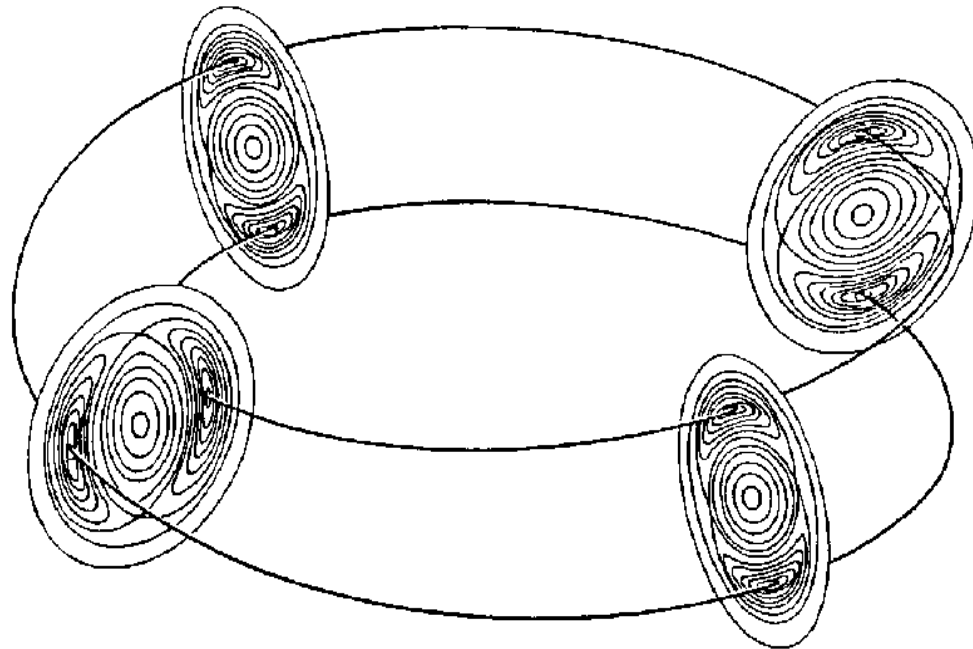
$m = 1$



$m = 2$



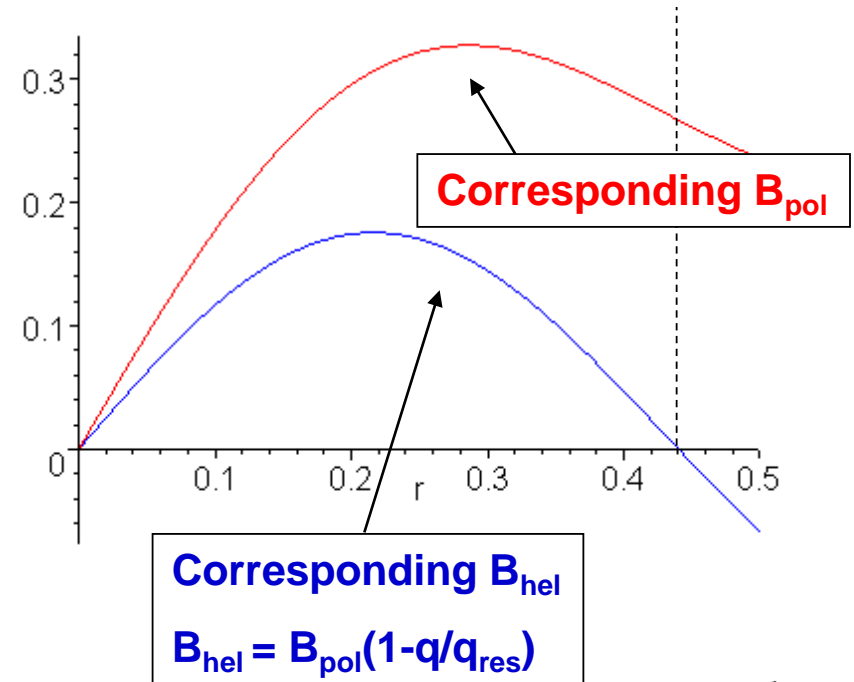
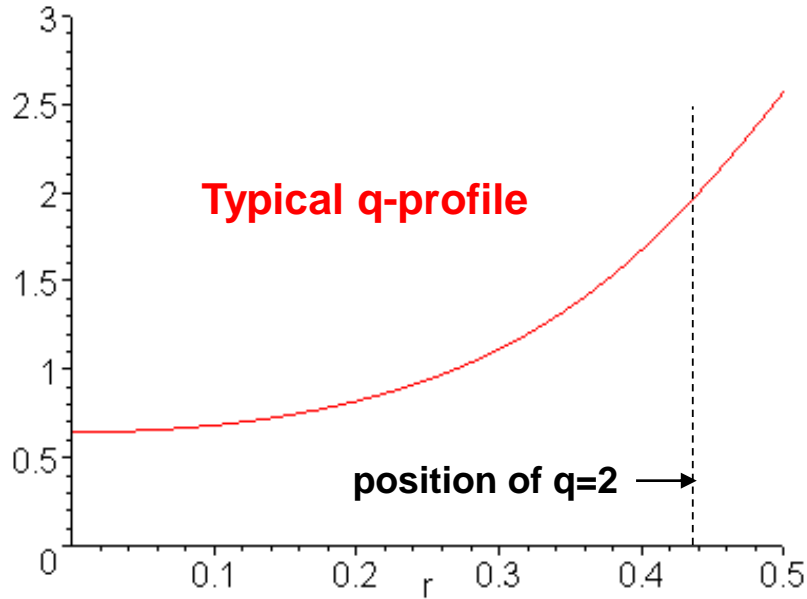
$m = 3$



„Resonant surfaces“ prone to instabilities with $q = m/n$

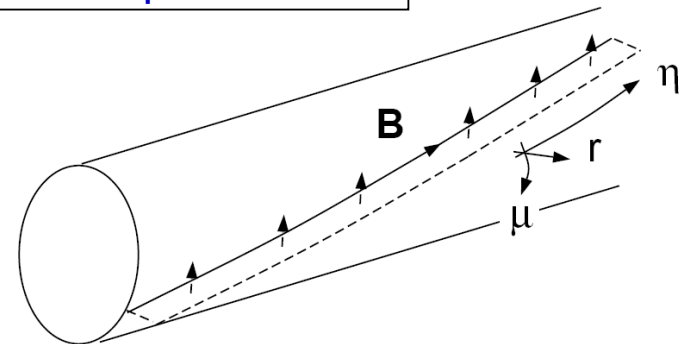


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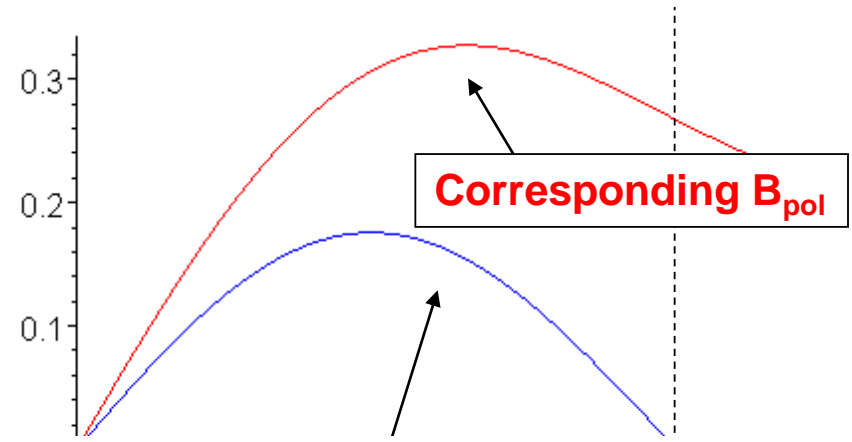
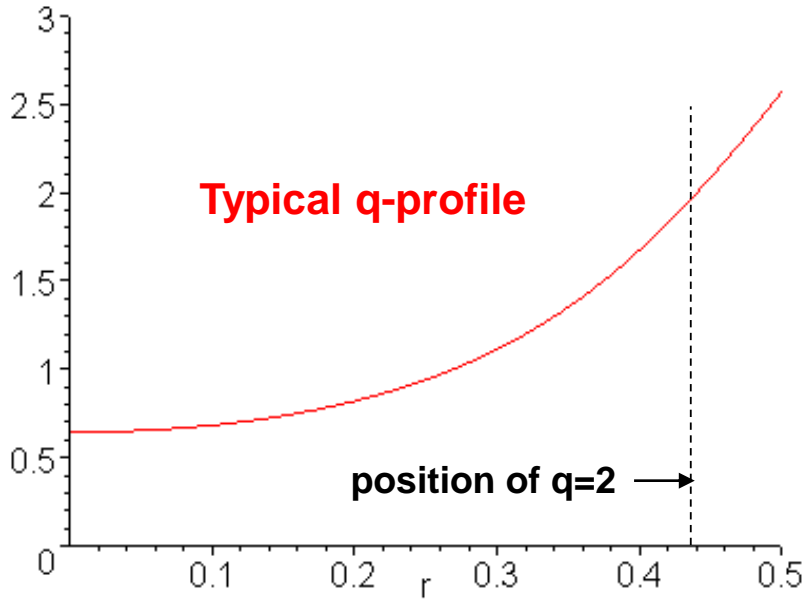
Helical field (i.e. ,poloidal' field relative to resonant surface) changes sign:

- reconnection of helical flux can form new topological objects - islands



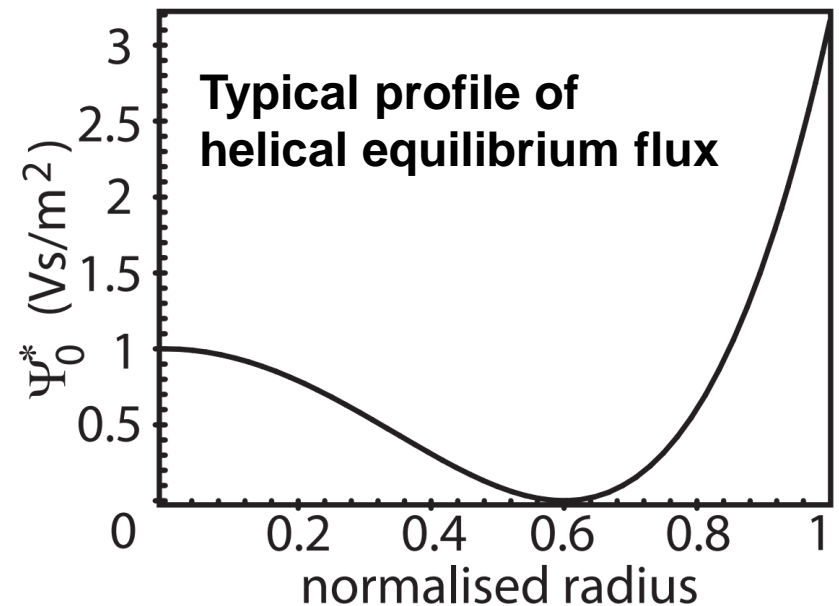


Reconnection on 'rational' magnetic surfaces



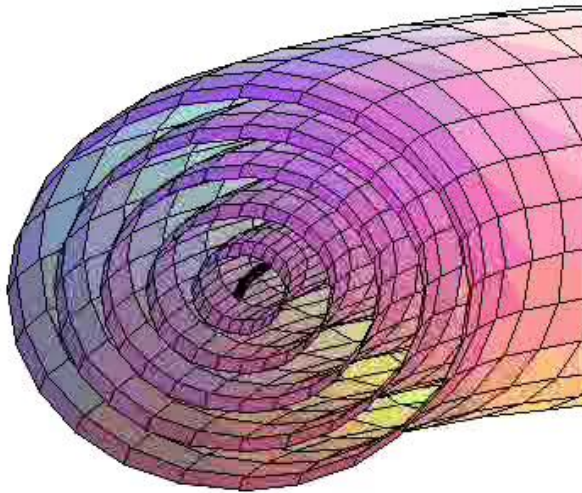
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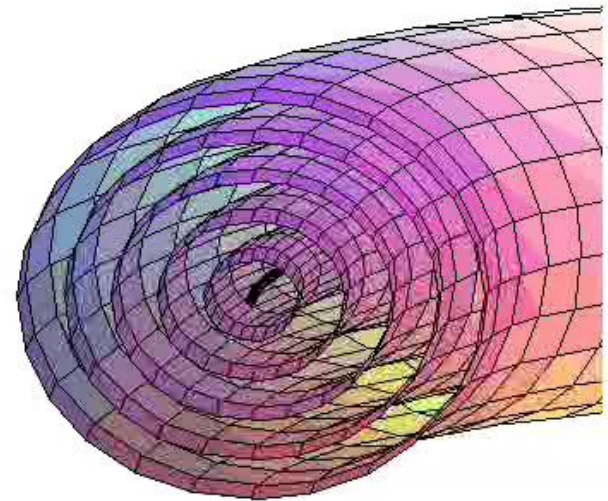




