

# Compressed sensing and radio interferometers to detect transients

université



**Julien Girard**  
AIM/IRFU/CEA-Saclay  
Université Paris Diderot



Microquasars workshop *RHODES UNIVERSITY*  
*Where leaders learn*



# **Outline**

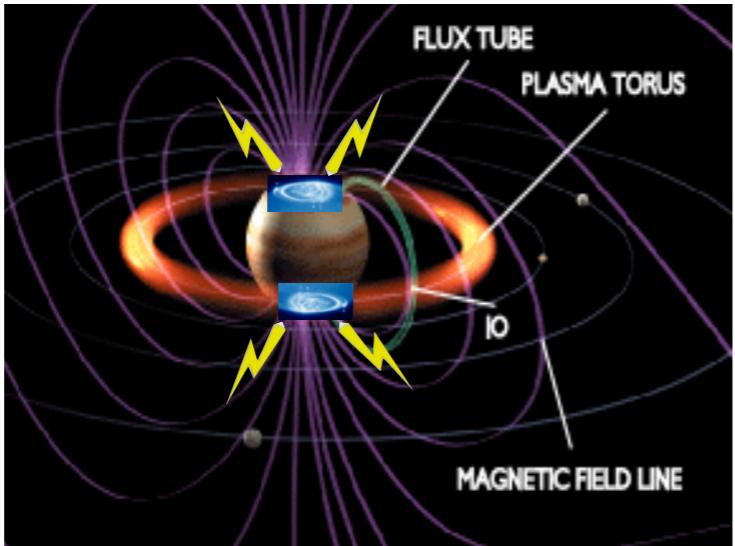
**New instrumentation, new problems  
(and new solutions !)**

**Sparsity and deconvolution**

**Application to transient imaging**

**Conclusions**

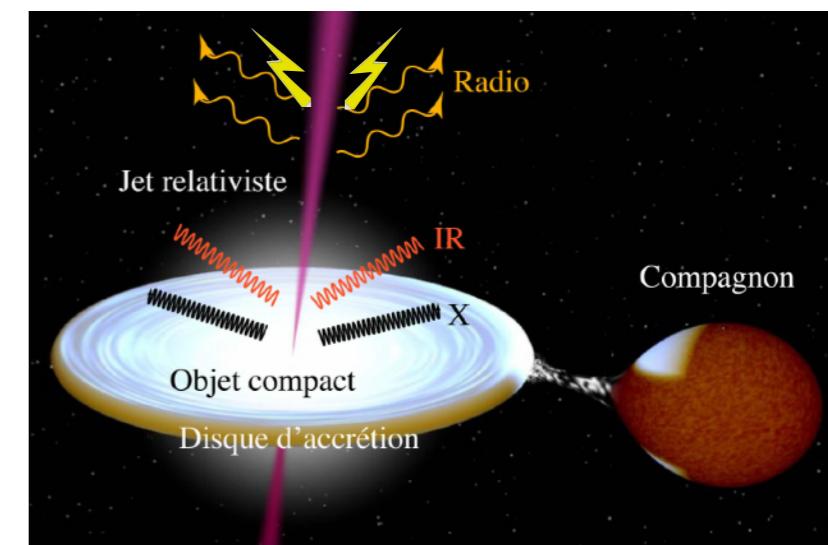
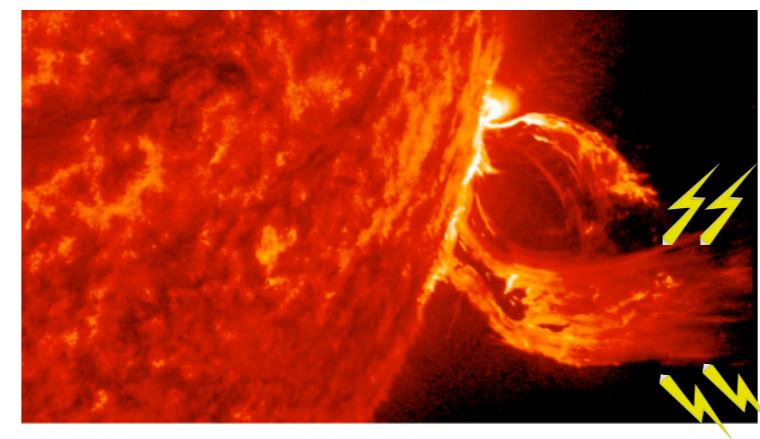
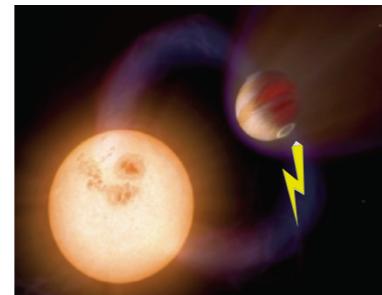
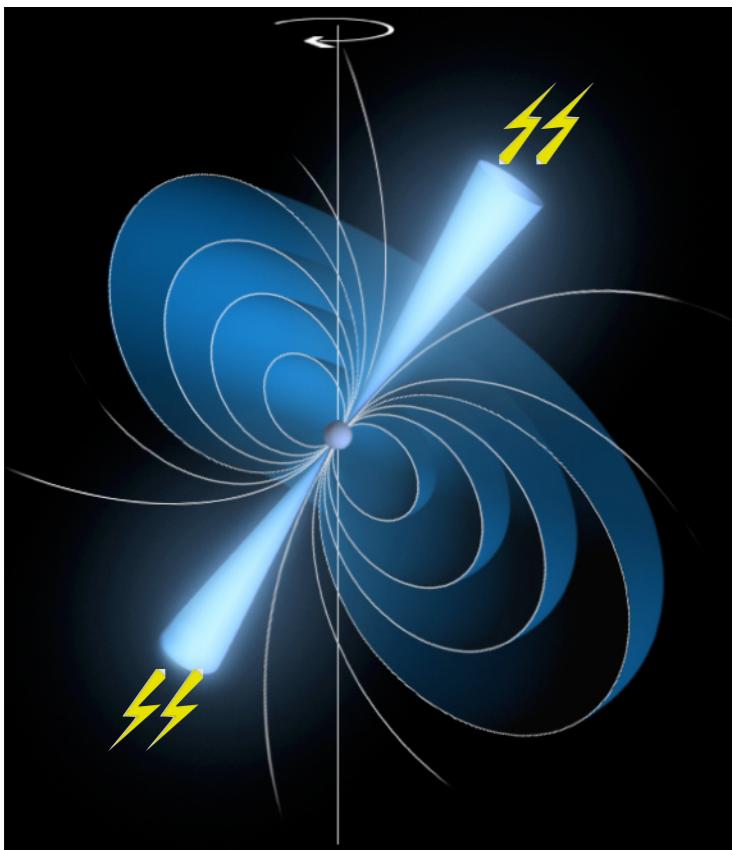
# Universality of transients in radio



