What can recurrence analysis tell us about accretion flows in XRBs?

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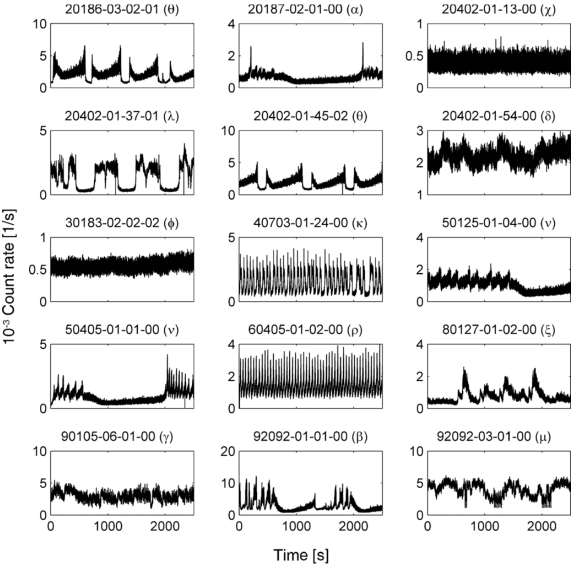
Motivation

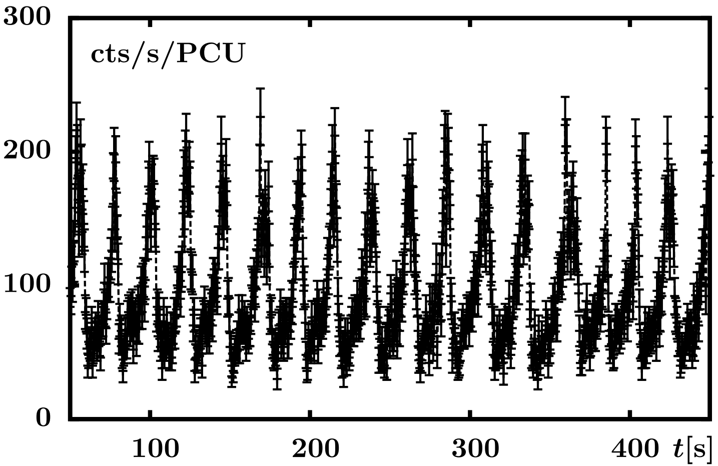
- Radiation from microquasars – shows variability on different time scales during outburst
- Different temporal and spectral states – changes during days
- Flares, QPOs, heart beat state, spectral evolution
- Low frequency QPOs – ~ 0.1 Hz up to few tens of hertz
  – more types, frequency evolves during the outburst
  – some possible explanation: shock in low angular momentum flow (LAF), propagatory oscillating shock model (POS), radiation pressure instability
- Searching for deterministic non-linearity and chaos:
  BH: GRS 1915+105: Misra et al., 2004 & 2006, Harikrishnan et al., 2006 & 2011; Jacob et al., 2017
  NS: Sco X-1, Cyg X-2: Karak et al., 2009
  Negative result: Mannattil et al., 2016
- Is recurrence analysis good tool for X-ray lightcurves?

Different states of GRS 1915+105



Polyakov et al, 2012

Heartbeat state of IGR 17091-3624 – 2011 outburst



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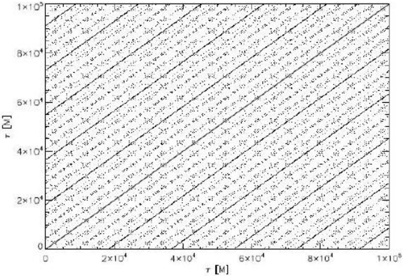
- input data – X ray light curve – flux $f(t)$ in time bin $\mathrm{d}t$
- reconstruction of m-dim phase space trajectory by time delay $\mathbf{x}(t) = \{f(t), f(t+\Delta t), f(t+2\Delta t), \ldots, f(t+(m-1)\Delta t)\}$, embedding delay: $\Delta t = k\mathrm{d}t, \quad k \in \mathbb{N}$
- RP = effective tool for visualisation of recurrences in the system from time series