Finite resistivity allows for changes in topology



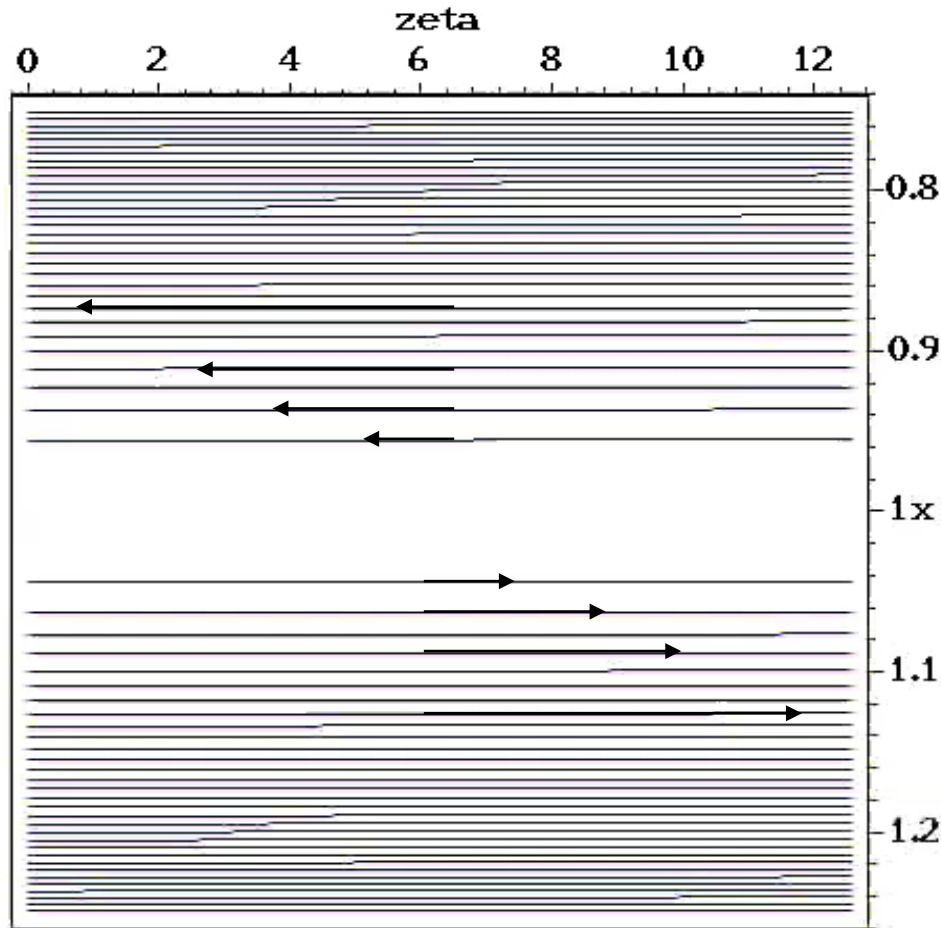
$\sigma \rightarrow \infty$: ideal Kink



$\sigma \neq \infty$: Tearing Mode



Resistivity gives access to a new class of instabilities

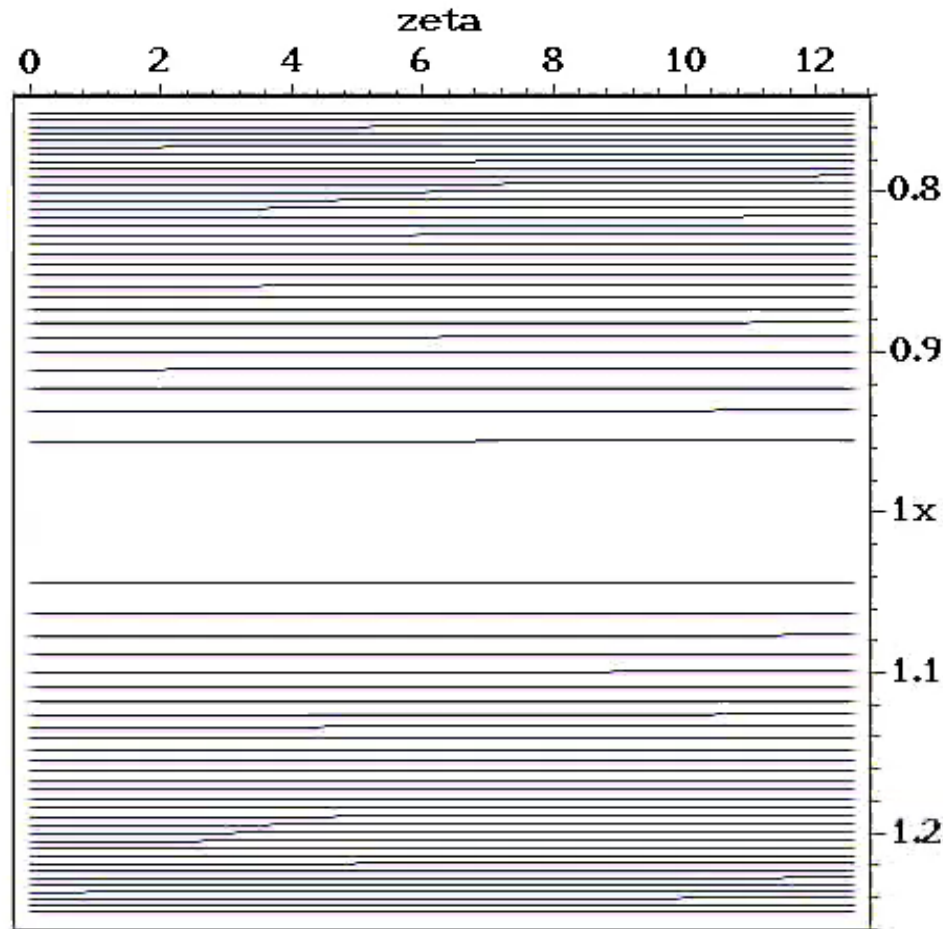


‚reconnection‘ of field lines at the ‚X-point‘

loss of thermal insulation across the island of width W at the ‚O-point‘



Resistivity gives access to a new class of instabilities



‚reconnection‘ of field lines at the ‚X-point‘

loss of thermal insulation across the island of width W at the ‚O-point‘

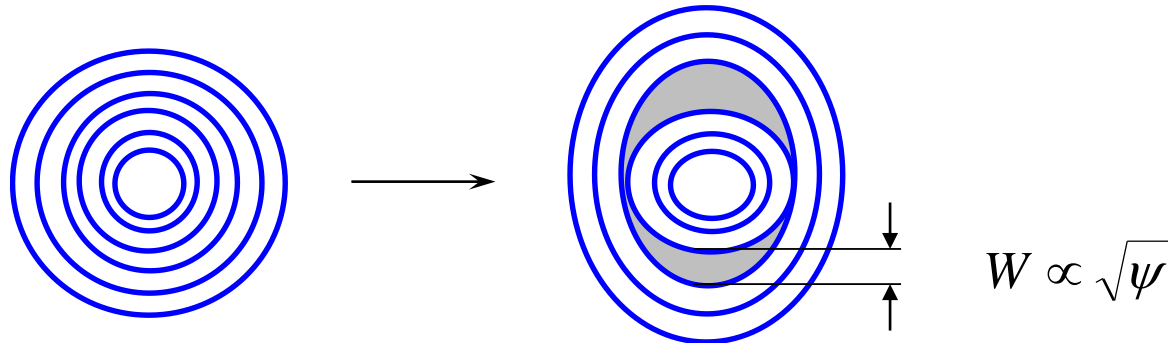


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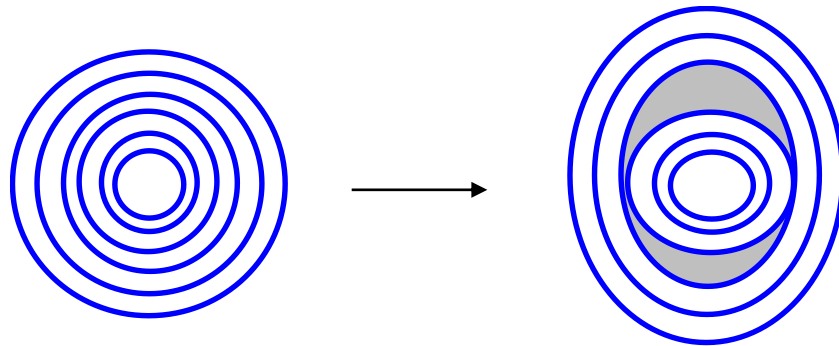
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Deformation of flux surfaces opens up island of width W

- derive tearing mode equation for helical magnetic flux ψ
- may be considered as series of ideal MHD equilibria ($\nabla p = j \times B$)
- ideal MHD may be used everywhere except rational surface

Resistive MHD only important at the rational surface – timescale \sim ms!



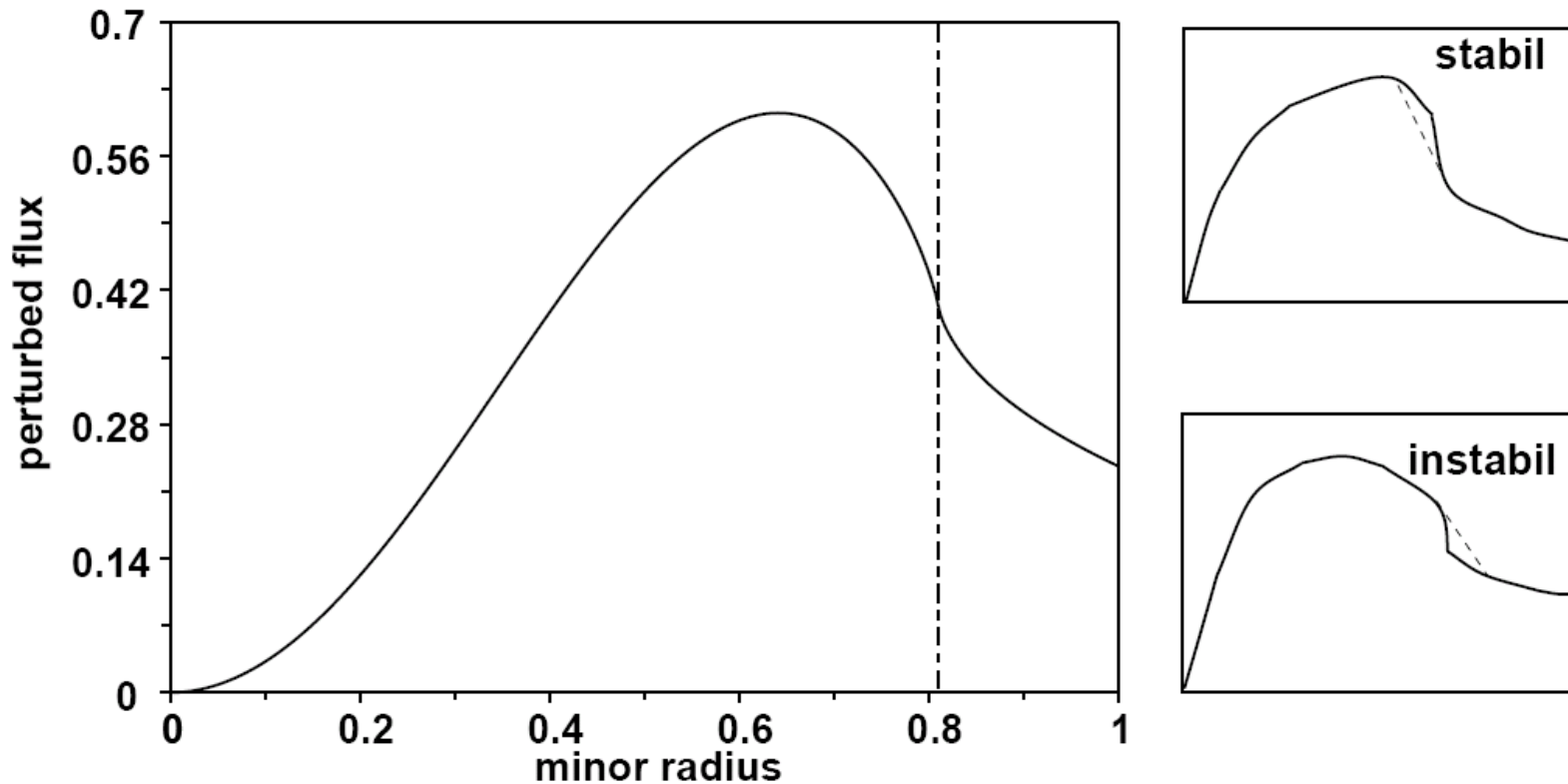
**,outer region equation'
from Nuno's talk**

$$\Delta \psi + \frac{\mu_0 \frac{dj(r)}{dr}}{B_\theta (1 - \frac{n}{m} q(r))} \psi = 0$$

Deformation of flux surfaces opens up island of width W

- Tearing Mode equation ($\nabla p = j \times B$) singular at resonant surface:
implies kink in magnetic flux ψ , jump in B
 \Rightarrow current sheet on the resonant surface

$\psi(r)$: helical magnetic flux
 $j(r)$: current profile
 $q(r)$: field line helicity profile
 m, n : mode quantum numbers

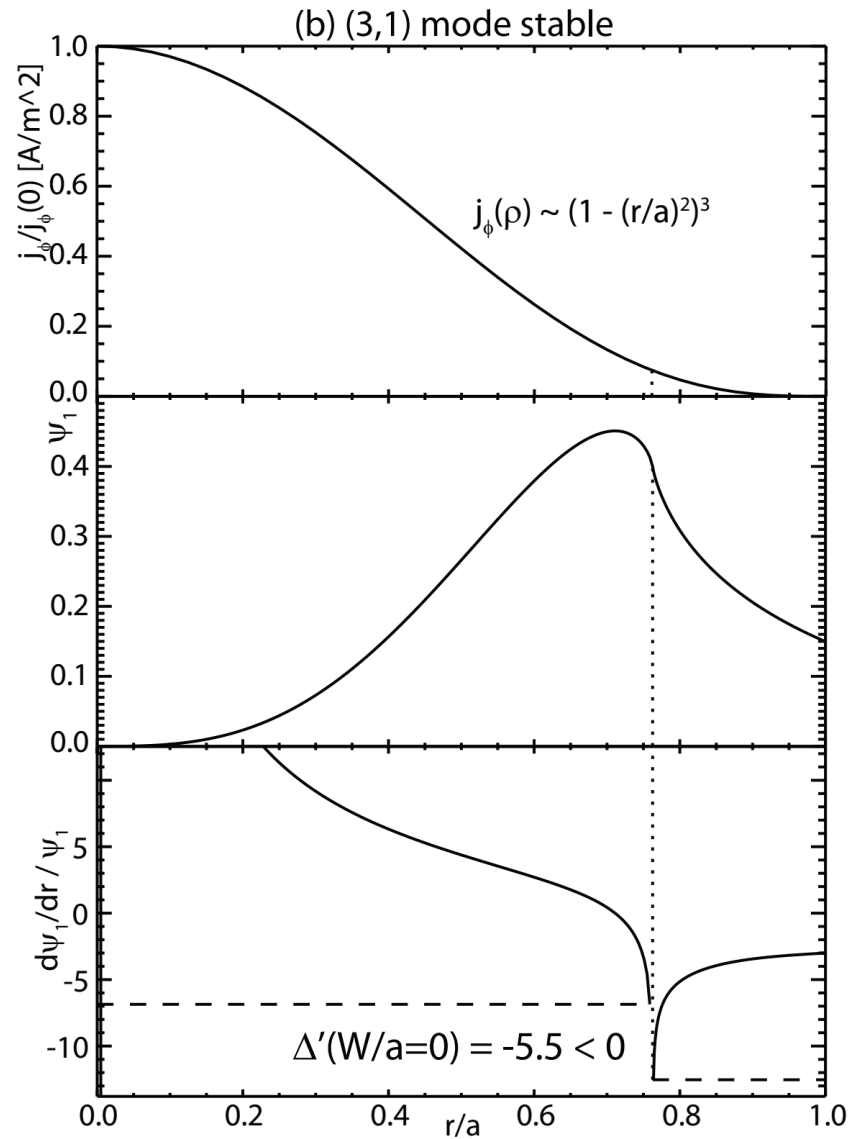
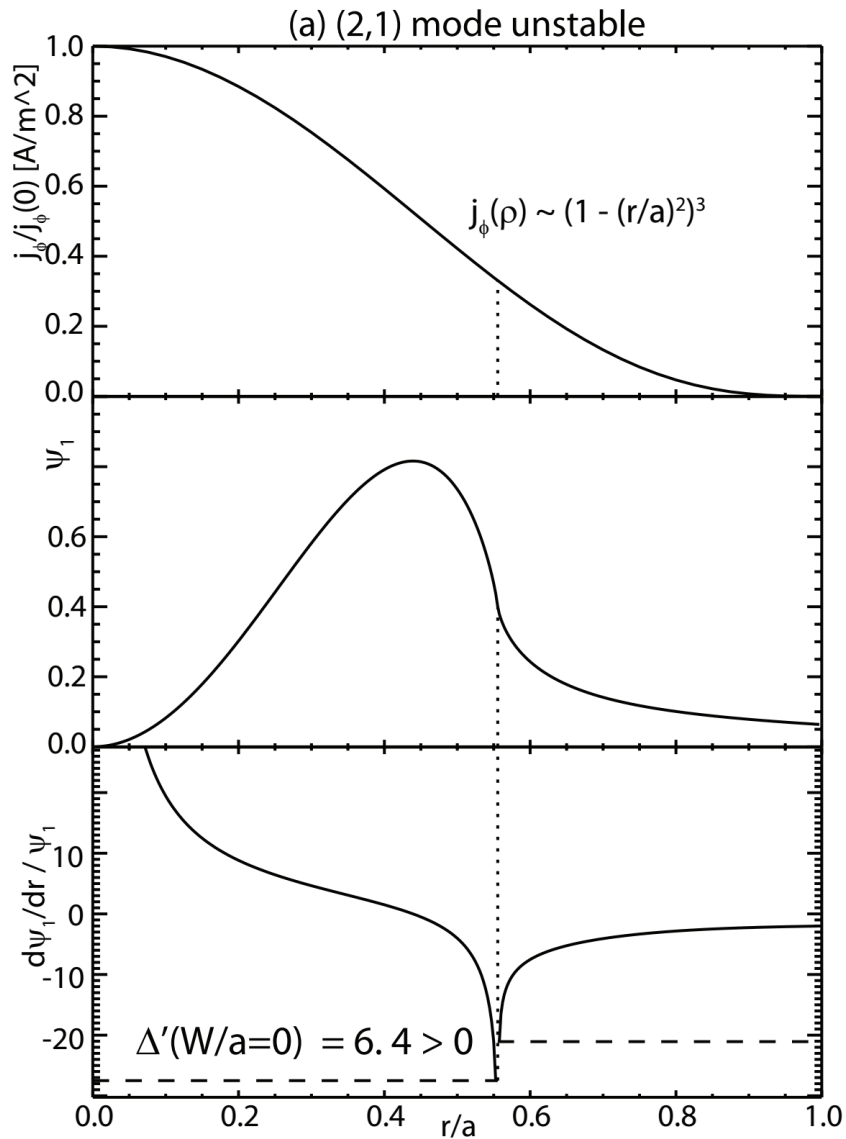


Solution of tearing mode equation can be made continuous, but has a kink

- implied surface current will grow or decay depending on equilibrium $j(r)$
- the parameter defining stability is $\Delta' = ((d\psi/dr)_{right} - (d\psi/dr)_{left}) / \psi$
- if $\Delta' > 0$, tearing mode is linearly unstable – this is related to $\nabla j_{resonant\ surface}$

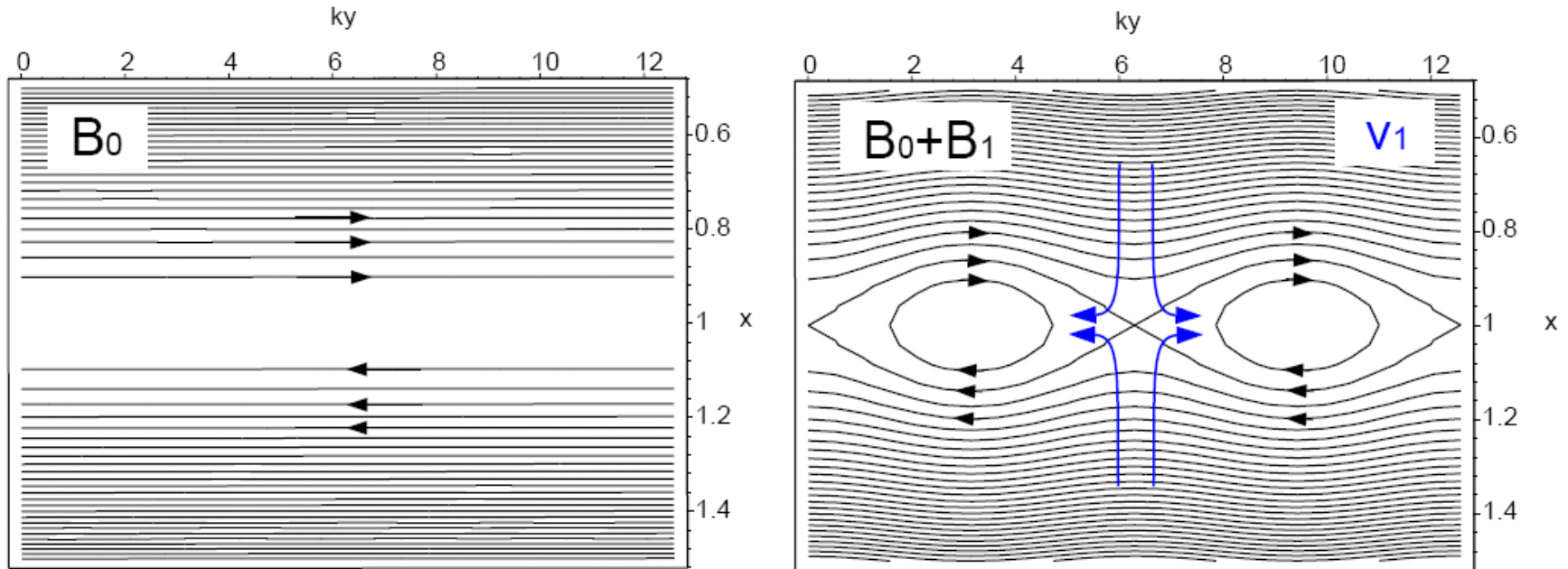


MHD description of tearing mode formation





Tearing Modes – linear growth



In the linear stage, the island growth is limited by resistive diffusion as well as the inertia of the plasma flowing into the island across the X-point

The corresponding growth rate is $\gamma = 0.55(\Delta' a)^{4/5} \tau_A^{-2/5} \tau_{res}^{-3/5}$

**remember
Nuno's talk**

However, once the island width is larger than the reconnection sheath, the flow becomes unimportant and the growth enters into a nonlinear regime



Tearing Modes – nonlinear growth



Consider various helical surface currents on resonant surface...