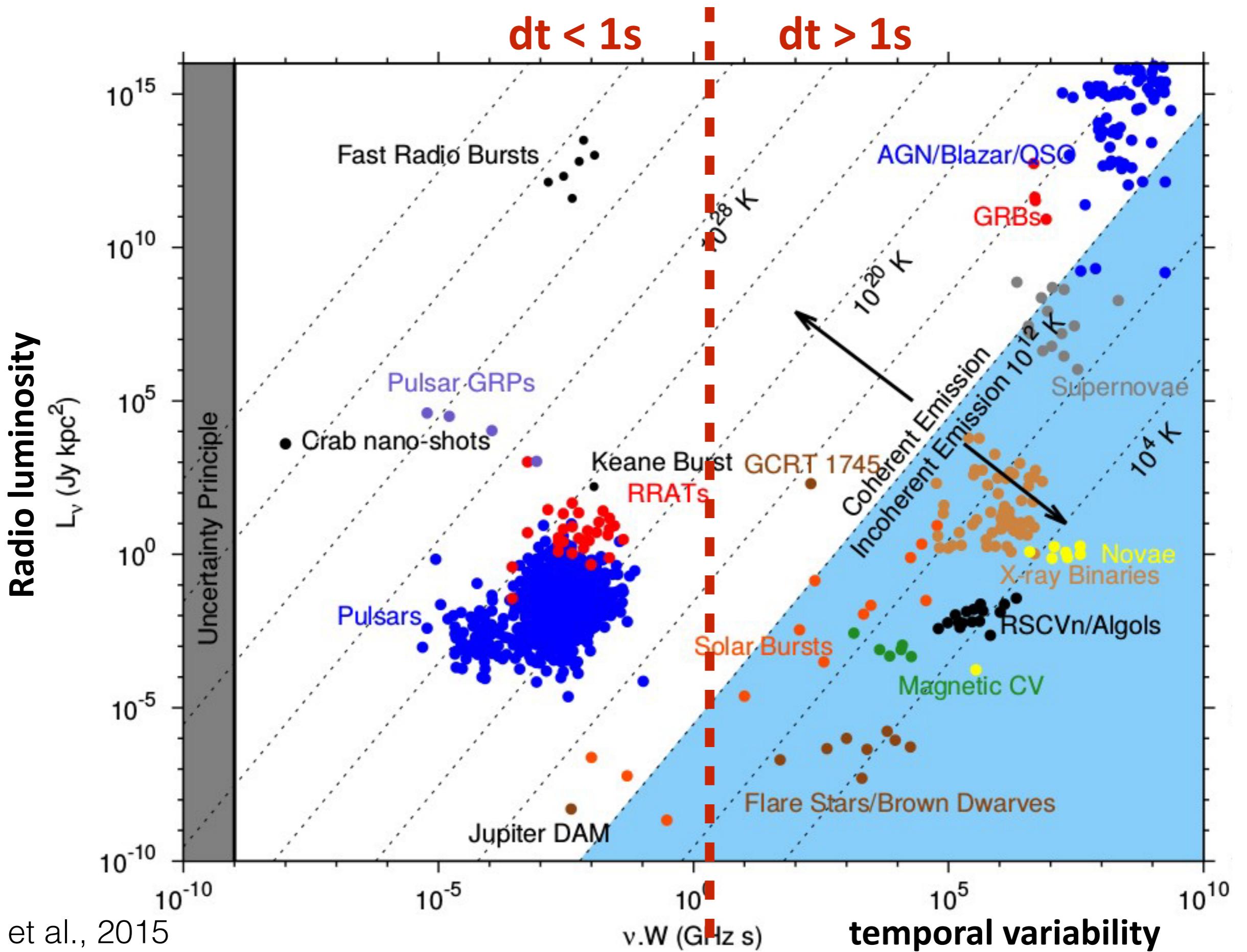
## Radio signatures

- mark the presence of magnetic fields
- have a rich spectral and temporal features
- associated to catastrophic events