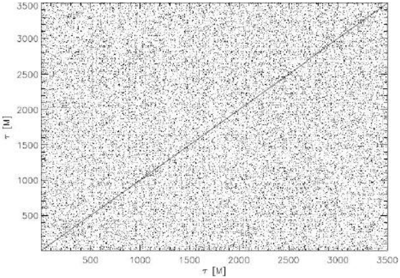
$$\mathbf{R}_{i,j}(\varepsilon) = \Theta(\varepsilon - \parallel \mathbf{x}_i - \mathbf{x}_j \parallel), \qquad i, j = 1, ..., N \tag{1}$$

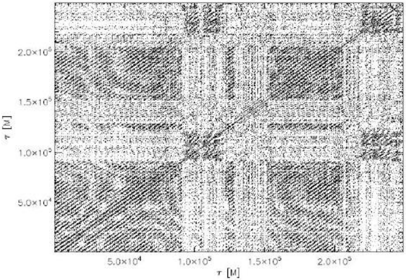
$\mathbf{x}_i = \mathbf{x}(t_i)$ point on the trajectory, ε recurrence threshold

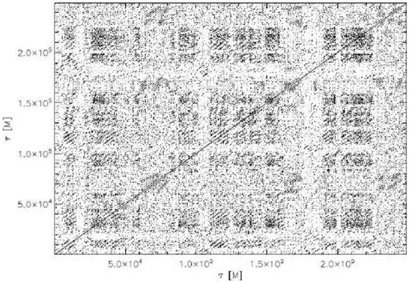
- $\mathbf{R}_{i,j} = 1$ – the point $\mathbf{x}(t_j)$ is in the ε-neighbourhood of the point $\mathbf{x}(t_i) \rightarrow$ black dot in the plot
  the trajectory returns near itself after the time $t_j - t_i$
- the pattern of recurrences in the dynamical system reveals important features of the motion – whether it is regular, chaotic or random

Examples of RP for regular, random and chaotic motion









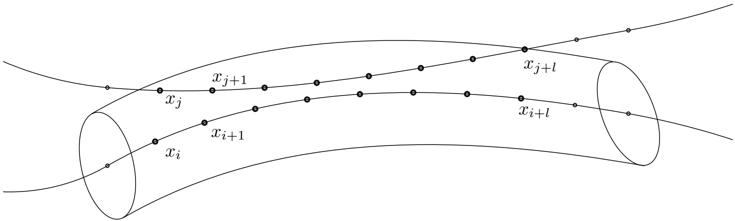


Illustration of origin of diagonal line in the recurrence plot. Two different parts of the trajectory are depicted, whose distance at the times t_i and t_j is lower than the threshold ε. Within the time $l\Delta t$ the pairs of points x_{i+k} and x_{j+k} remain closer then ε. The diagonal line ends when the successive points of the segments move away from each other.

Quantification of recurrence plots

- Recurrence Quantification Analysis (RQA)
  – method of nonlinear data analysis which quantifies the number and duration of recurrences of a dynamical system from the recurrence matrix (`Marwan et al.,2007`)
- $RR(\varepsilon) = \frac{1}{N^2} \sum_{i,j=1}^{N} R_{i,j}(\varepsilon)$ – recurrence rate
- histogram of diagonal lines of a certain prescribed length l,

$$P(\varepsilon, l) = \sum_{i,j=1}^{N} (1 - R_{i-1,j-1}(\varepsilon))(1 - R_{i+l,j+l}(\varepsilon)) \prod_{k=0}^{l-1} R_{i+k,j+k}(\varepsilon)$$

- how many recurrence points form diagonal lines with $l \geq l_{\min}$

$$DET(\varepsilon) = \sum_{l=l_{min}}^{N} lP(\varepsilon, l) / \sum_{l=1}^{N} lP(\varepsilon, l)$$

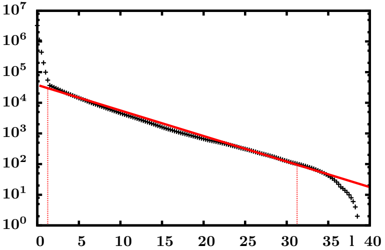
- $L_{\max} = \max(\{l_i\}_{i=1}^{N})$ – the longest diagonal line
- $DIV = 1/L_{\max}$ – crude estimate for Lyapunov exponents