$$B_{\theta}(r_s^+) - B_{\theta}(r_s^-) \propto \delta I = I_{Ohm} + I_{bs} + I_{extern}$$

$$I_{Ohm} \propto j_{Ohm} W \propto \sigma W \frac{d\psi}{dt} \propto \sigma W^2 \frac{dW}{dt}$$

inductive

$$I_{bs} \propto j_{bs} W \propto -\frac{\nabla p}{B_{\theta}} W$$

pressure driven

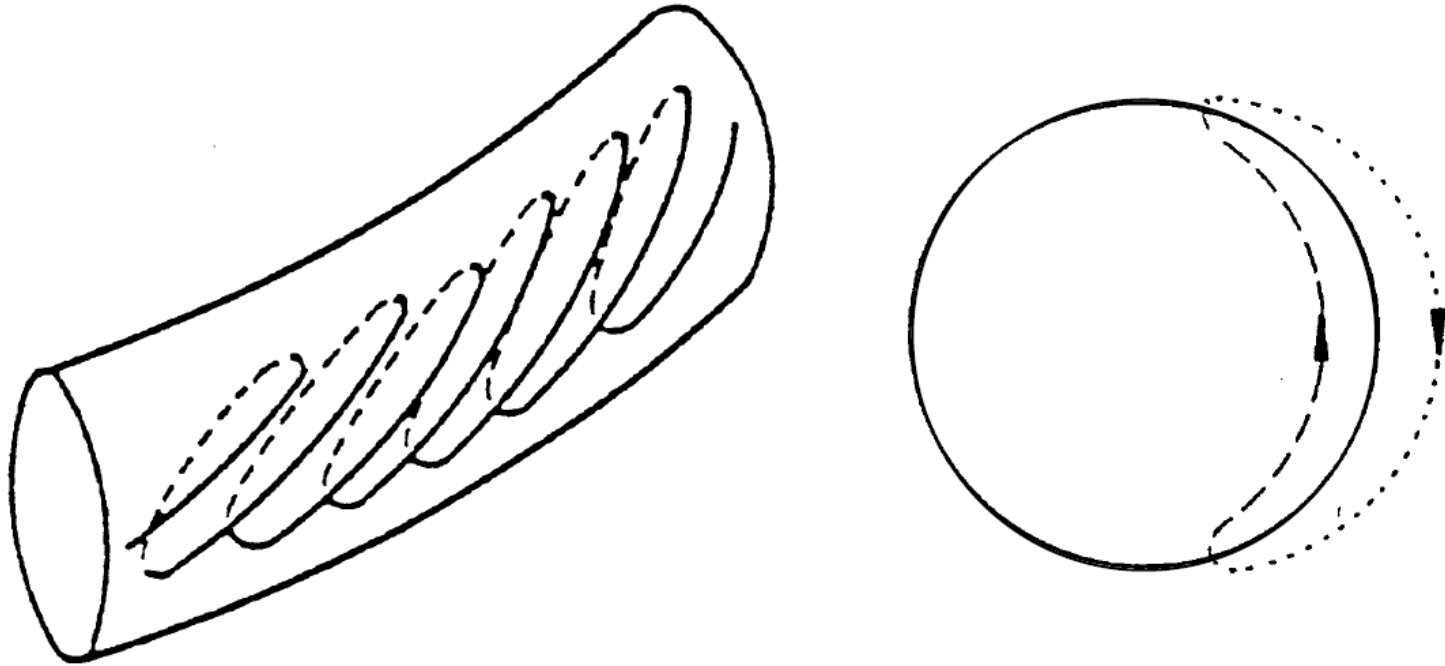
$$I_{extern}$$

externally driven

...leads to the so-called Rutherford equation (equivalent to Ohm's law)

$$\tau_{res} \frac{dW}{dt} = a_1 \Delta' + a_2 \frac{\nabla p}{W} - a_3 \frac{I_{extern}}{W^2}$$

where $\Delta' = (B_{\theta}(r_s^+) - B_{\theta}(r_s^-)) / \psi$



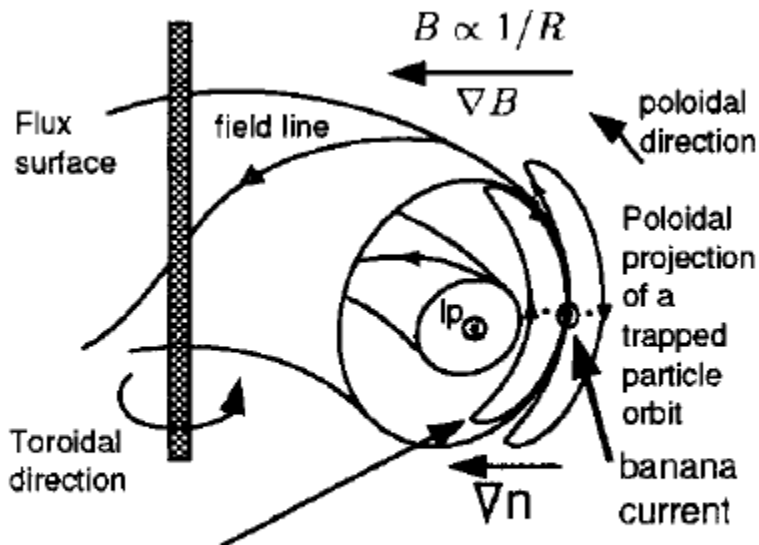
- Due to the $1/R$ decay of a B-field in a torus, there is a magnetic mirror
- particles with low v_{\parallel} are trapped in this mirror, bounce back and forth
 - poloidal projection of orbit resembles banana – ‚banana orbit‘



Bootstrap current



A.G. Peeters, Plasma Phys. Controlled Fusion 2000



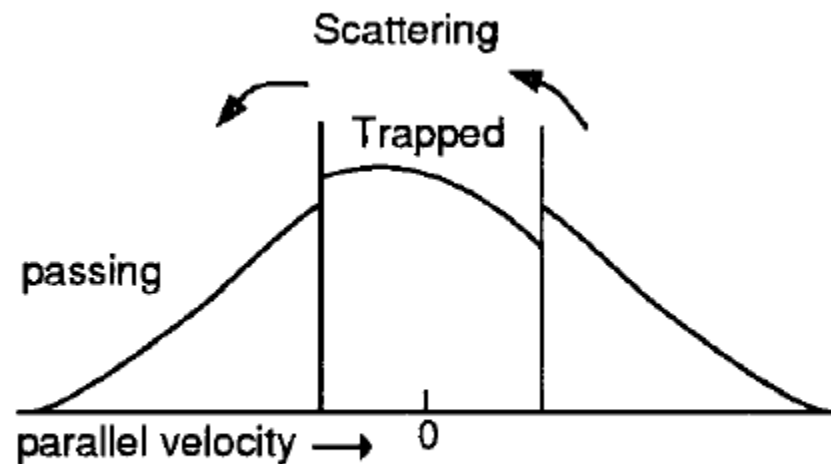
This orbit lies further inside more particles follow this orbit than the orbit further out.

For finite pressure gradient, there is a net current of trapped particles along field lines

banana current



bootstrap current



Distribution function as a function of the parallel velocity for constant perpendicular velocity.

Distortion of distribution function due to trapped particles leads to a net current of passing electrons



Interpretation of the different terms

$$\tau_{res} \frac{dW}{dt} = a_1 \Delta' + a_2 \frac{\nabla p}{W} - a_3 \frac{I_{extern}}{W^2}$$

- for small ∇p , current gradient (Δ') dominates
⇒ 'classical Tearing Mode', current driven
- for larger ∇p , pressure gradient dominates:
⇒ 'neoclassical Tearing Mode', pressure driven
- adding an externally driven helical current can stabilise



‘Modified Rutherford equation’ for the temporal evolution of island width W :

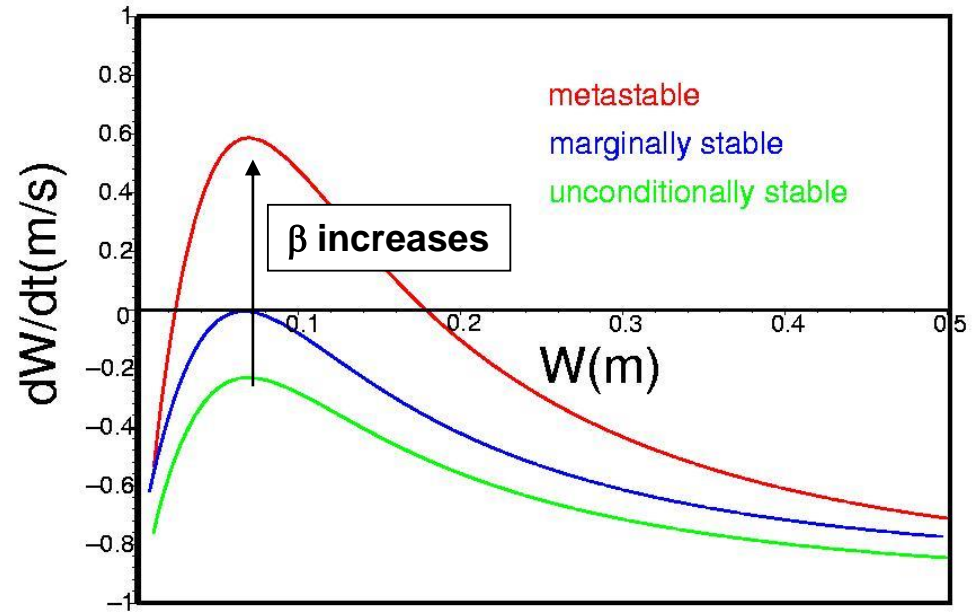
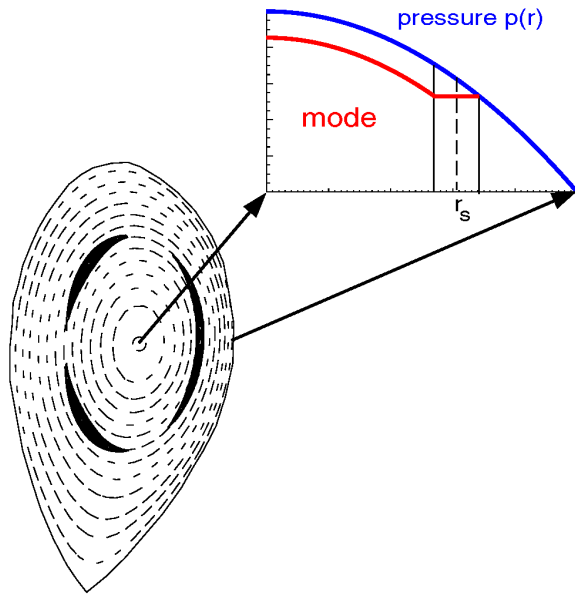
$$\frac{\tau}{r_s} \frac{dW}{dt} = r_s \Delta' + 6.34 r_s \frac{\mu_0 L_q}{B_p} f_{GGJ} j_{bs} \frac{W}{W^2 + W_0^2} - 32 \frac{\mu_0 r_s L_q d}{B_p} j_{extern} \frac{1}{W^2}$$

- here, j_{extern} is an externally generated *helical current* in the island
- effect of external current on stability via Δ' has been neglected
- intrinsic stabilisation at small island width ($W < W_0$)

Important: there is a number of additional terms that describe the stability at small island width. These are quite important for stability, But there is no agreed picture what the most important physics is...



Tearing Modes – nonlinear growth



- NTMs = magnetic islands driven by lost bootstrap current inside island
- island shortcuts radial transport: flat pressure \Rightarrow loss of bootstrap current
 - at $\beta > \beta_{\text{marg}}$, finite seed island is sustained by plasma
 - for negative Δ' (stable to classical tearing modes), NTM is metastable



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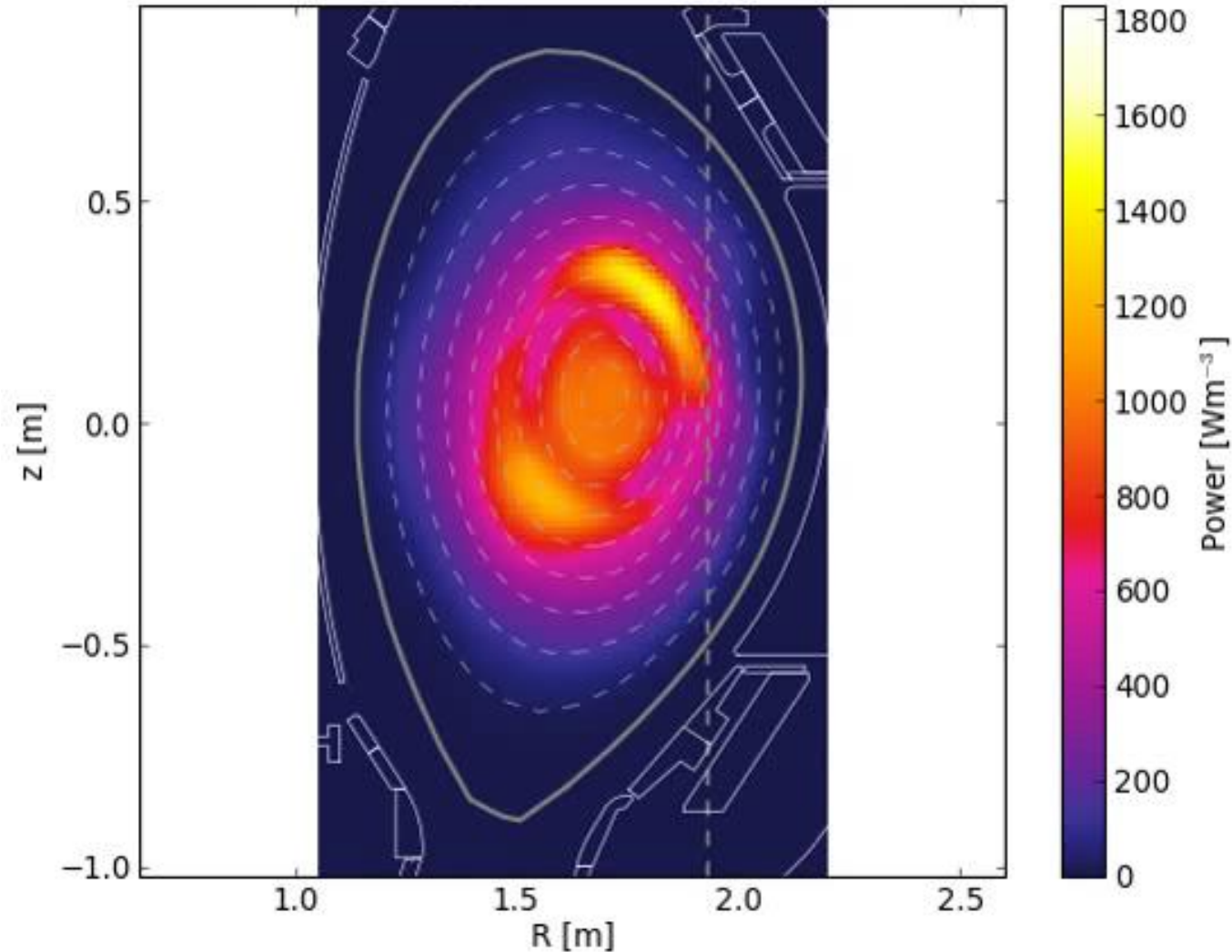
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Magnetic islands are real ;-)



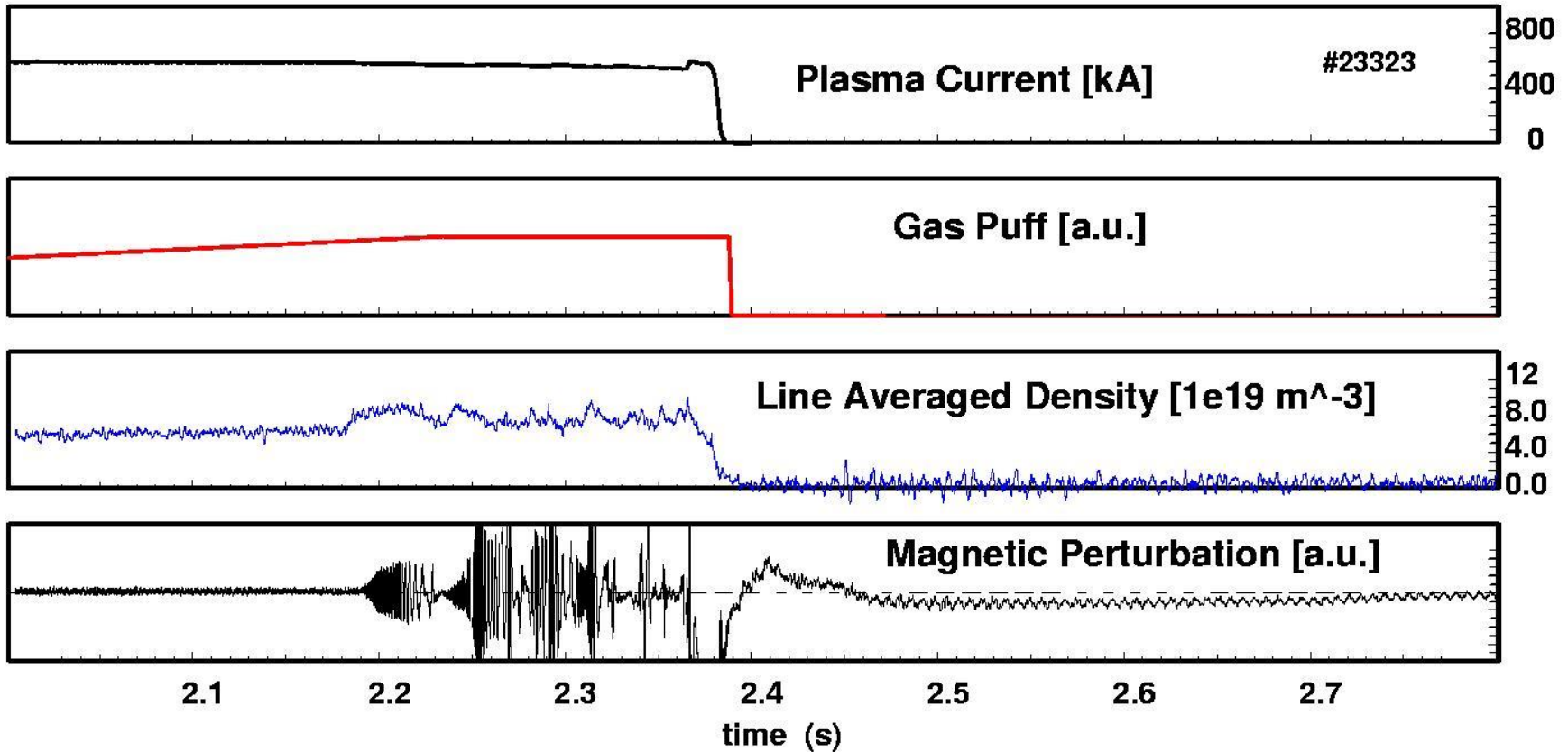
Shot: 30530, time 4.4899, $\chi^2 = 2.86$