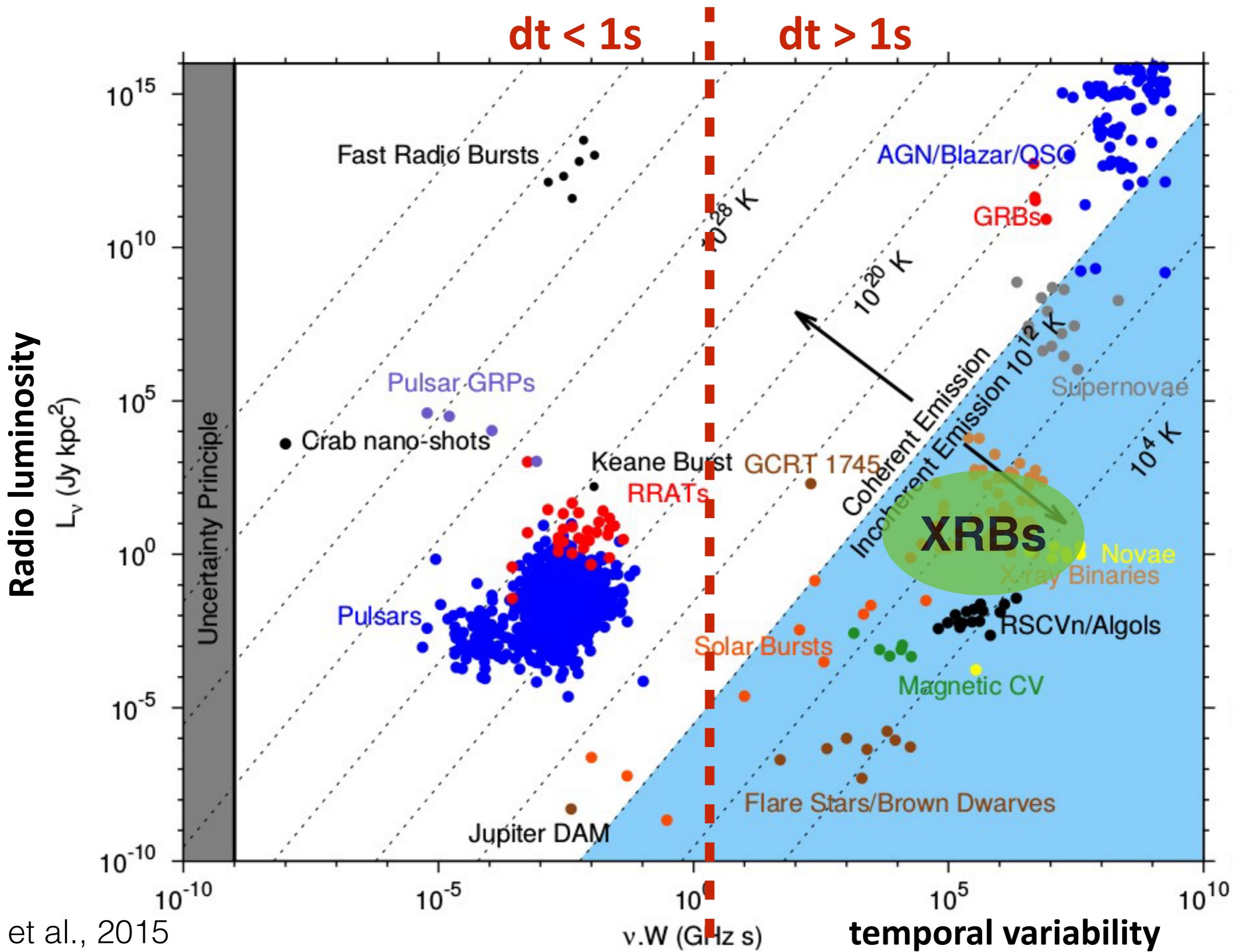
at all scales in energy, distances, durations...