- the second order Rényi entropy (correlation entropy) – dynamical invariant

$$K_2 = -\lim_{\Delta t \to 0} \lim_{\varepsilon \to 0} \lim_{l \to \infty} \frac{1}{l \Delta t} \ln \sum_{i_1,\ldots,i_l} p^2_{i_1,\ldots,i_l}(\varepsilon) \qquad (2)$$

- we plot ln (number of diagonal lines of length $\geq l$) versus $l \Rightarrow$ straight line with the slope $-K_2(\varepsilon)\Delta t$

b)



- correlation entropy gives the lower estimate for the sum of positive Lyapunov exponents

$$K_2 \leq \sum_{\lambda_i > 0} \lambda_i \qquad (3)$$

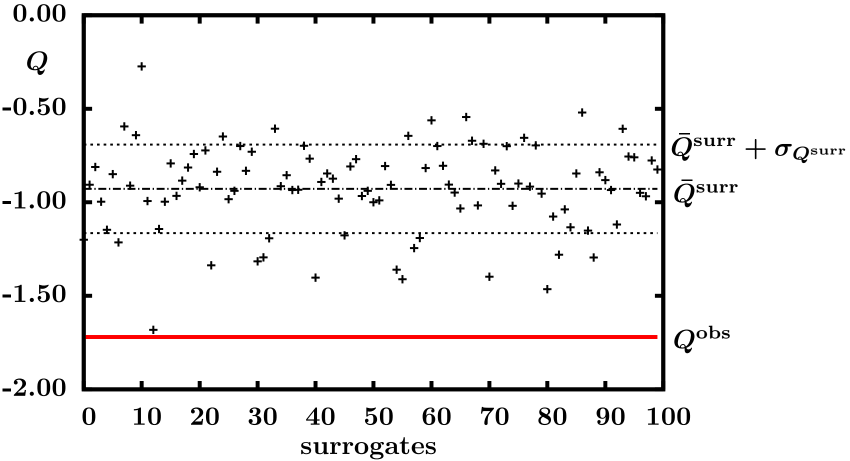
Surrogate data method combined with recurrence analysis

- surrogate data method (`Theiler et al., 1991`) – artificial data series sharing some properties of observed time series according to given null hypothesis, otherwise random
- discriminating statistics - number for comparison of observed and surrogate data ($Q = \ln(K_2)$)
- null hypothesis:
  a) temporally independent, identically distributed gaussian noise – no dynamics at all, surrogates share mean and variance
  – generated by shuffling the order of the data points
  b) linearly autocorrelated gaussian noise – linear process
  $x_n = a_0 + \sum_{k=1}^{q} a_k x_{n-k} + \sigma e_n$
  $\rightarrow$ all structures in time series given by Fourier power spectrum, surrogates share power spectrum
  – generated by Iterative amplitude adjusted Fourier transform algorithm (IAAFT)

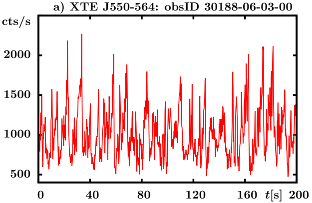
- extracted light curve rescale to zero mean and unit variance
- 100 surrogates made by `surrogates` from `TISEAN`
- $\varepsilon_{\min}$ and $\varepsilon_{\max}$ such that RR $\in$ (1% and 25%), set $\{\varepsilon_i\}_{i=1}^{N_\varepsilon}$
- $\forall\varepsilon_i$: cumulative histogram of diagonal lines – `rp` (`Marwan 2007`) $\rightarrow$ estimate of $Q = \ln K_2 \rightarrow Q^{\mathrm{obs}}(\varepsilon_i)$, $Q_j^{\mathrm{surr}}(\varepsilon_i)$
- We compute the non-linear dynamics indicator (NLD)

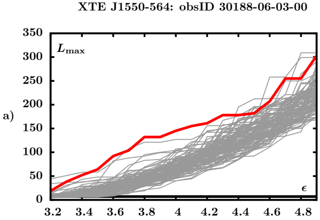
$$
\begin{aligned}
\bar{\mathcal{S}} &= \frac{1}{N_\varepsilon}\sum_{i=1}^{N_\varepsilon}\mathcal{S}(\varepsilon_i) \\
\mathcal{S}(\varepsilon) &= \frac{N_{\mathrm{sl}}}{N^{\mathrm{surr}}}\mathcal{S}_{\mathrm{sl}} - \mathrm{sign}(Q^{\mathrm{obs}}(\varepsilon) - \bar{Q}^{\mathrm{surr}}(\varepsilon))\frac{N_{\mathcal{S}_K}}{N^{\mathrm{surr}}}\mathcal{S}_{K_2}(\varepsilon) \\
\mathcal{S}_{\mathrm{sl}} &= 3 \\
\mathcal{S}_{K_2}(\varepsilon) &= \frac{|Q^{\mathrm{obs}}(\varepsilon) - \bar{Q}^{\mathrm{surr}}(\varepsilon)|}{\sigma_{Q^{\mathrm{surr}}(\varepsilon)}}
\end{aligned}
$$