Usually rotate in the lab frame, but may be ,locked' to error fields



Classical Tearing Modes: the disruptive instability

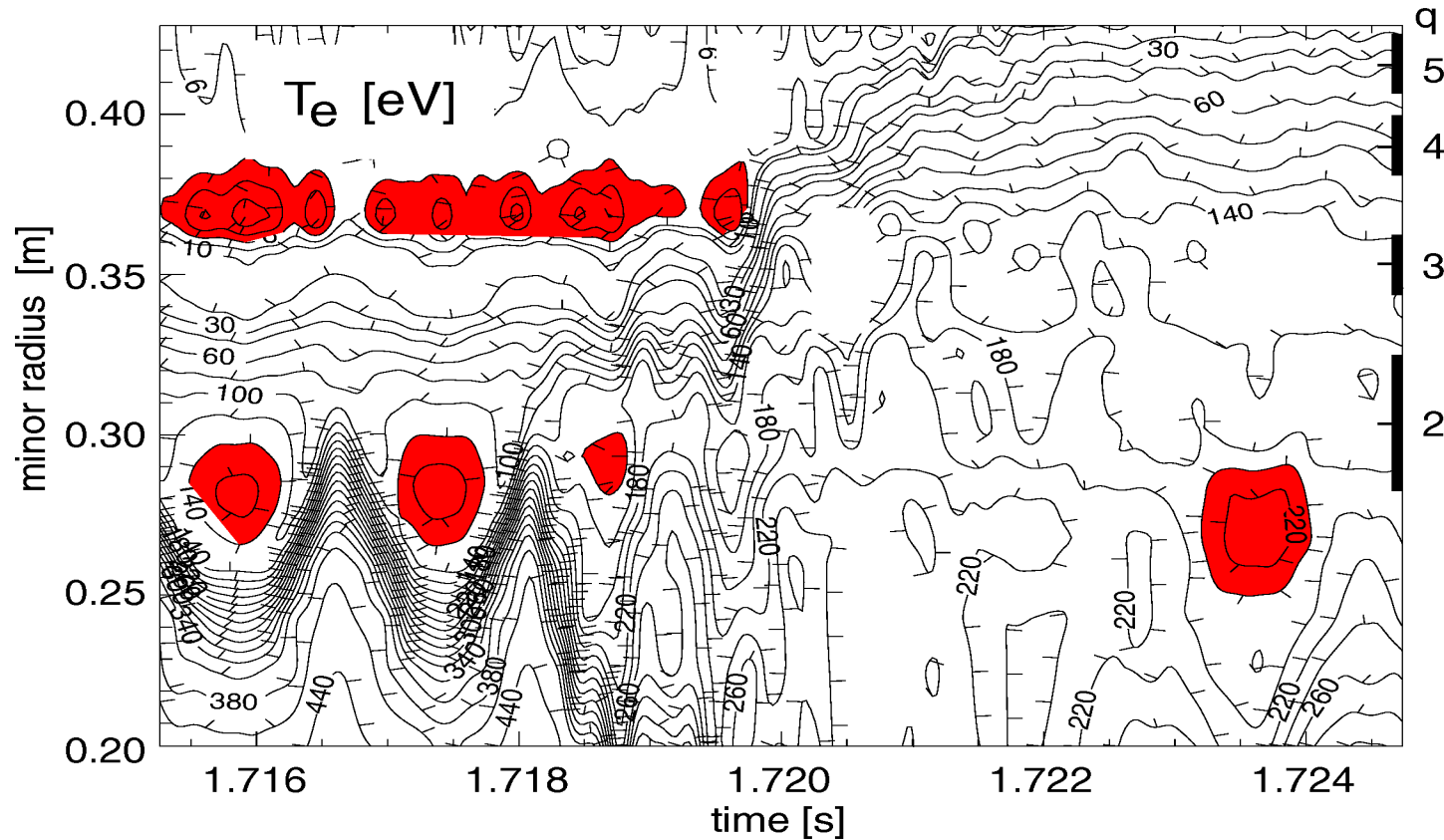


Here: try to increase the density by gas puff

- edge cooling increases $\nabla j(r)$, triggers (3,1), then (2,1) tearing mode
- related to the (edge) radiative instability discussed by X. Garbet yesterday



W. Suttrop et al., *Nuclear Fusion* 1997

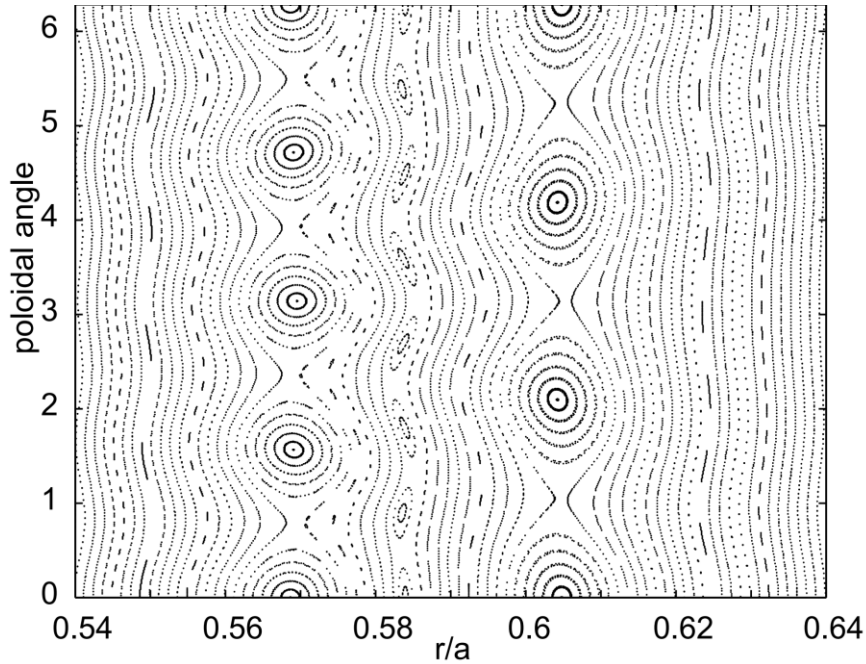


coupling between island chains (possibly stochastic regions)

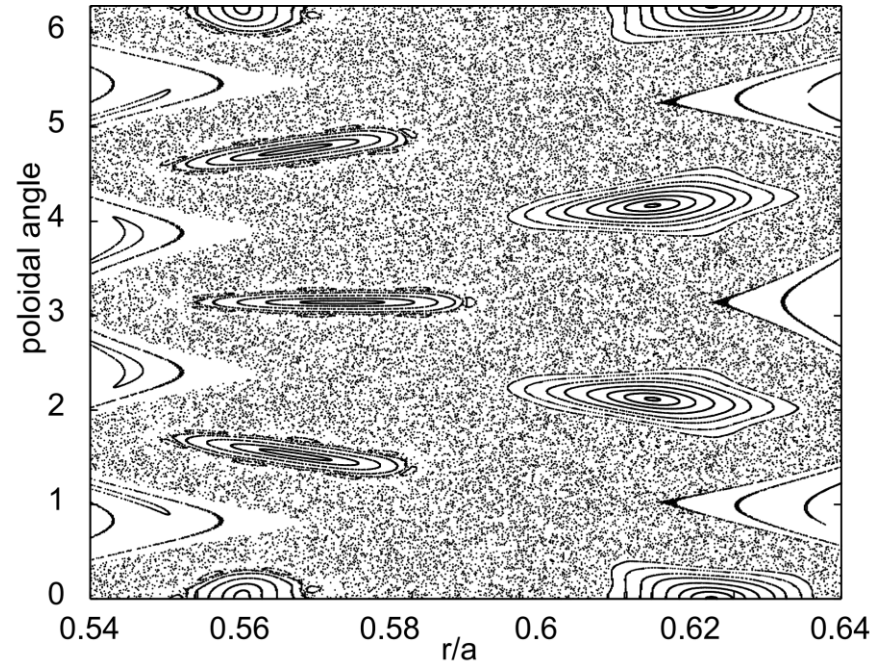
⇒ sudden loss of heat insulation ('disruptive instability')



Q. Yu et al., *Phys. Plasmas* 2006



**,Intermediate' amplitude:
Separated island chains**



**,Large' amplitude:
Stochastisation**

coupling between island chains (possibly stochastic regions)

⇒ sudden loss of heat insulation ('disruptive instability')



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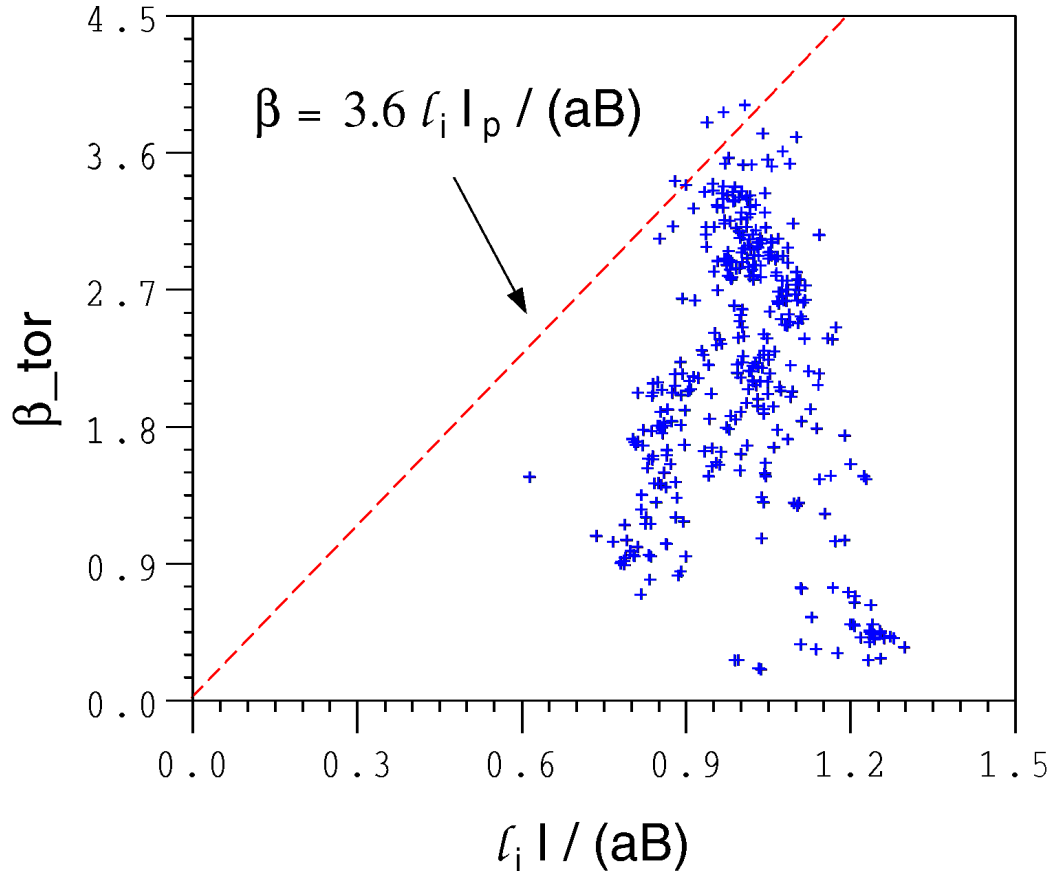
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Tearing modes and the β -limit



Reminder: ideal MHD instabilities limit normalised pressure $\beta_{max} \sim I_p / (aB)$



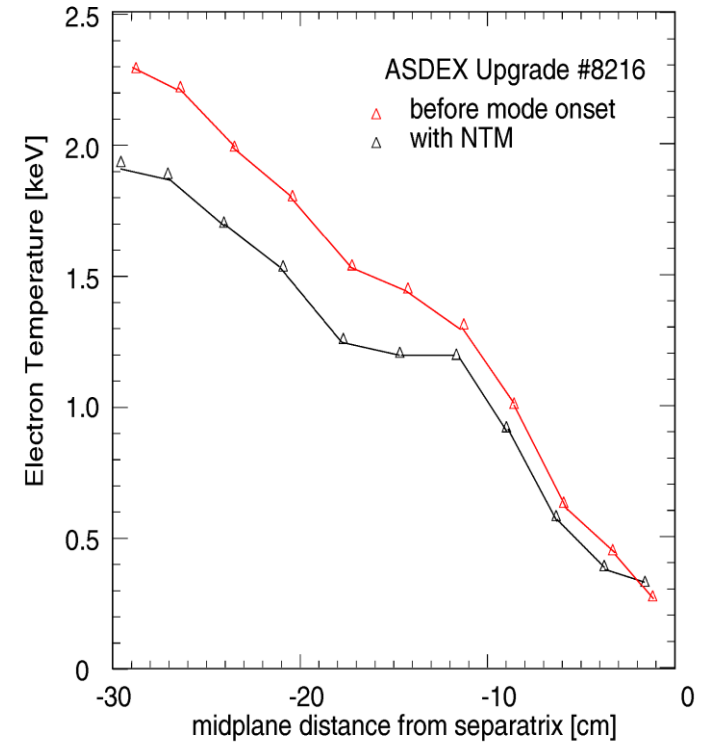
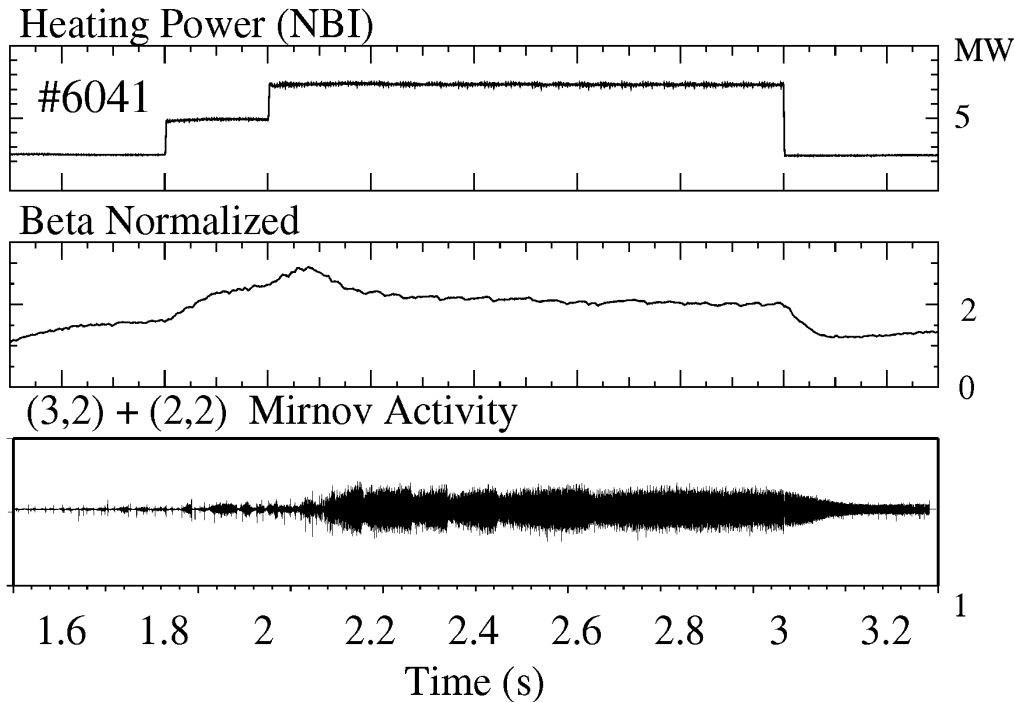
'Troyon-Limit,, leading to the definition of $\beta_N = \beta / (I / (aB))$



...but resistive limits may be lower...