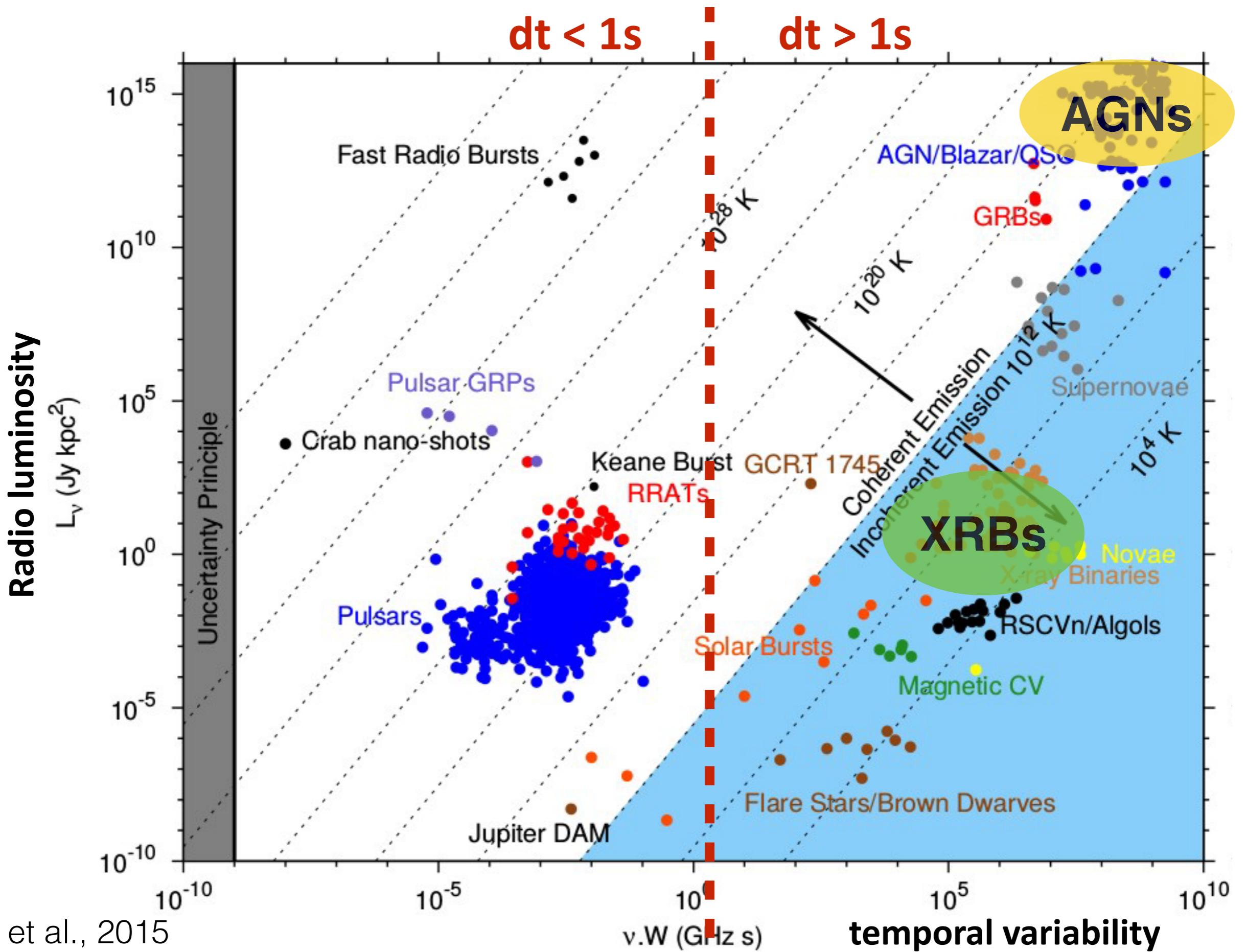
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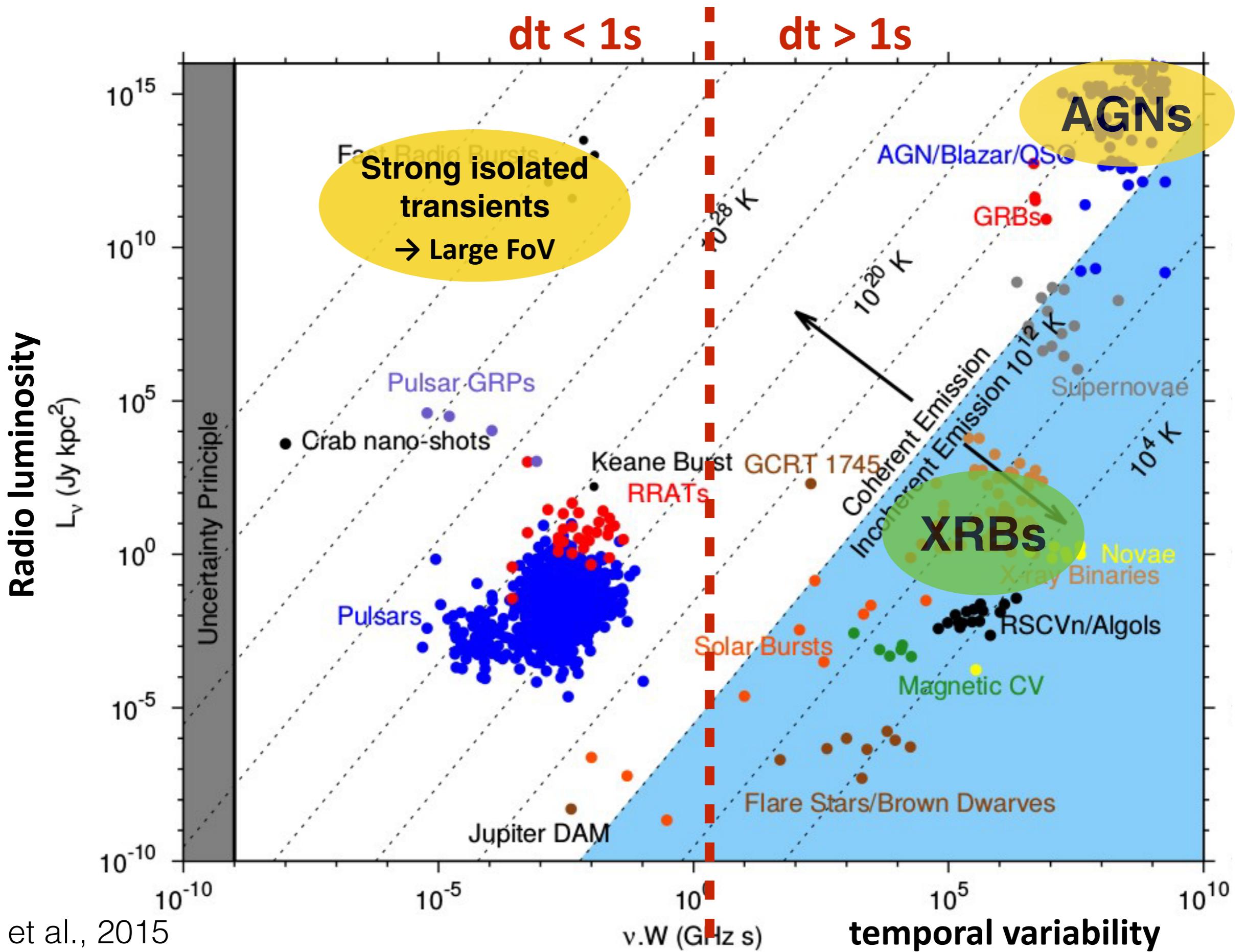
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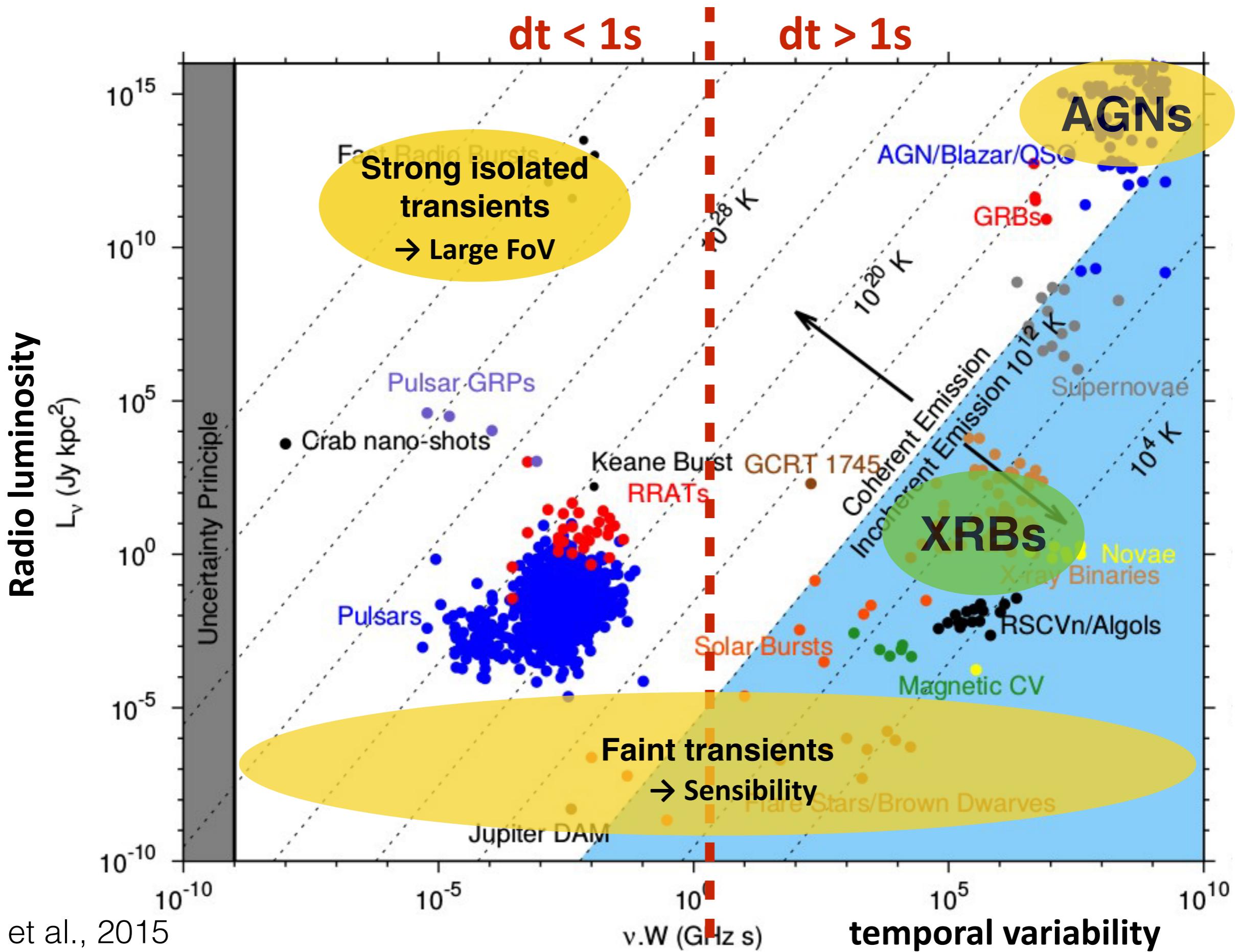
# Universality of transients in radio