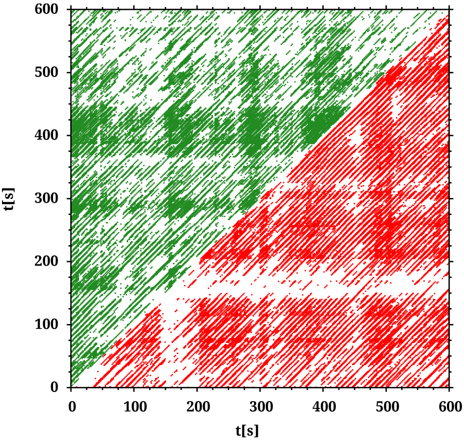
`Suková, Grzedzielski & Janiuk (2016) A&A, 586, A143`



$\mathcal{S}(4.6) = 3.35, \bar{\mathcal{S}} = 2.7$ for $\varepsilon \in [3.9, 5.5]$, $m = 20$, $k = 7$


a) XTE J550-564: obsID 30188-06-03-00
cts/s
2000
1500
1000
500
0
40
80
120
160
t[s]
200

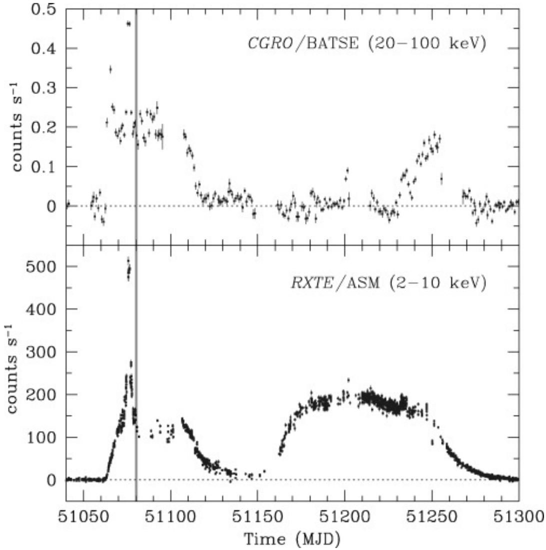

XTE J1550-564: obsID 30188-06-03-00
$L_{\max}$
a)
350
300
250
200
150
100
50
0
ϵ
3.2
3.4
3.6
3.8
4
4.2
4.4
4.6
4.8


t[s]
600
500
400
300
200
100
0
0
100
200
300
400
500
600
t[s]

ObsID	N	5% L_{max}	10% L_{max}	20% L_{max}	$\bar{S}$	N_ϵ
01-00	3380	6	7	10	–	–
01-02	5536	6	7	11	–	–
02-00	3174	4	6	10	–	–
02-02	6383	6	7	10	–	–
02-03	3305	4	6	8	–	–
03-00	3030	4	6	8	–	–
03-01	6682	6	7	12	–	–
04-00	5472	73	144	198	1.93	32
04-01	2297	6	8	13	–	–
04-02	5939	33	117	274	3.21	27
04-03	5499	6	10	14	–	–
05-00	5444	12	22	100	2.35	18
05-01	2872	10	26	89	1.08	15
05-02	6620	6	6	10	–	–
05-03	4683	13	72	126	2.45	19
05-04	5684	13	68	164	3.18	19
06-00	5227	43	130	215	2.73	28
06-01	6408	125	280	344	3.29	35
06-02	5611	134	207	391	3.72	35
06-03	3617	19	77	79	1.21	18
07-00	3201	137	223	290	2.26	34
07-01	6389	125	171	289	3.04	34
07-02	6388	106	180	285	3.13	36
08-00	6365	35	51	100	1.63	17