H. Zohm et al., *Plasma Phys. Contr. Fusion* 37 (1995)

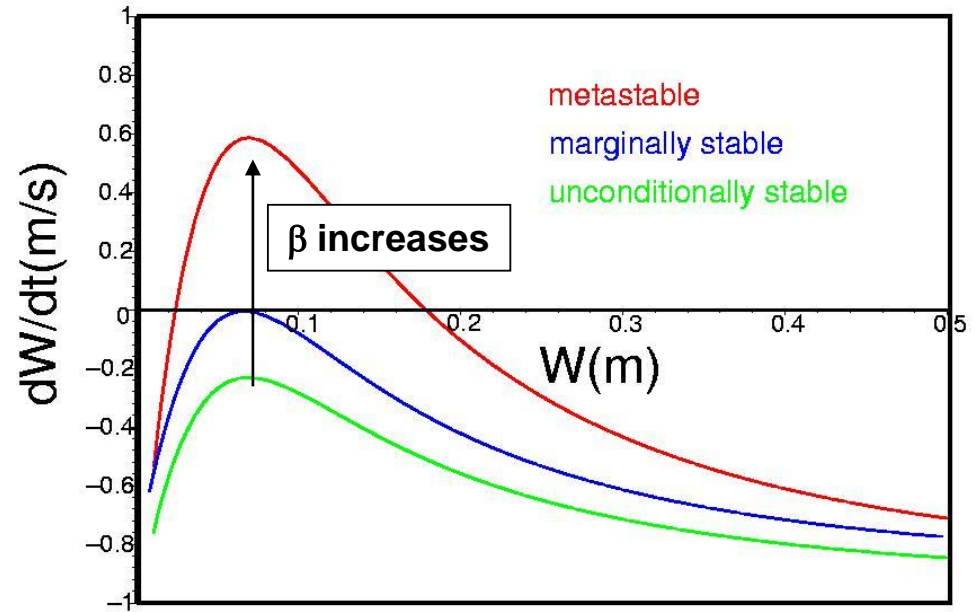
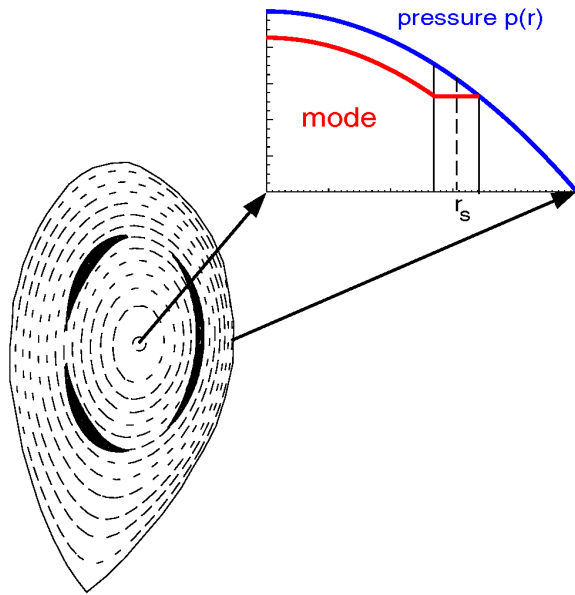


Neoclassical tearing modes can occur well below ideal limit

- ,practical β -limit' in ITER standard scenario (ELMy H-mode)
- note: can also lead to disruptive termination (especially at low q)



Tearing Modes – nonlinear growth



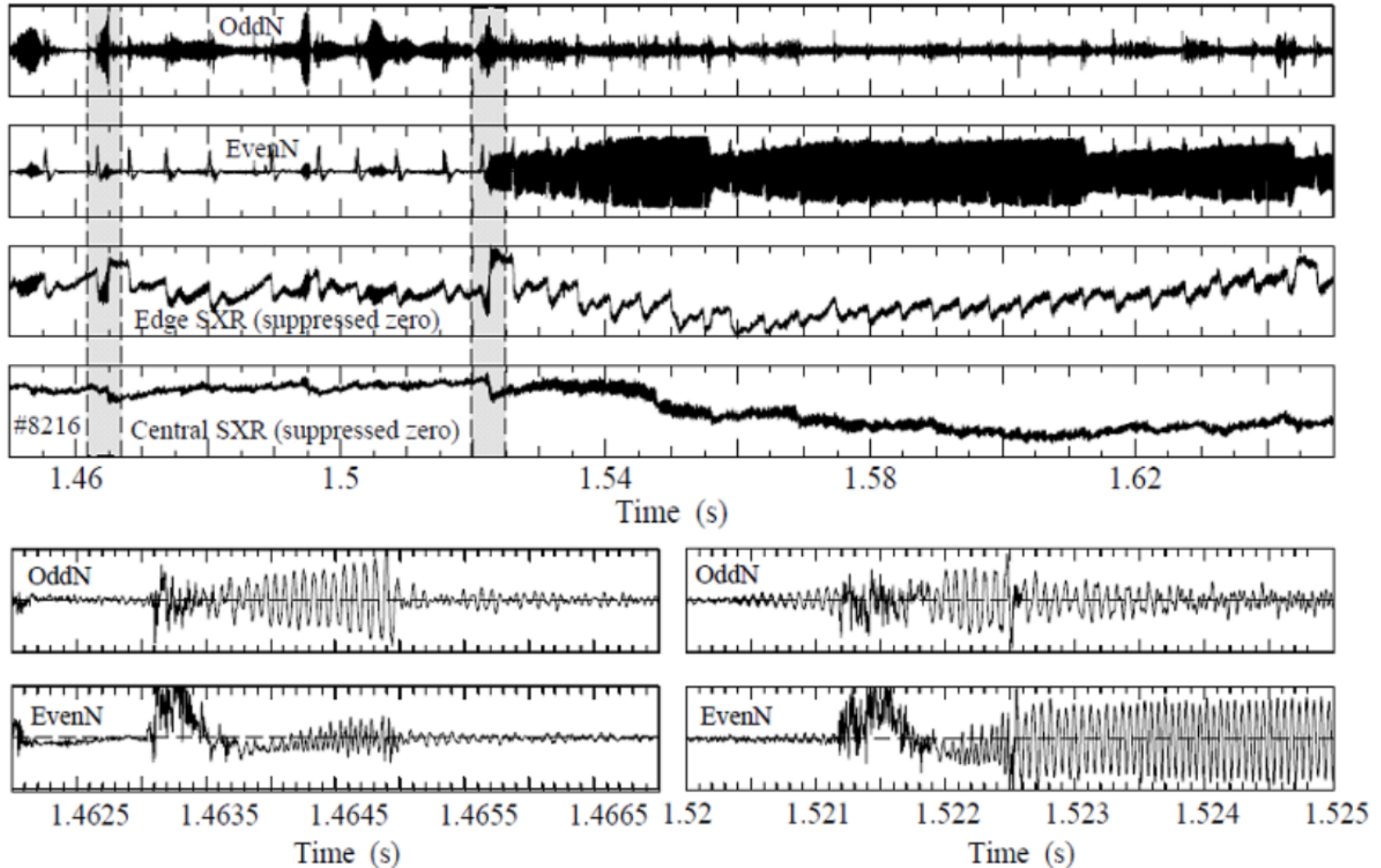
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Triggering of NTMs by 'seed' islands



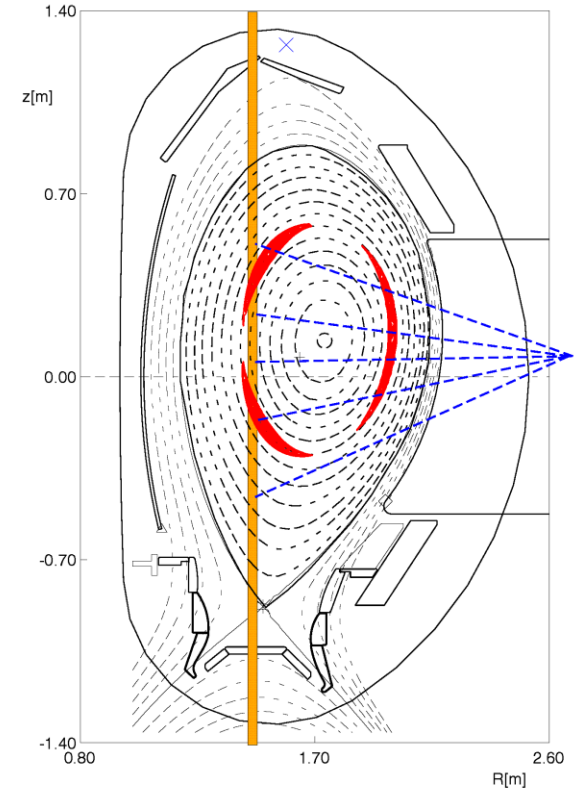
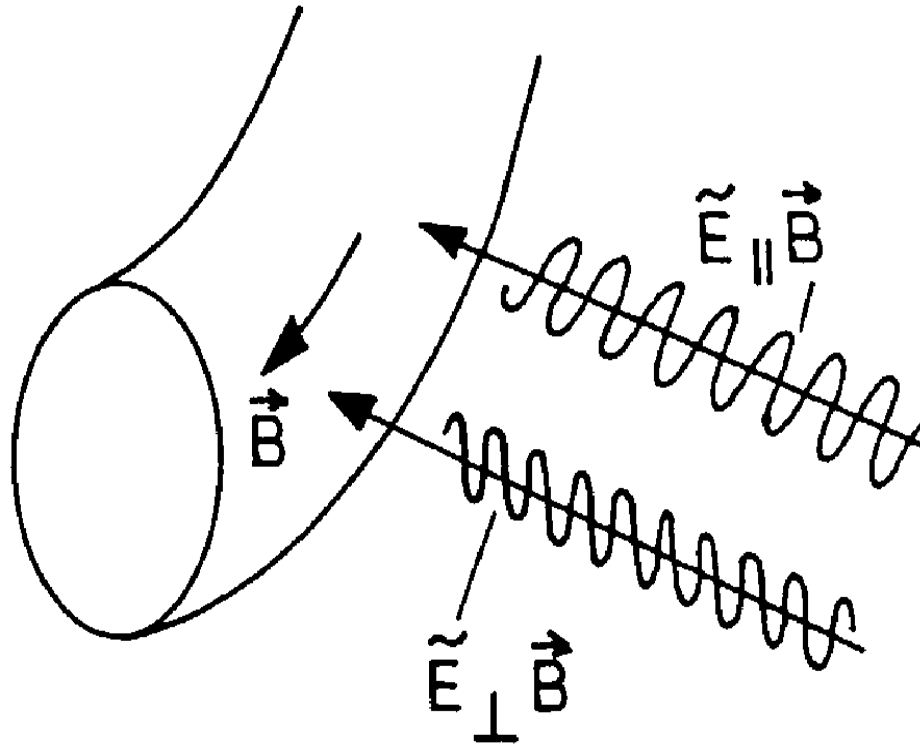
H. Zohm et al., Plasma Phys. Control. Fusion 1997



,'Metastable' states of NTMs: stable at $W=0$, unstable at $W>W_{seed}$



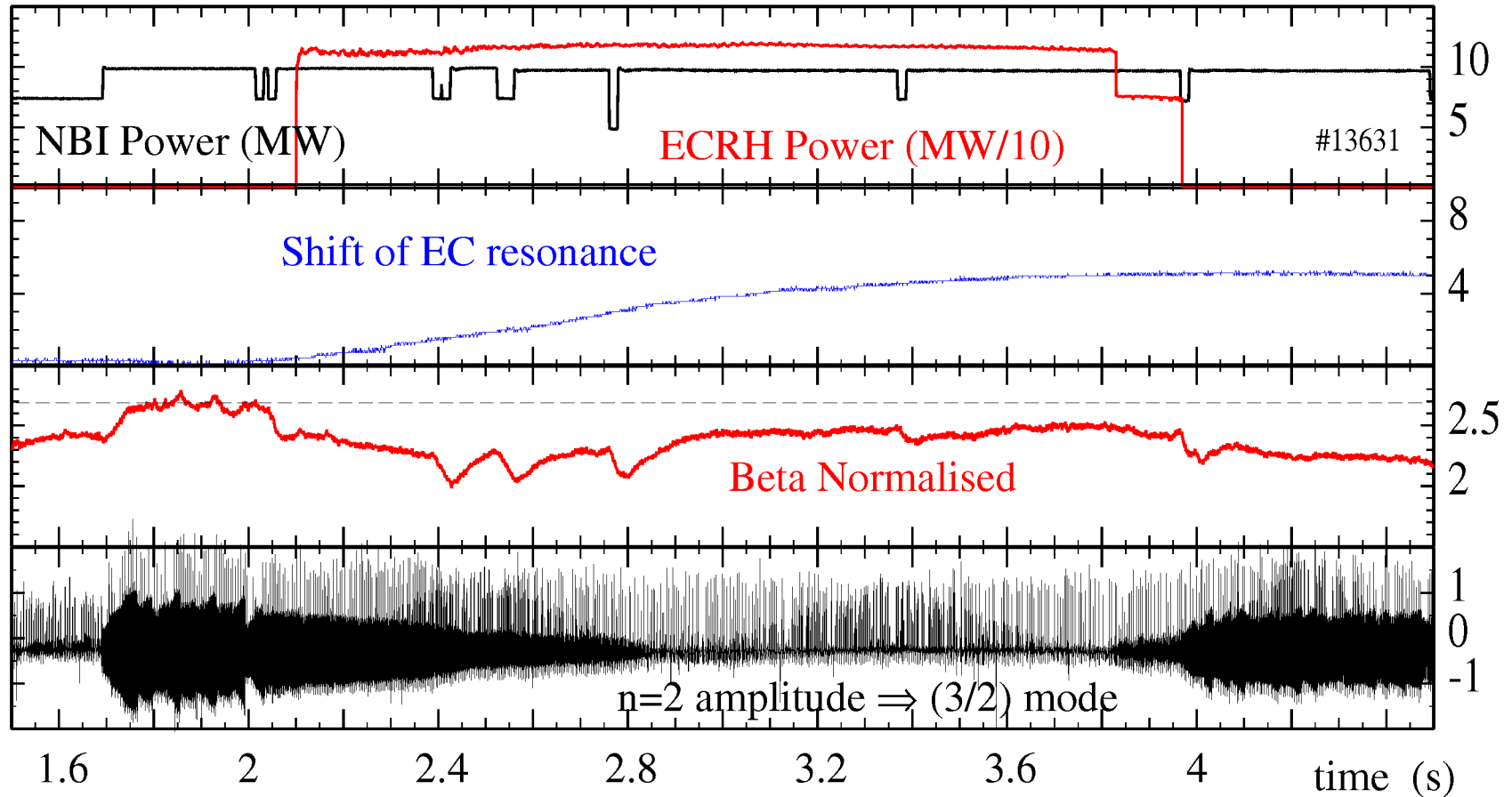
Removal of magnetic islands by microwaves (ECCD)



- Helical current can be driven by electron cyclotron resonance waves
- Deposition controlled by local B-field \Rightarrow very good localisation
- Feedback control of position possible via launch angle of ECCD beam



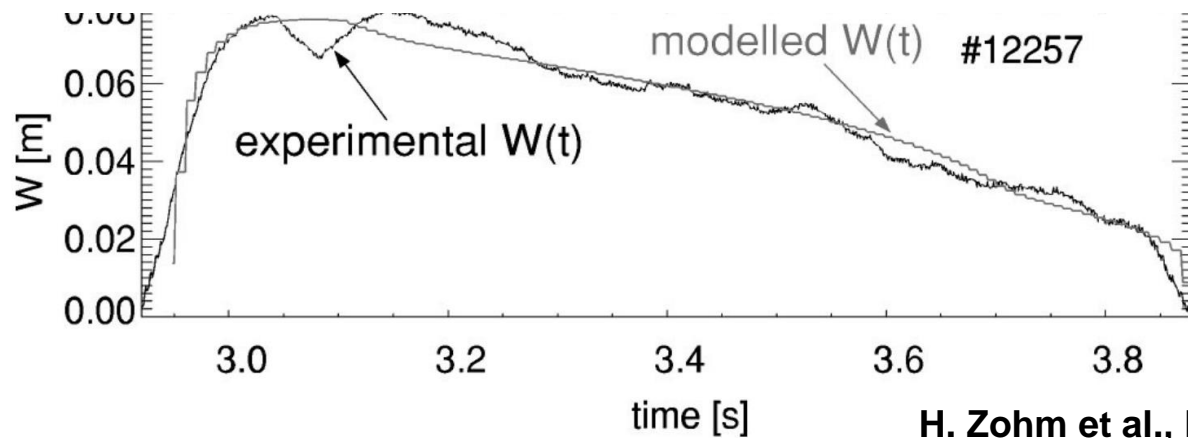
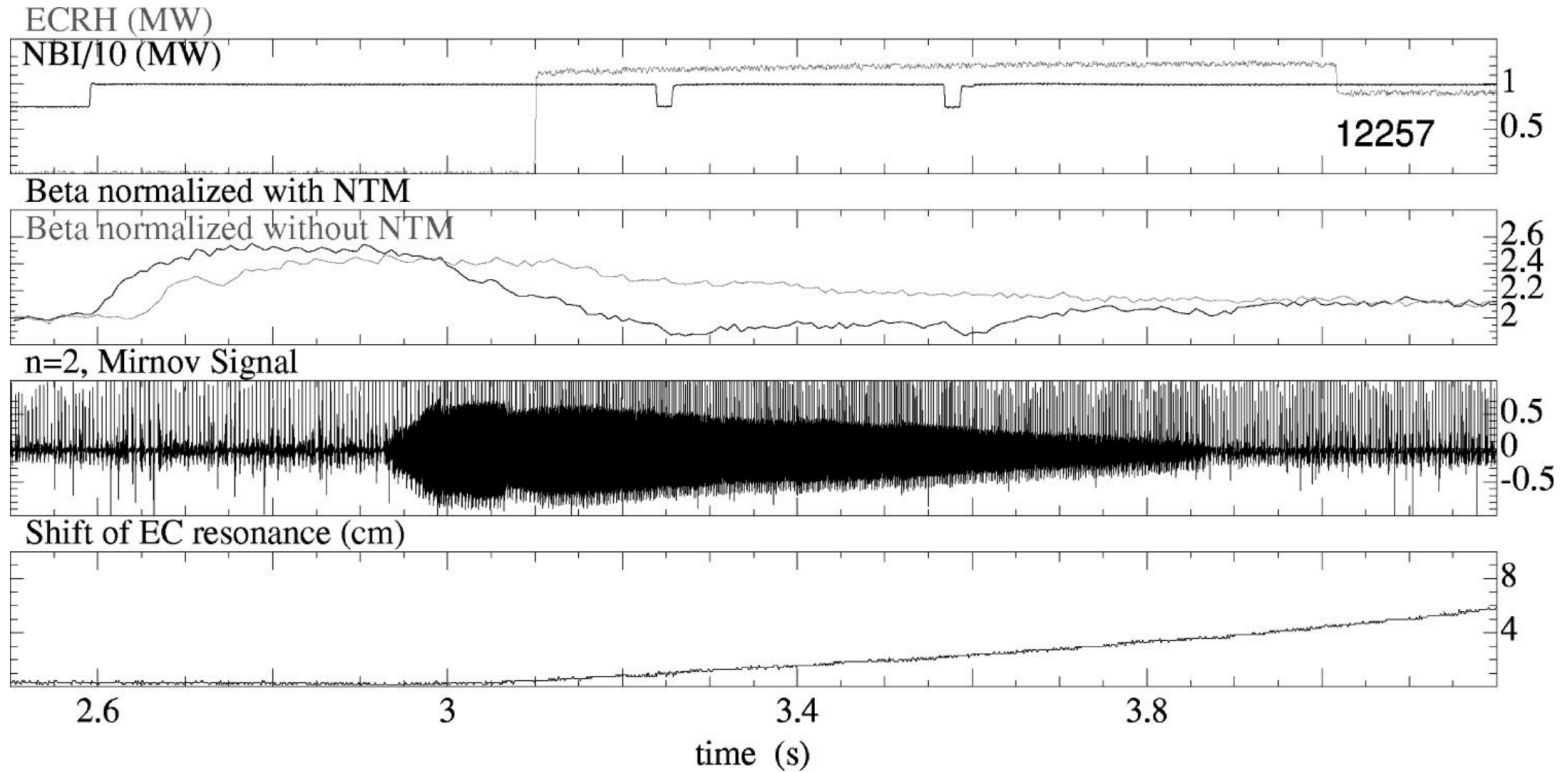
Removal of magnetic islands by microwaves (ECCD)



Adding a helical current ($P_{\text{ECRH}} / P_{\text{total}} = 10\%$) results in removal
method has the potential for reactor applications...