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# Continental-scale instruments

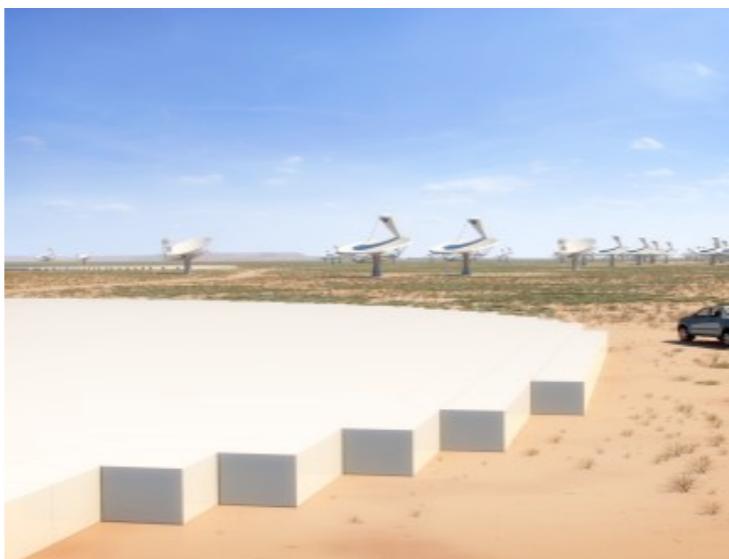
(see Stephane's review)



- Full polarimetric measurement
- Large field of views  $10s^\circ$
- High sensitivity  $\mu Jy$
- High time resolution  $ns$   
spectral resolution  $< kHz$   
angular resolution  $< arcsec$

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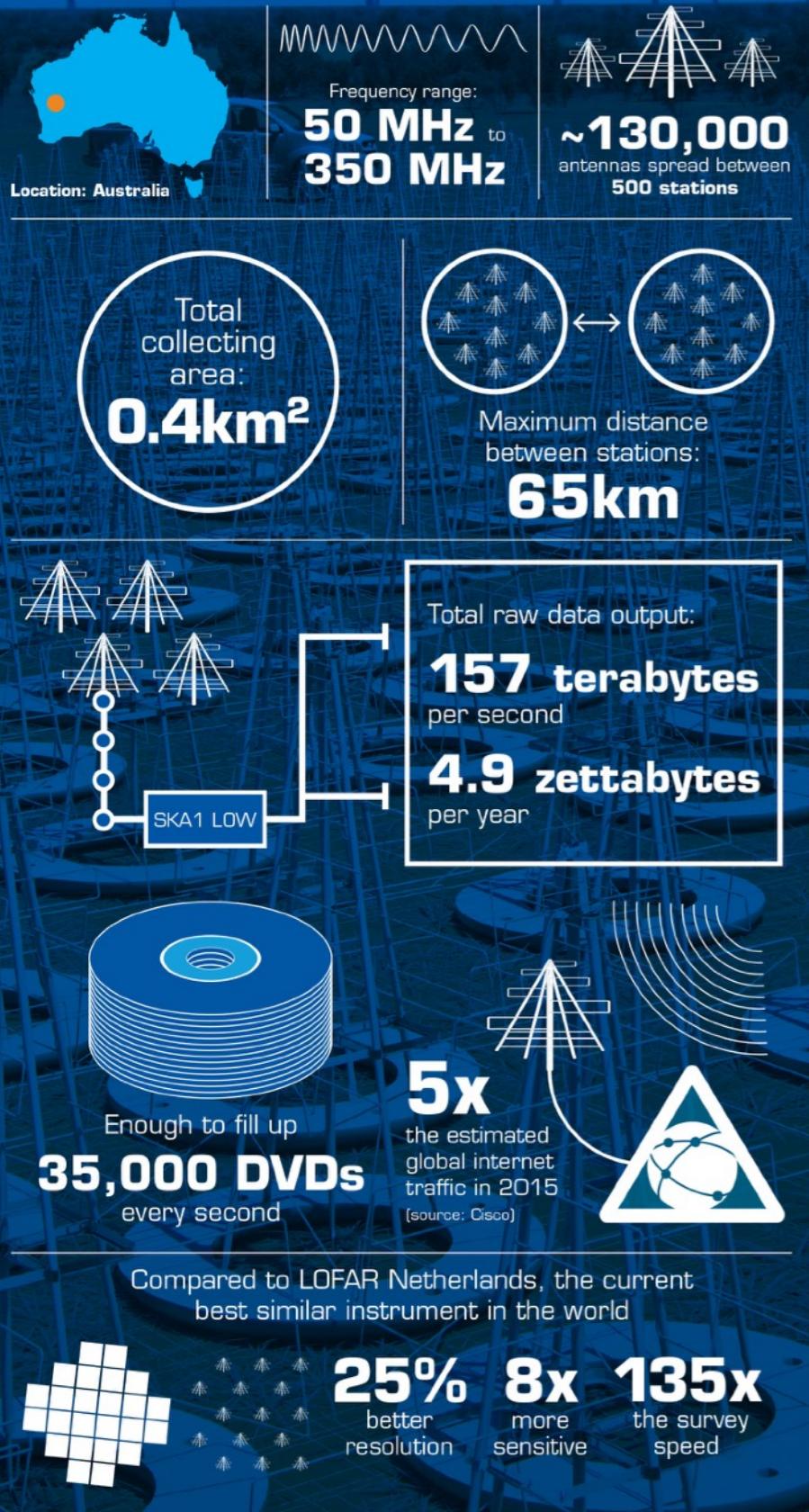
$< arcsec$