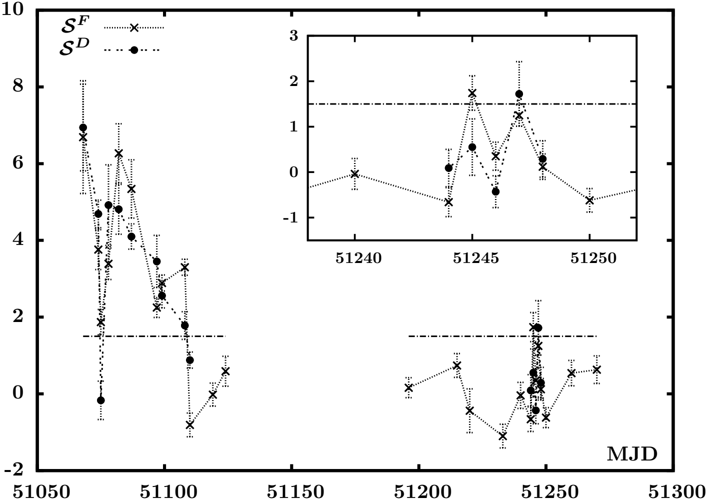
Notes. The prefix of the ObsID is 96420-01-. We used the RP parameters $m = 10, \Delta t = 7$s for every observation. The number of points N

NLD indicator during XTE J1550-564 outburst 1998/1999



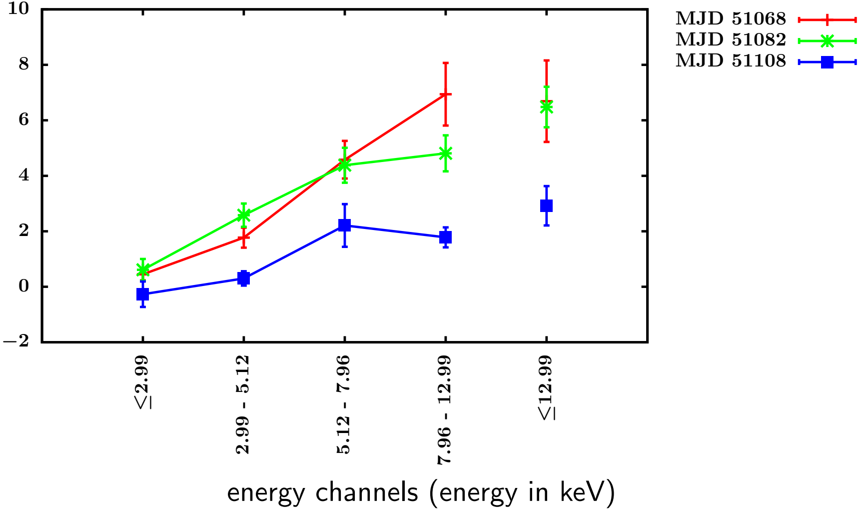
Gierliński & Done, 2003

NLD indicator during XTE J1550-564 outburst



Suková & Janiuk (2016) A&A, 591, A77

Spectral dependence of NLD



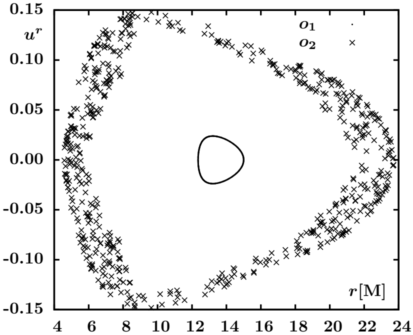
Outburst scenario

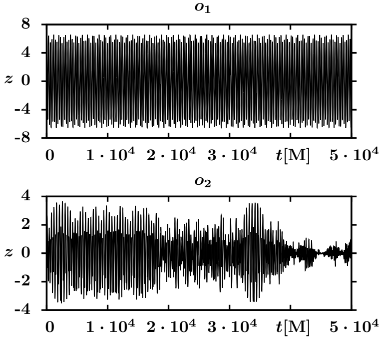
- `Wu et al., 2002; Kubota & Done, 2004`
- low angular momentum flow (LAF) released from magnetic trap near L1
- Keplerian flow through Roche lobe overflow
- LAF - radial velocity close to free fall far away from the center – electrons gain high energy ($\gtrapprox$ 100 keV) – inverse Compton scattering of radiation – fast rise of hard X-ray
- LAF meets the centrifugal barier close to BH – shock may form and oscillate – NLD high in high energy band
- Keplerian component slowly propagates inward (viscous time scale) – truncated disc can invoke/infer/reflect oscillations – slow rise of soft X-rays, NLD shifts to lower energies
- Keplerian disc reaches ISCO - stabilises – thermal emission, LAF component – oscillations ceases, depleted within days
- second luminosity rise – no clear signs of non-linear dynamics
- Other non-linear mechanisms possible