N.B.: good agreement with Rutherford equation





Theory of reconnection in magnetically confined fusion plasmas

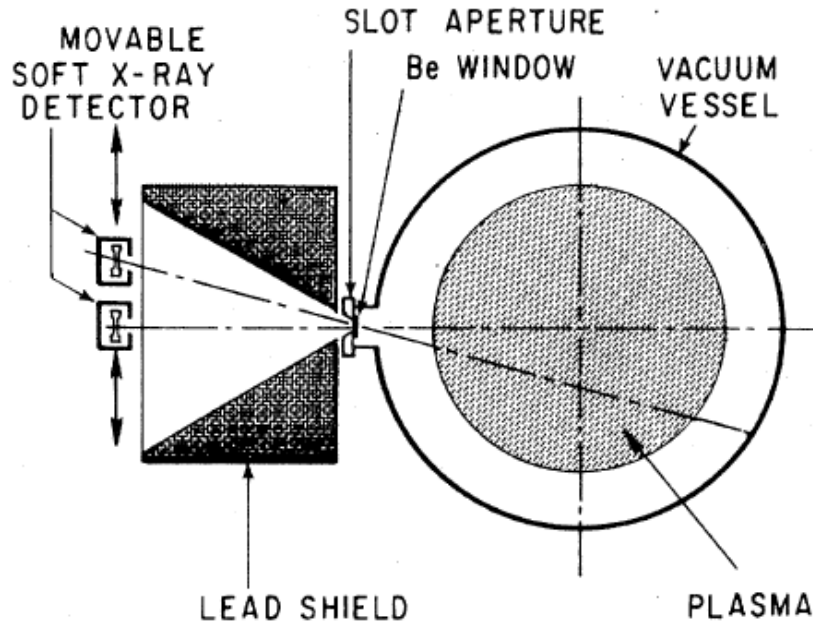
- recap of MHD equations and the role of resistivity
- formation of magnetic islands in tokamaks
- linear and nonlinear growth

Experimental examples of reconnection in tokamaks

- classical and neoclassical tearing modes
- **rapid reconnection events: sawteeth**
- others: MHD 'dynamo' through tearing modes



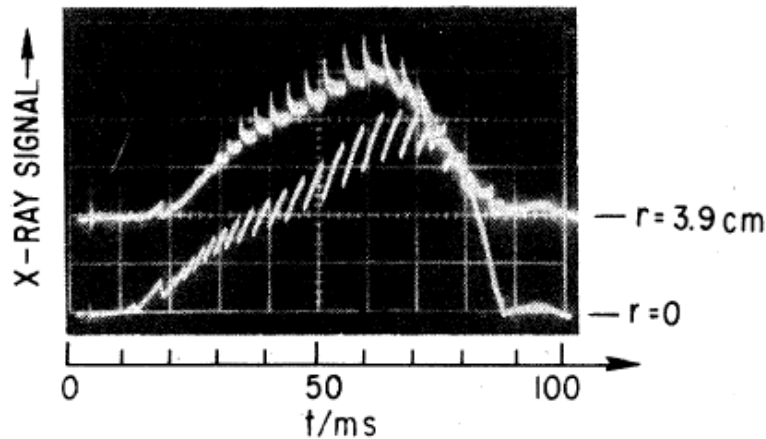
Sawtooth cycles as discovered on ST Tokamak



S. Von Goeler et al.,
PRL (1974)

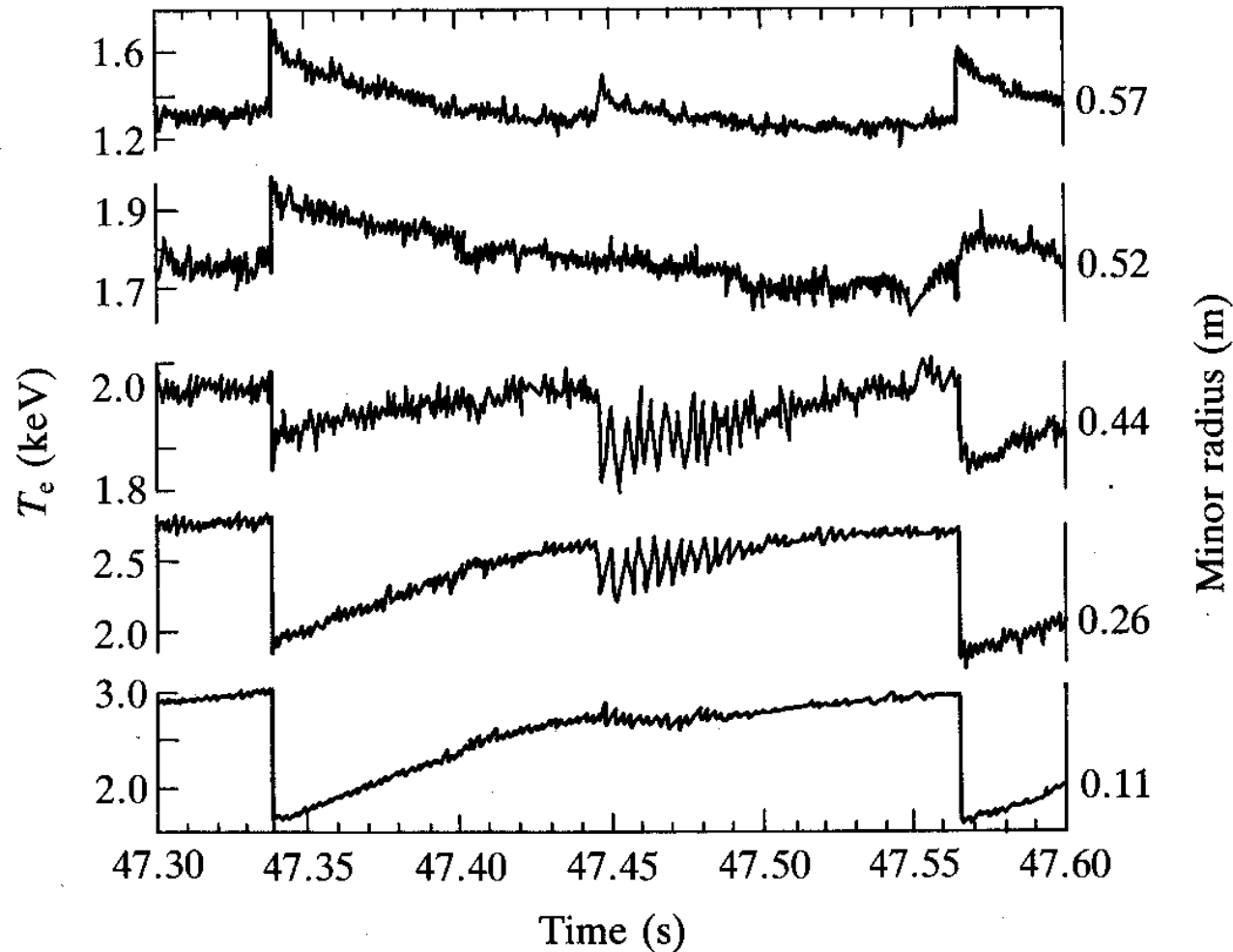
Repetitive increase and crash of central temperature

- increase slow, crash fast
- signal has the shape of a sawtooth
- note inversion of signal shape for $r > r_{inv}$





Sawtooth cycles as seen in T_e (JET)

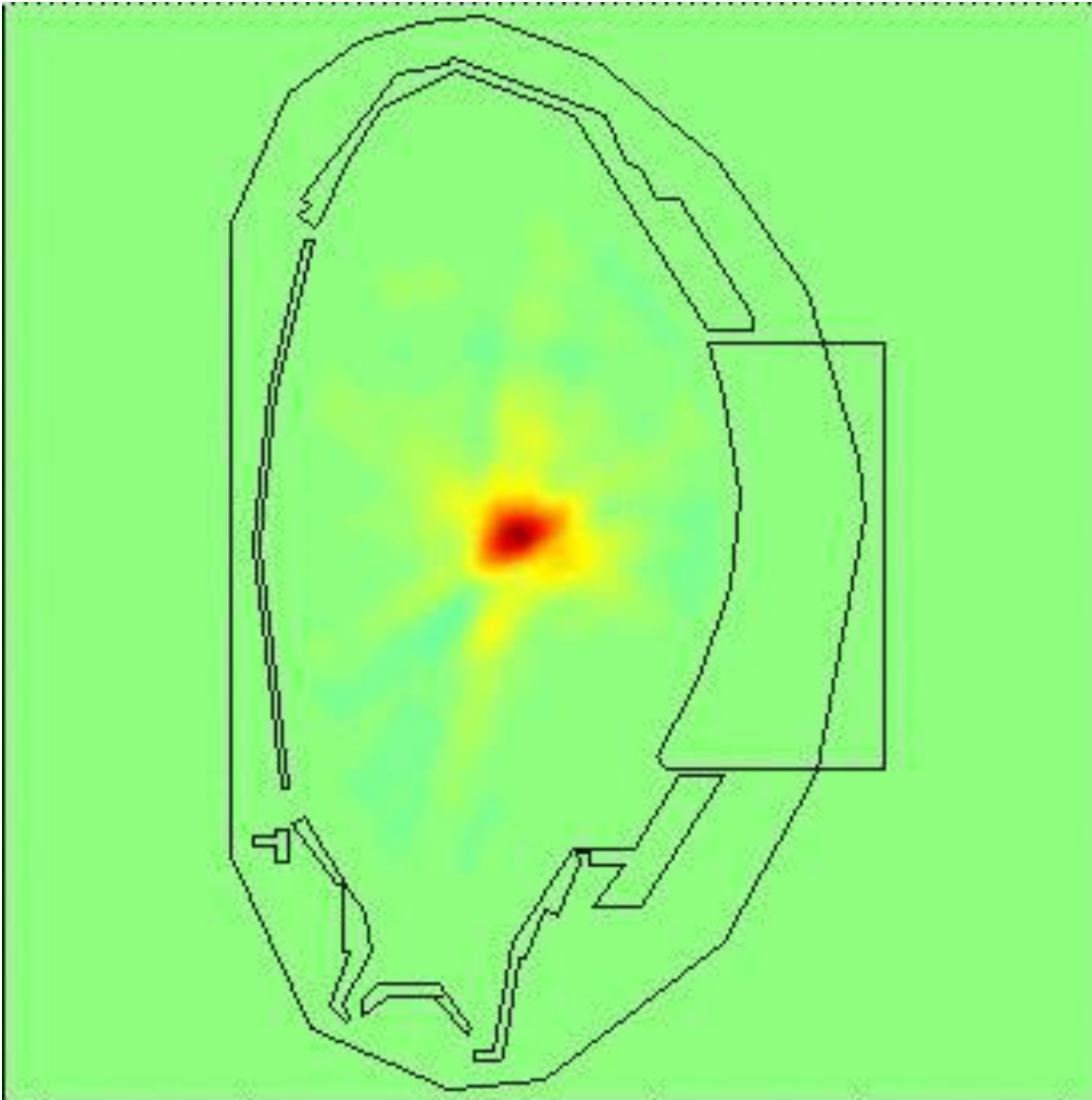


Bigger machine – longer sawtooth period

- consistent with global current re-distribution time scale ($\sim r^2 T^{3/2}$)



Sawtooth crashes as seen in SXR (ASDEX Upgrade)

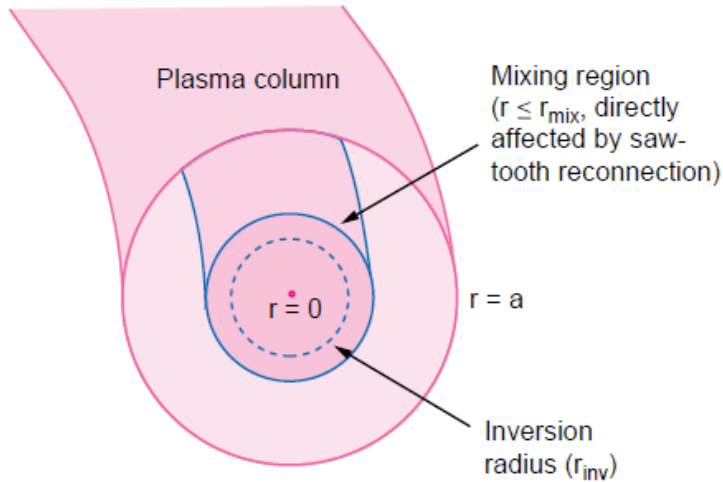


MHD signature of the mode preceding the crash is that of a (1,1) internal kink

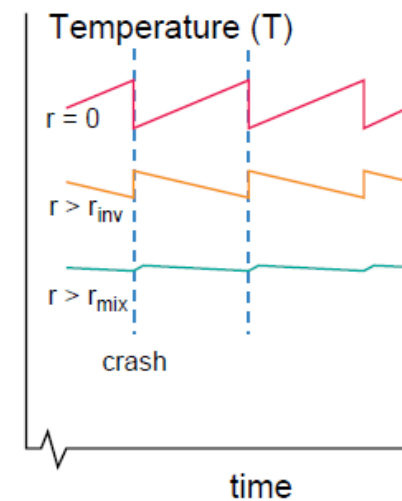
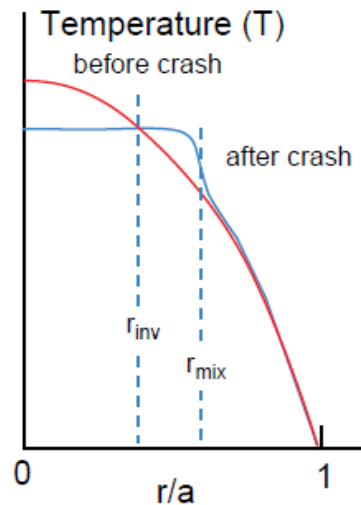
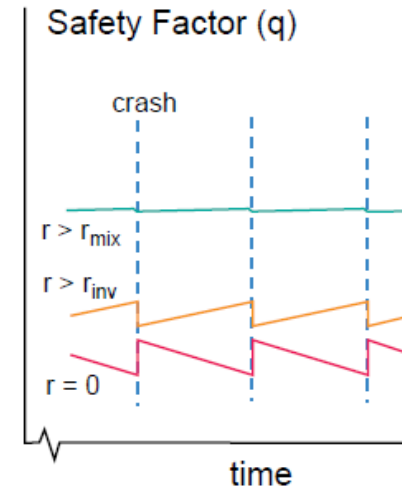
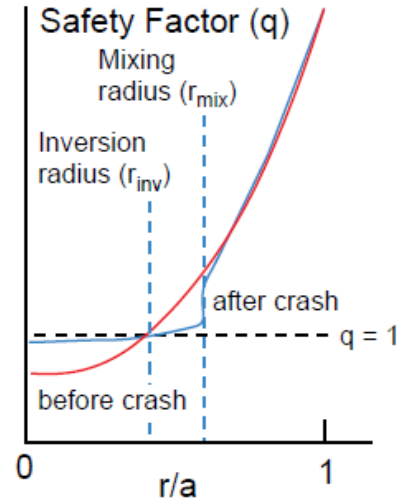
Suggests that $q(0)$ falls below 1 due to current diffusion and this triggers the crash

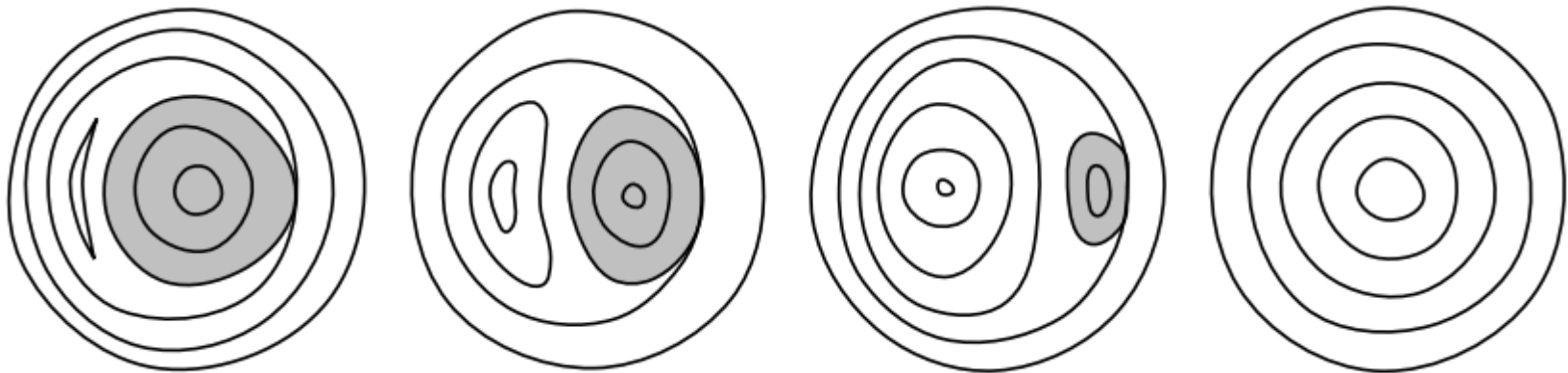


Sawtooth cycles – effect on kinetic and q-profiles



ITER physics base ,
Nuclear Fusion (1999)