**Logistical complexity**  
**Increased data rates**  
**Data proc complexity**  
**Instrumental effects**

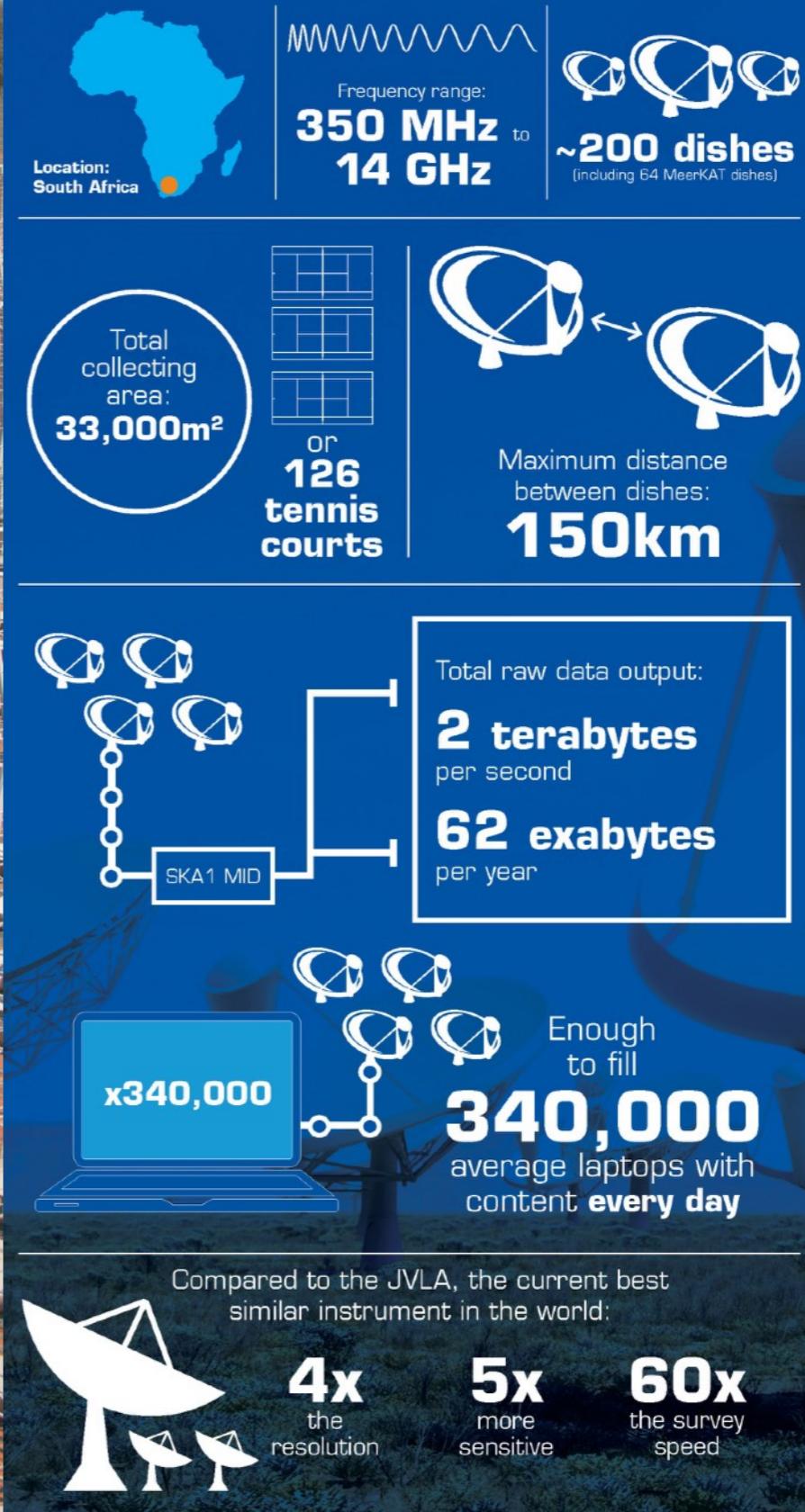
## SKA1 LOW - the SKA's low-frequency instrument

The Square Kilometre Array (SKA) will be the world's largest radio telescope, revolutionising our understanding of the Universe. The SKA will be built in two phases - SKA1 and SKA2 - starting in 2018, with SKA1 representing a fraction of the full SKA. SKA1 will include two instruments - SKA1 MID and SKA1 LOW - observing the Universe at different frequencies.



## SKA1 MID - the SKA's mid-frequency instrument

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Astronomers ideally want the whole sky at 0.001 nsec x 0.001 Hz  
x 0.000000001" resolution  
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## Impact on transient search

### *Slow transients*

Store long term light curves / reduced data  
to allow revisiting the data  
(e.g. FRBs...)