Conclusions

- We used the recurrence analysis with the X-ray light curves of microquasars during outburst
- We combined it with the surrogate data method and propose an algorithm for non-linear dynamics indicator (NLD)
- The method was build on the data from IGR J17091-3624 2011 outburst - especially the heart-beat state
- We tested the method with simulated chaotic data and we studied the dependence on parameters and presence of noise
- Some earlier results for GRS 1915+105 were confirmed
- Few examples for other sources (GX 339-4, GRO J1655-40)
- We studied the 1998/1999 outburst of XTE J1550-564 – nonlinear dynamics found in the early stage of the outburst in highest energy band (except of the strong flare), NLD decreases with time and shifts to lower energies

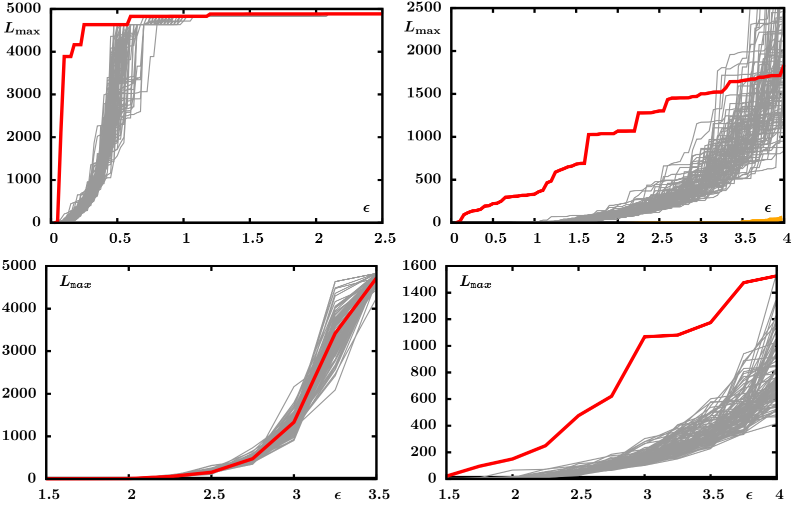
Testing the method on simulated data

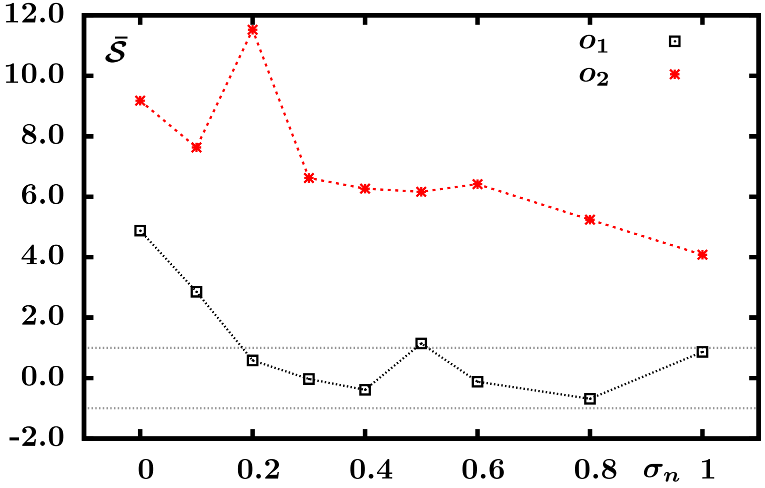




Suková & Janiuk (2017), Workshop Series of the Argentinian Astronomical Society, conference HEPRO V - 2015

Testing the method on simulated data




12.0
10.0
8.0
6.0
4.0
2.0
0.0
-2.0
$\bar{\mathcal{S}}$
o_1
o_2
0
0.2
0.4
0.6
0.8
σ_n
1