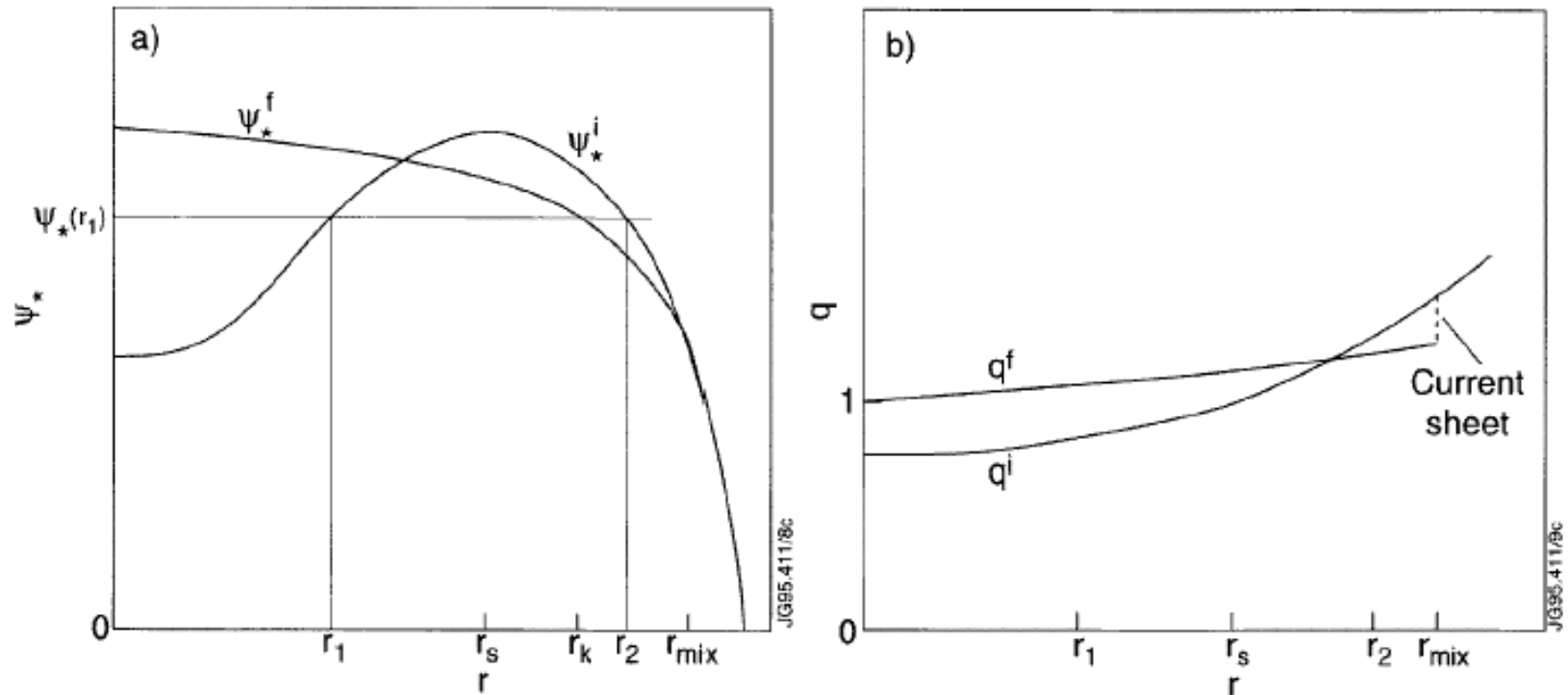


Kadomtsev proposed a reconnection of (1,1) helical flux

- note: not a tearing instability, rather a ,driven reconnection' by the ideal (1,1) mode
- consistent with the experimental signature described so far



Kadomtsev model for sawtooth crash

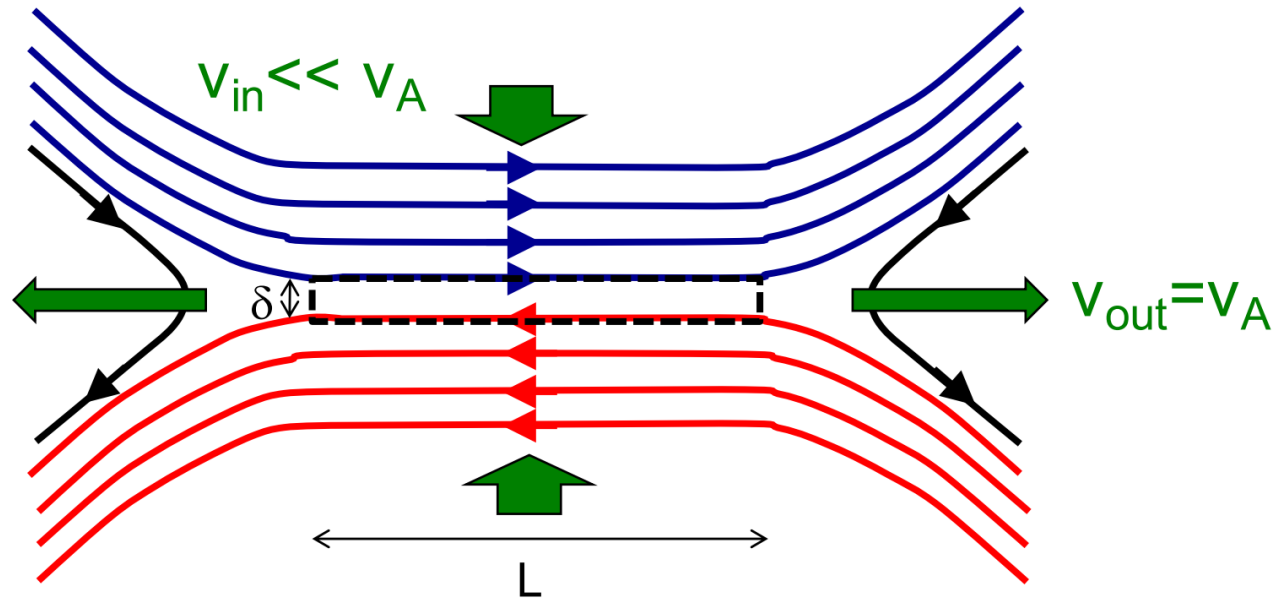


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Kadomtsev model for sawtooth crash



- Length L (system size) \gg width $\delta \sim L/S^{1/2}$ $S = \tau_R/\tau_A$
- Sweet-Parker Reconnection time $\tau_{SP} \sim \sqrt{\tau_R \tau_A} \sim S^{1/2} \tau_A$

remember
Nuno's talk

Good for early experiments, but with increasing S , $\tau_{\text{crash}} \sim \text{const.}$ (!)



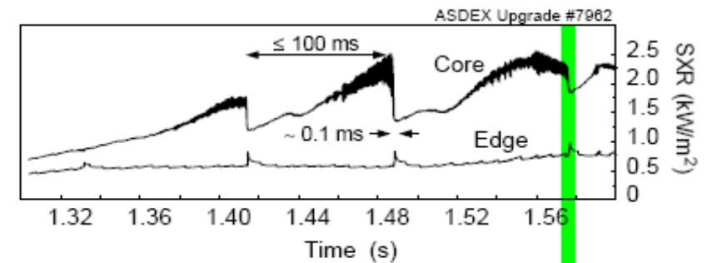
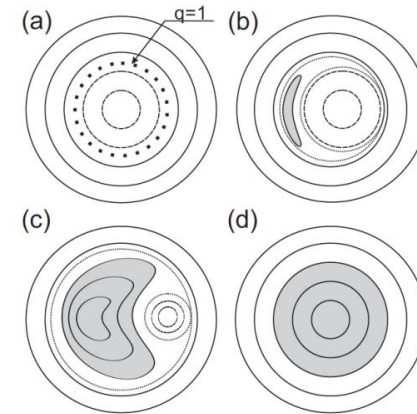
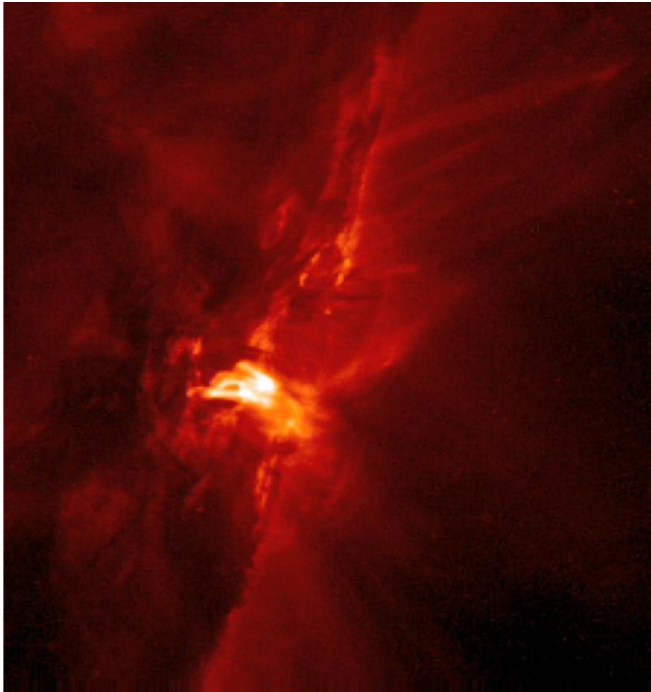
Kadomtsev model for sawtooth crash



Sweet Parker (MHD) reconnection time $\sim S^{1/2}\tau_A$ much longer than observed for large S numbers

Solar flares: SP time $\sim 10^6$ s

Crash times up to a factor of 100 faster than predicted by Kadomtsev model



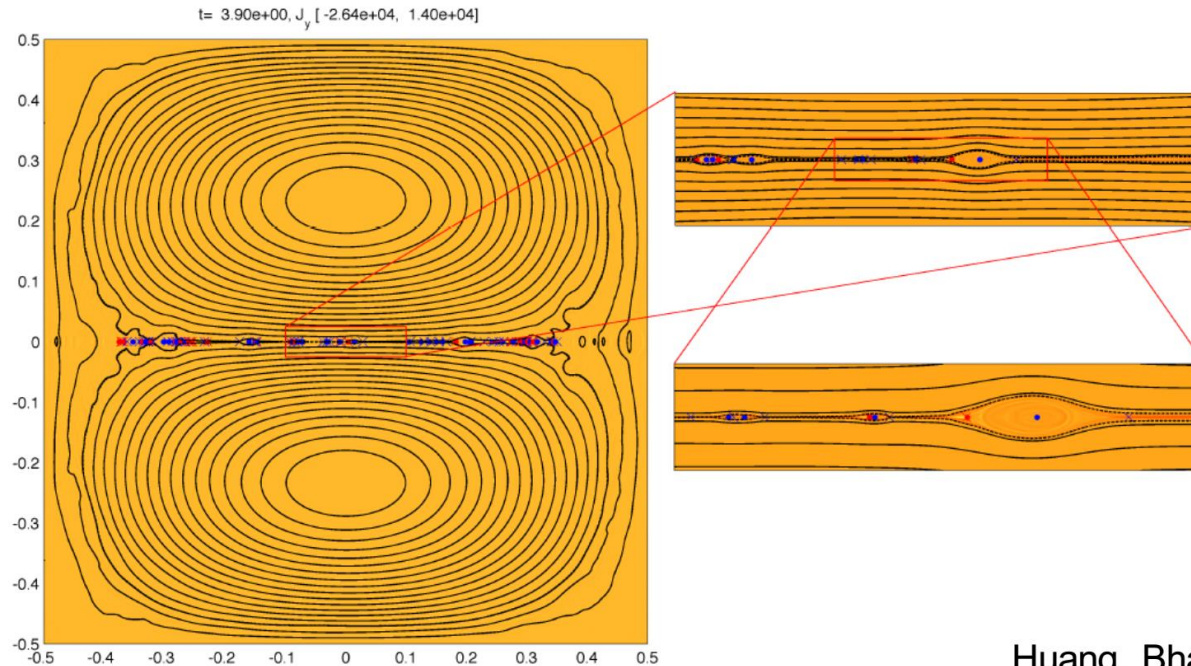


Current layer itself is not stable to start with!



For 2d reconnection without magnetic guide field:

- Critical aspect ratio ~ 100 (corresponding to $S_c \sim 10^4$) (e.g. Biskamp 1986)
- Linear growth rate and number of plasmoids increase with S (e.g. Loureiro et al. 2007)
- **Plasmoids significantly accelerate reconnection**



Huang, Bhattacharjee 2010



Generalised Ohm's law:

$$\frac{d\Psi}{dt} = -\eta \vec{j} + \frac{1}{en_e} (\vec{j} \times \vec{B}) + \frac{1}{en_e} \nabla p_e - \frac{m_e}{e^2 n_e} \frac{d\vec{j}}{dt}$$

Hall effect
Whistler waves

Electron pressure
gradient
Kinetic Alfvén waves

Electron inertia

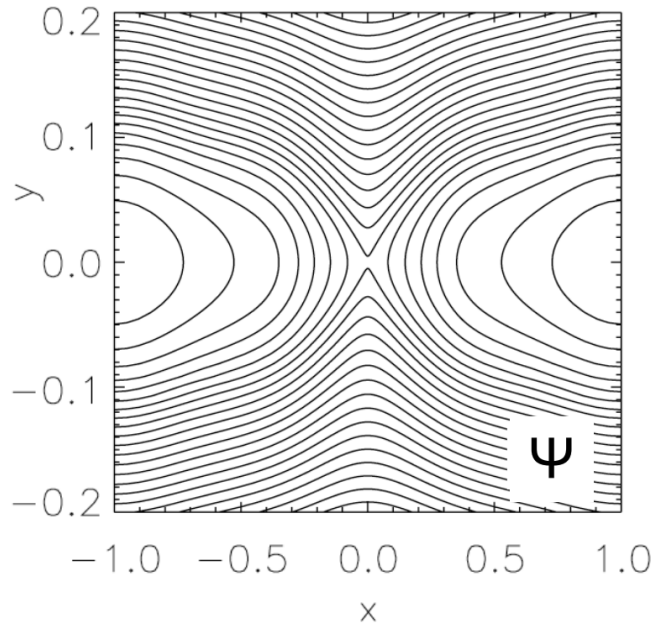
- Hall effect important in astrophysics, but currents perpendicular to magnetic fields are not relevant in reconnection in fusion plasmas
- Electron pressure gradient has qualitatively similar effect in fusion plasmas



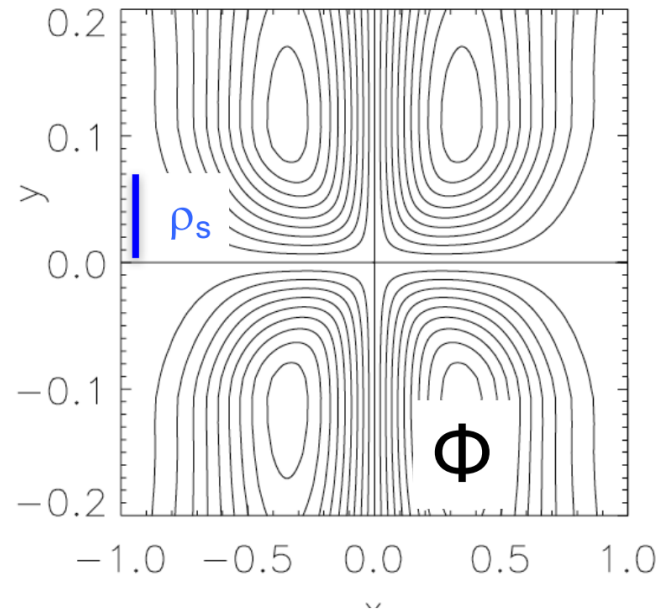
Using two-fluid MHD provides further speed-up



Magnetic flux:



Ion flow velocity: $\vec{u} = \vec{e}_z \times \nabla \Phi$



$$\rho_s = \sqrt{T_e/m_i}/\omega_{ci}$$

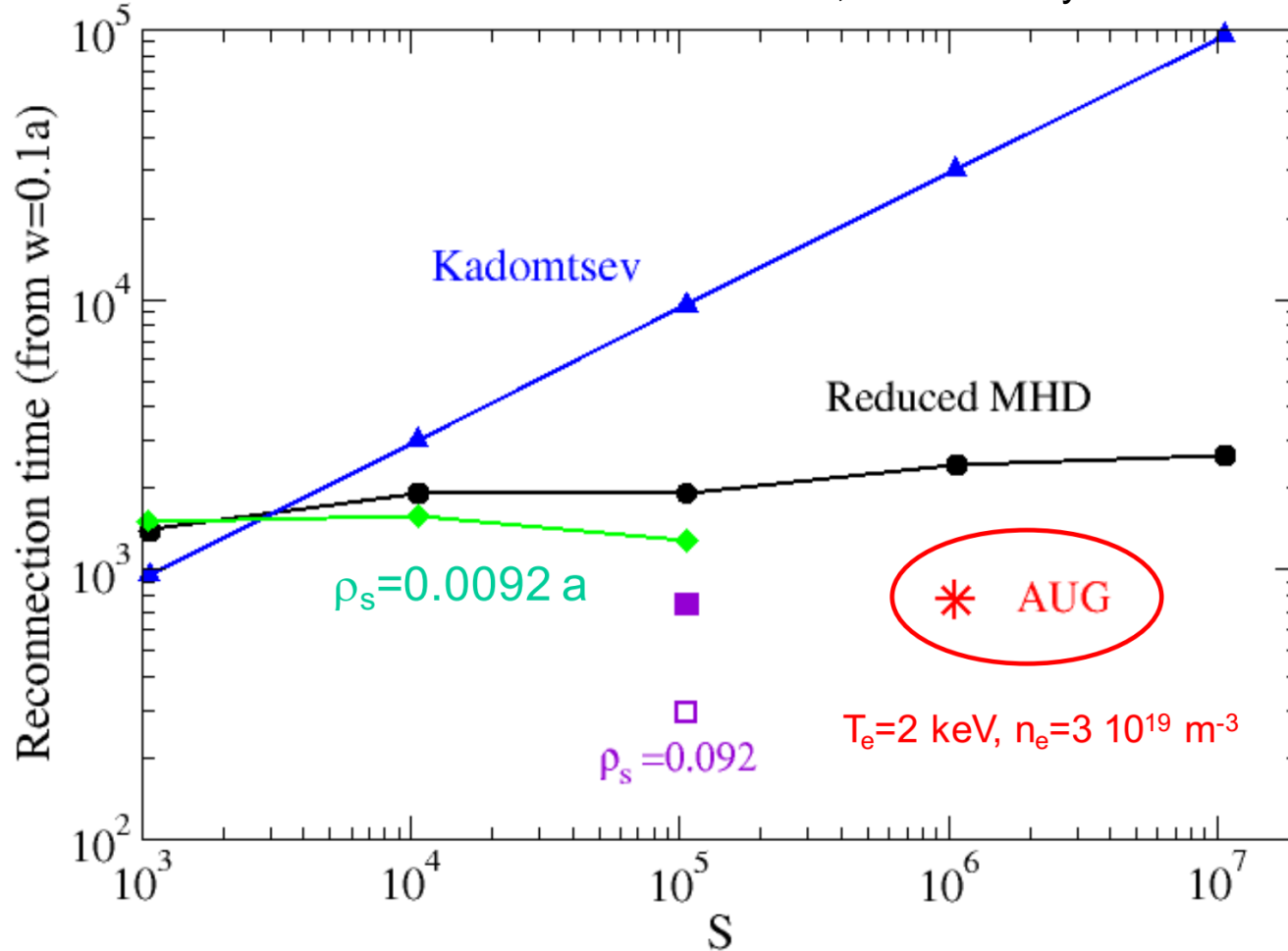
Ions are decoupled from magnetic field on a scale ρ_s