### *Fast transients*

Trade-off between SNR and time  
smearing of transient events  
(e.g. LOFAR TraP)

Good spectral coverage

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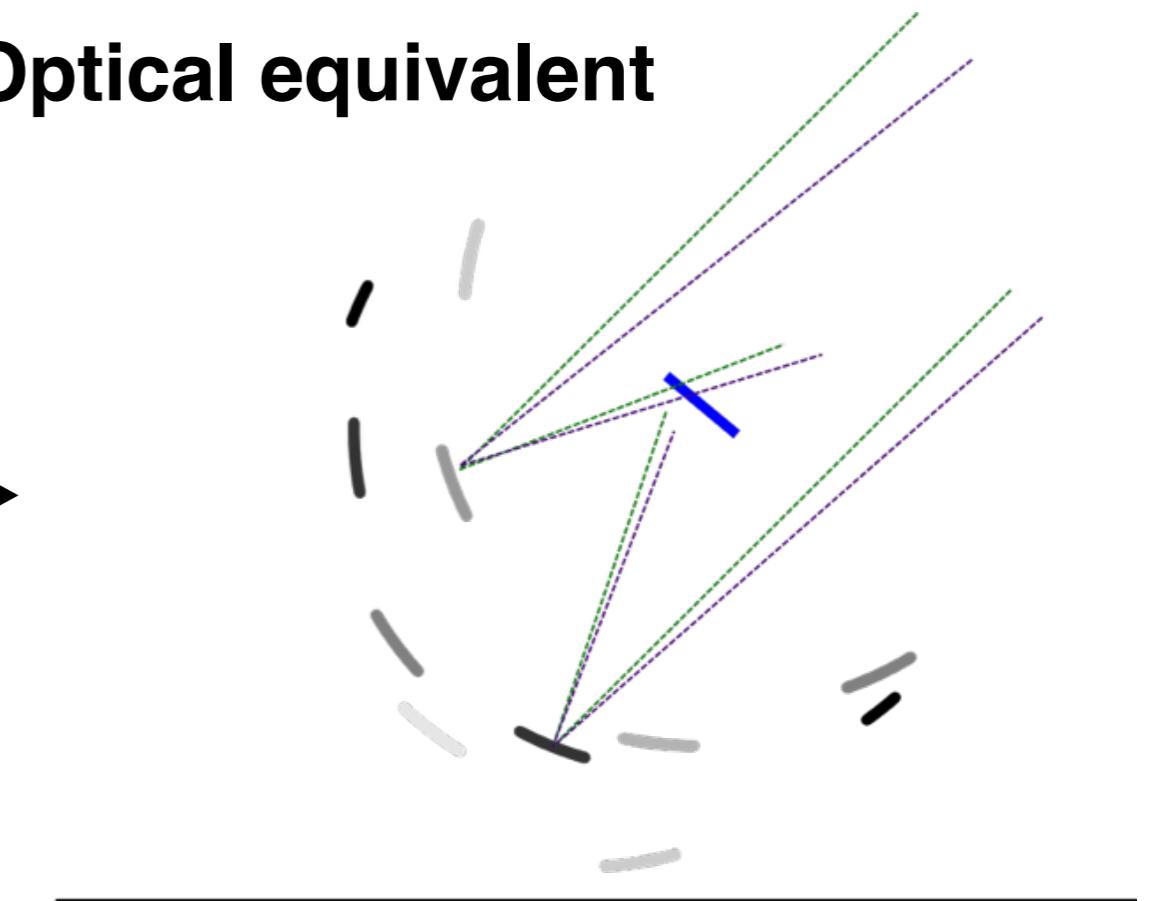
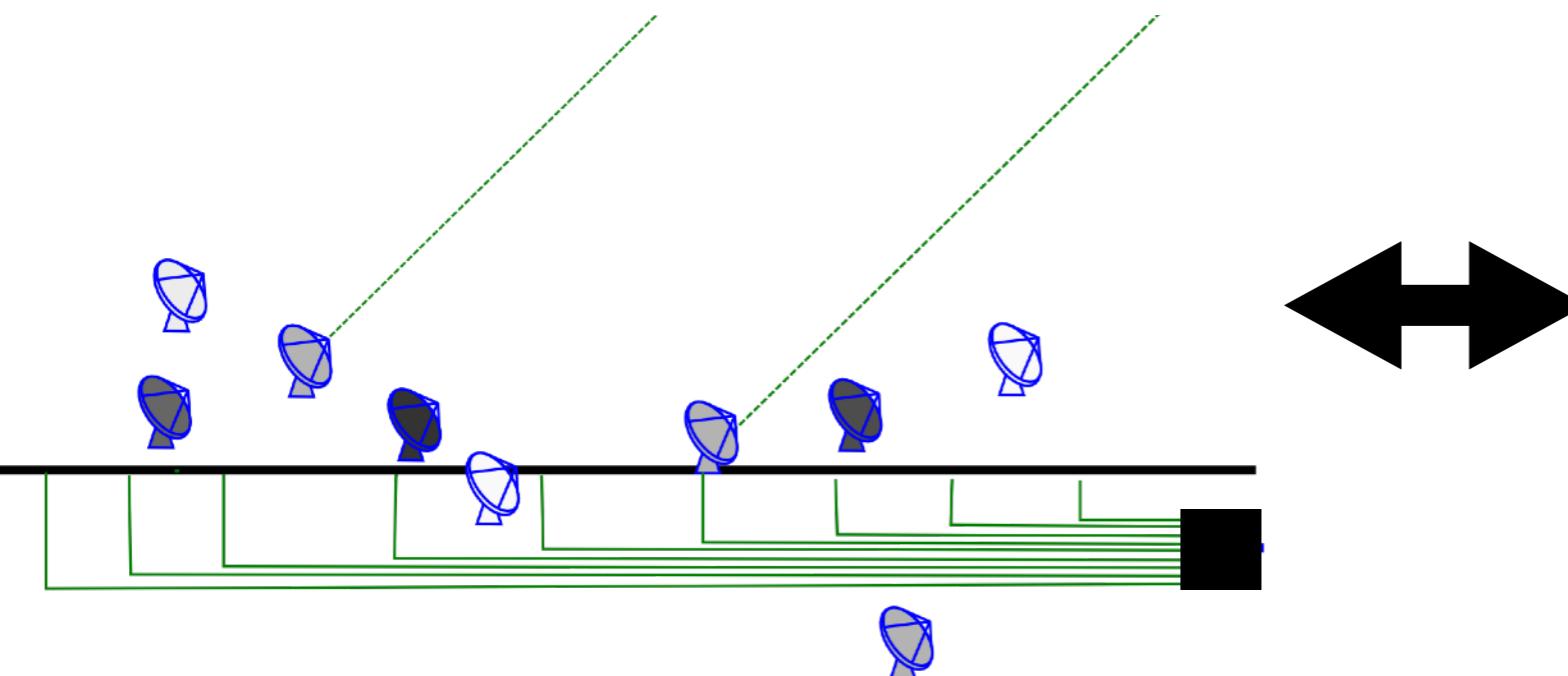
Most transient emissions are not resolved in radio so  
... Why should I care?...