Using two-fluid MHD provides further speed-up



S. Günter et al., Plasma Phys. Controlled Fusion 2014



Decoupling of ions from magnetic field leads to saturation of crash times and values comparable to those observed e.g. in ASDEX Upgrade



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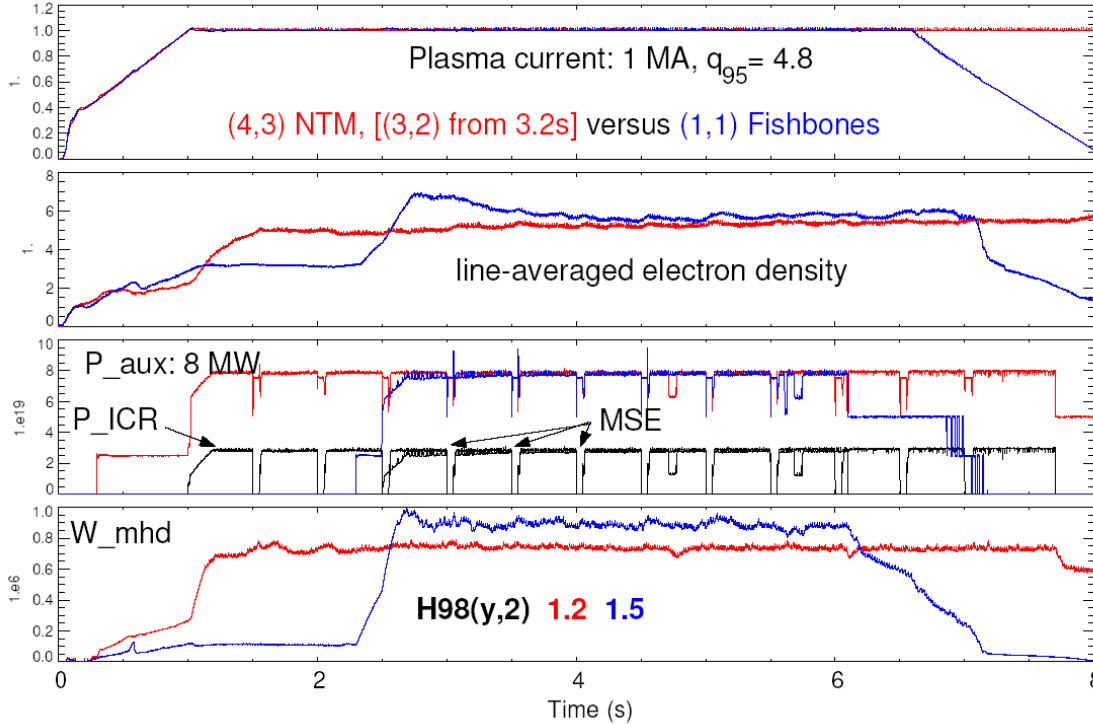


Advanced Scenarios for ITER: Improved H-mode



J. Stober et al., Nucl. Fusion 2011

Improved H-Mode: **early (#20993)** versus **late (#20995)** heating

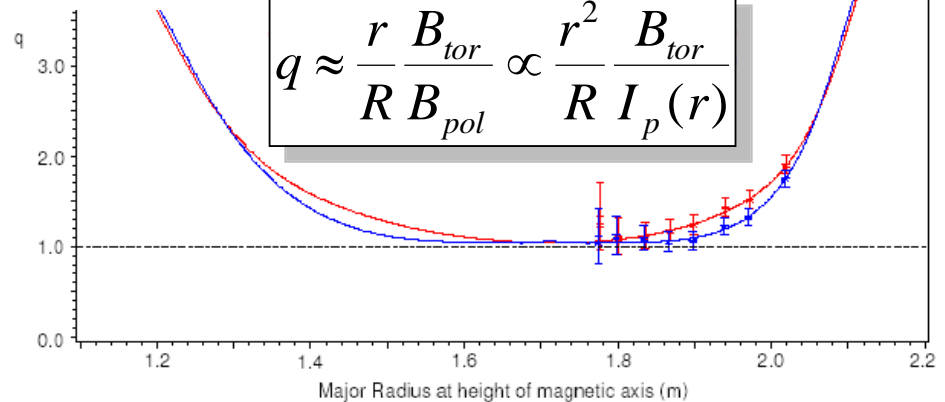


Clue: subtle changes in the current profile have large impact on performance

q-profiles

CLISTE result with MSE and local MSE estimates with error bars

$$q \approx \frac{r B_{tor}}{R B_{pol}} \propto \frac{r^2 B_{tor}}{R I_p(r)}$$

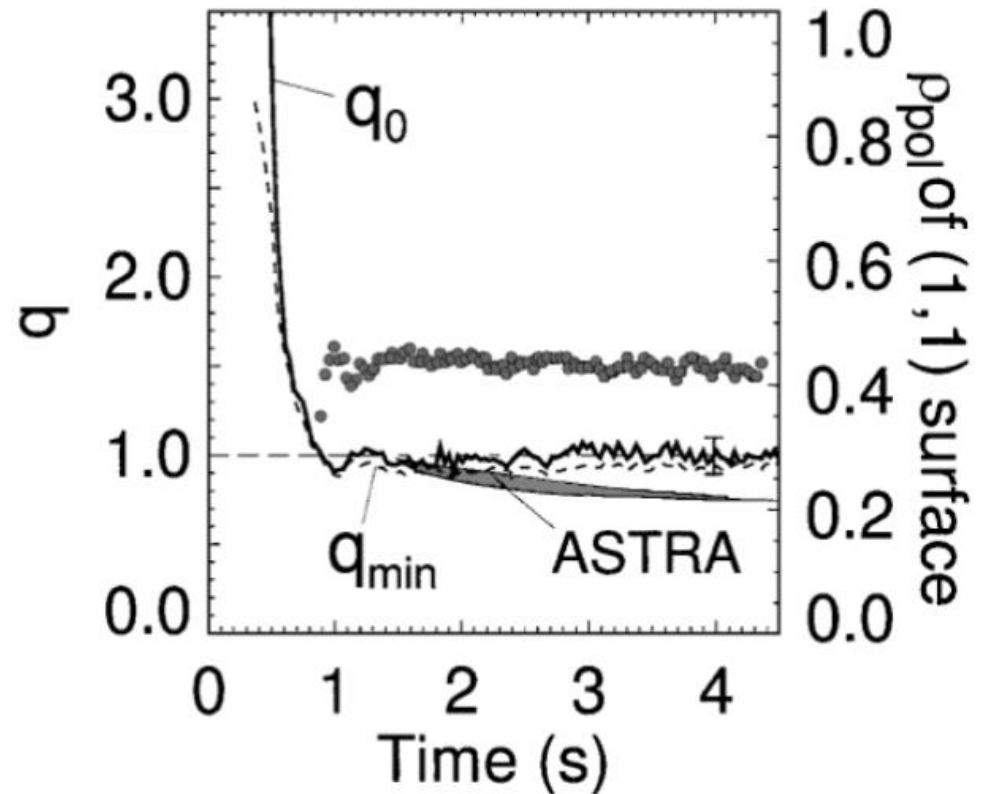
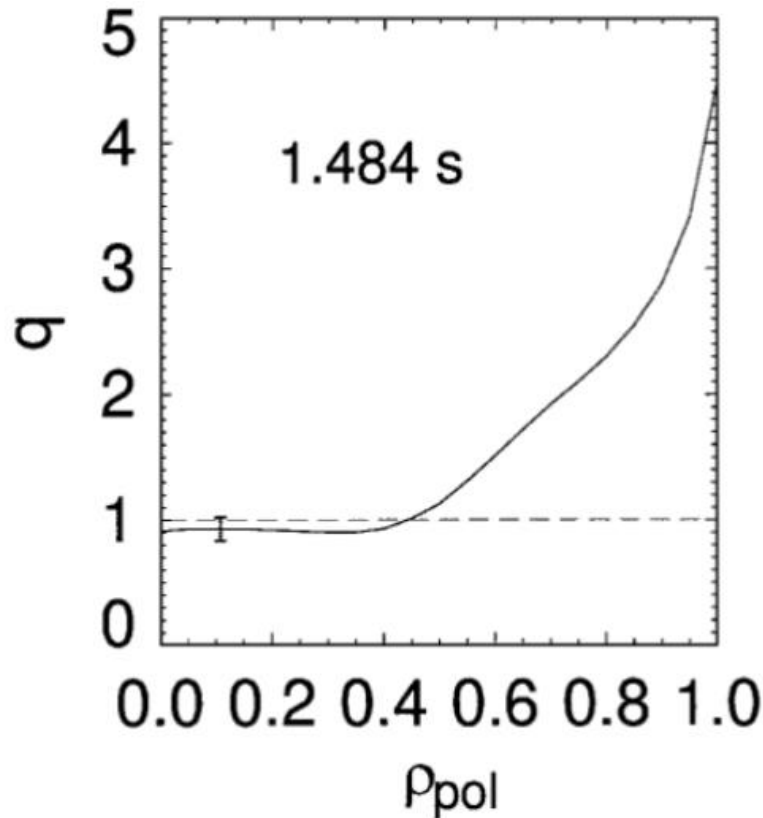


Example for Advanced Scenario:

- Improved H-mode, a.k.a. 'hybrid' scenario
- Confinement (H_{98}) and stability (β_N) improved w.r.t standard H-mode



Improved H-mode: temporal evolution of $q(0)$

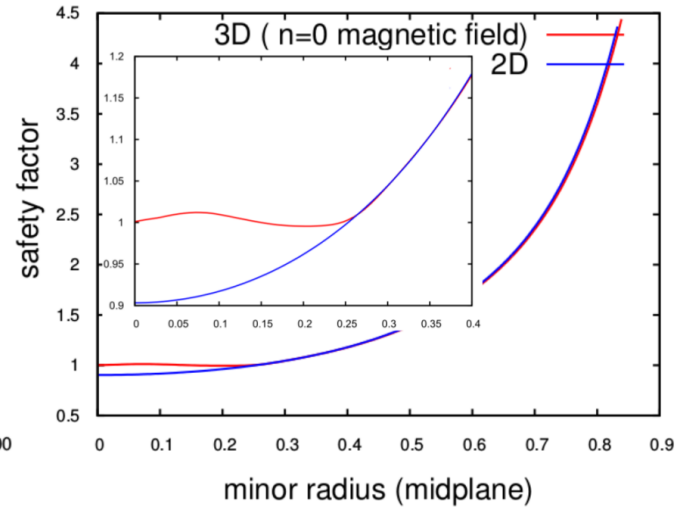
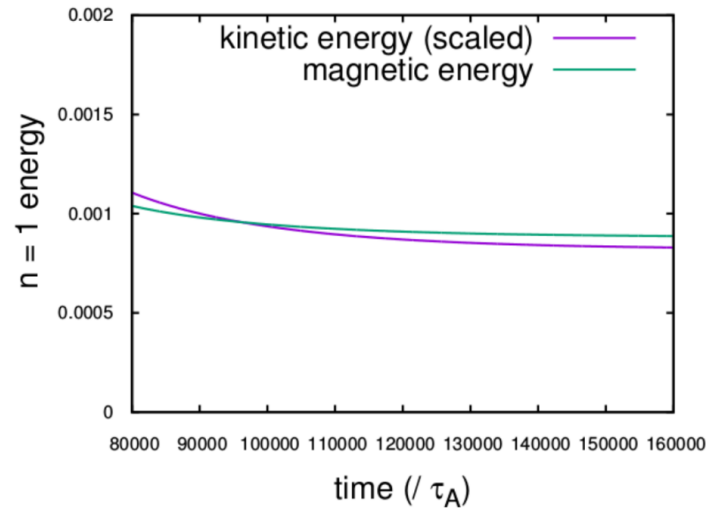


Central safety factor clamped to ~ 1 even in the absence of sawteeth

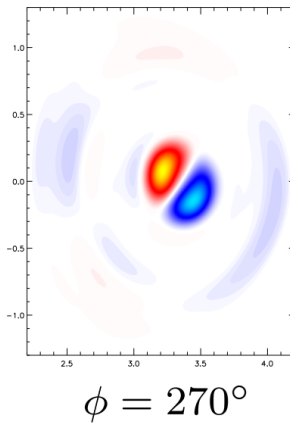
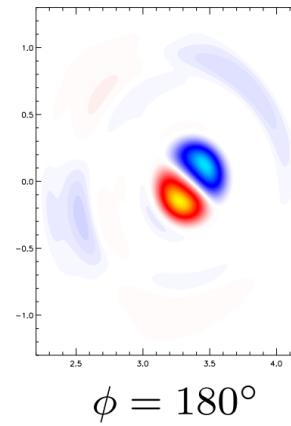
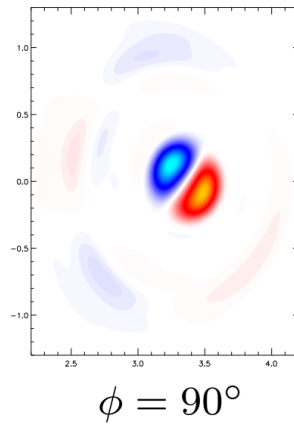
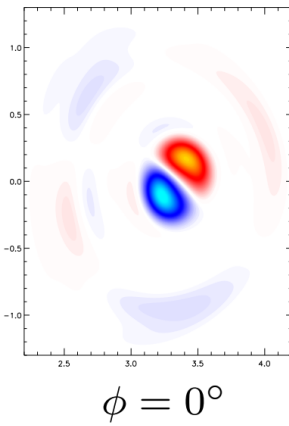
- instead, strong (1,1) mode activity is observed
- note: similar story reported from DIII-D for (3,2) NTMs



Sawtooth-free state with helical core



Poloidal velocity stream function (3D-2D)



Quasi-stationary state with helical core found in non-linear MHD simulations



Sawtooth-free state with helical core

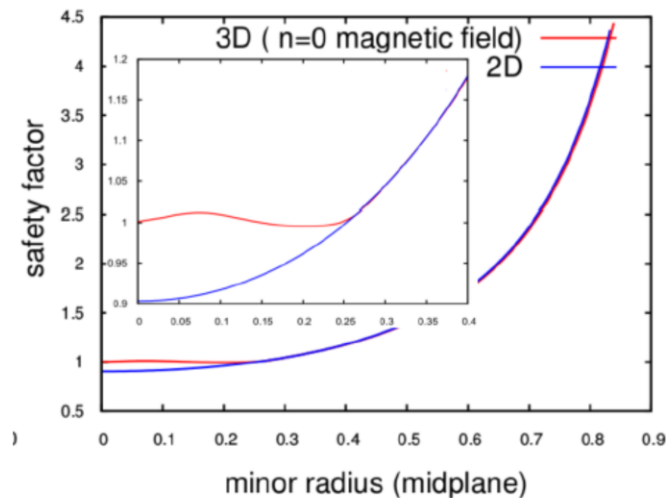
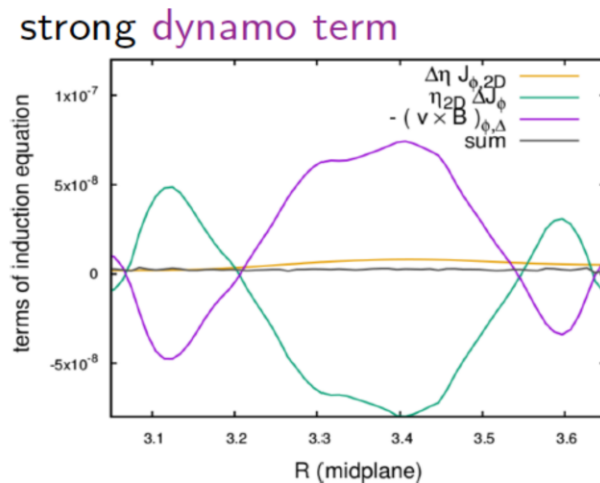


Sawtooth free discharges due to (1,1) mode activity and associated plasma flows

- (1,1) mode driven by pressure gradient in weak shear region
- Associated with strong (poloidal) flows giving rise to a dynamo (loop) voltage

$$\partial_t \Psi_0 = - \boxed{R \Delta \eta J_{\phi, 2D}} - \boxed{R \eta_{2D} \Delta J_{\phi}} + \boxed{R \hat{\phi} \cdot (\mathbf{v}_1 \times \mathbf{B}_1)}$$

I. Krebs (PPPL)



Extra (dynamo) loop voltage counteracted by Δj_{ϕ} – flattening of $j_{\phi}(r)$

Drive is the ideal instability of the ideal kink mode with weak magnetic shear

Note: dynamo quite common in Reversed Field Pinch (RFP)



Reconnection plays an important role in tokamak instabilities

- magnetic islands degrade confinement
- multiple magnetic islands can lead to disruptions

Magnetic islands in tokamaks are a testbed of our understanding of reconnection in hot (‘collisionless’) plasmas)

- relatively well diagnosed (compared to Astrophysical Plasmas)

Single islands are often well described by simple one-fluid MHD

- (neo)classical tearing mode largely consistent with Rutherford equations

There are other phenomena that clearly need a description beyond simple one-fluid MHD

- fast reconnection as in sawteeth (or ELMs, that I have not talked about)