# Uncalibrated instrument



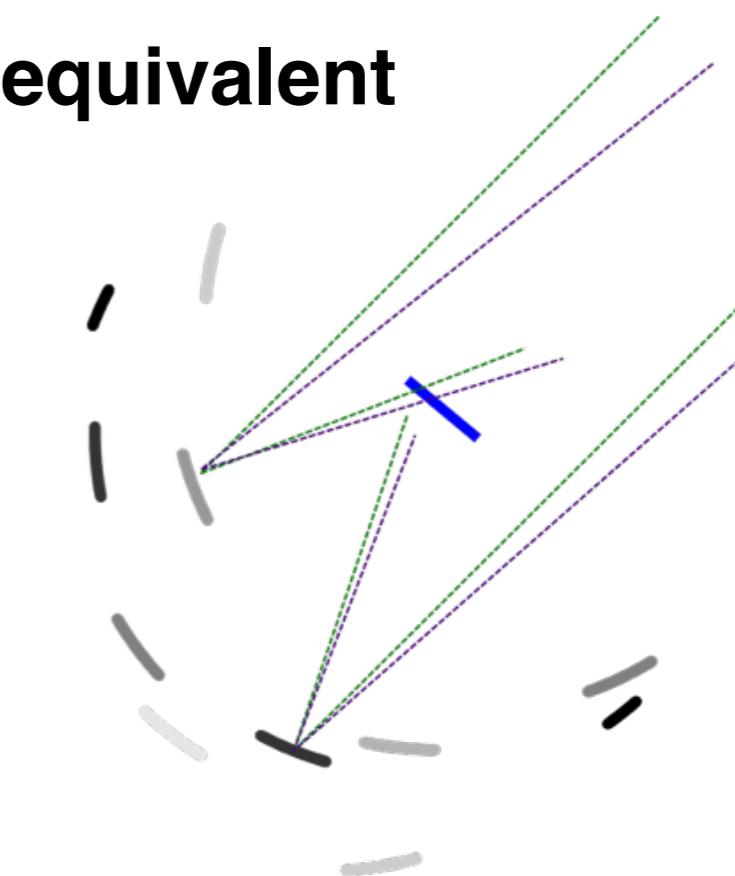
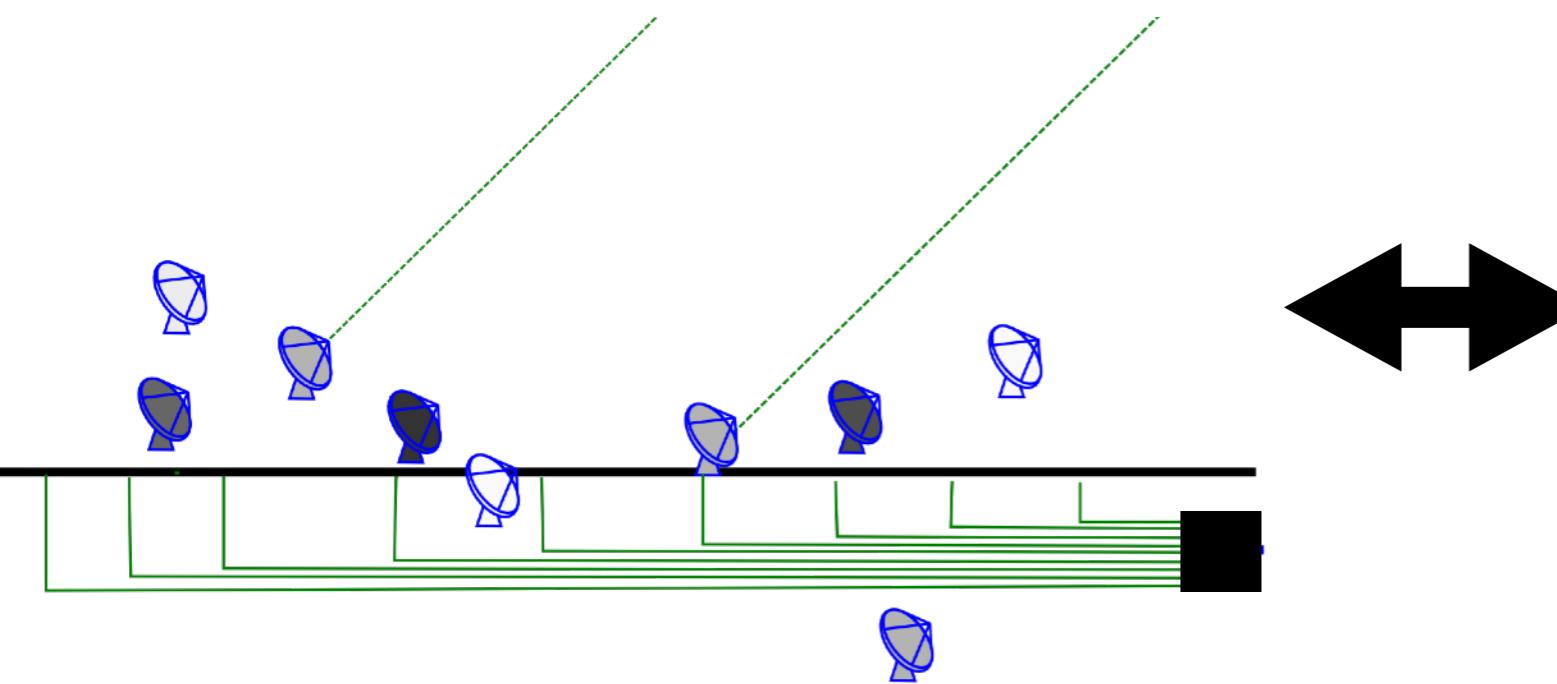
# Uncalibrated instrument

# Optical equivalent



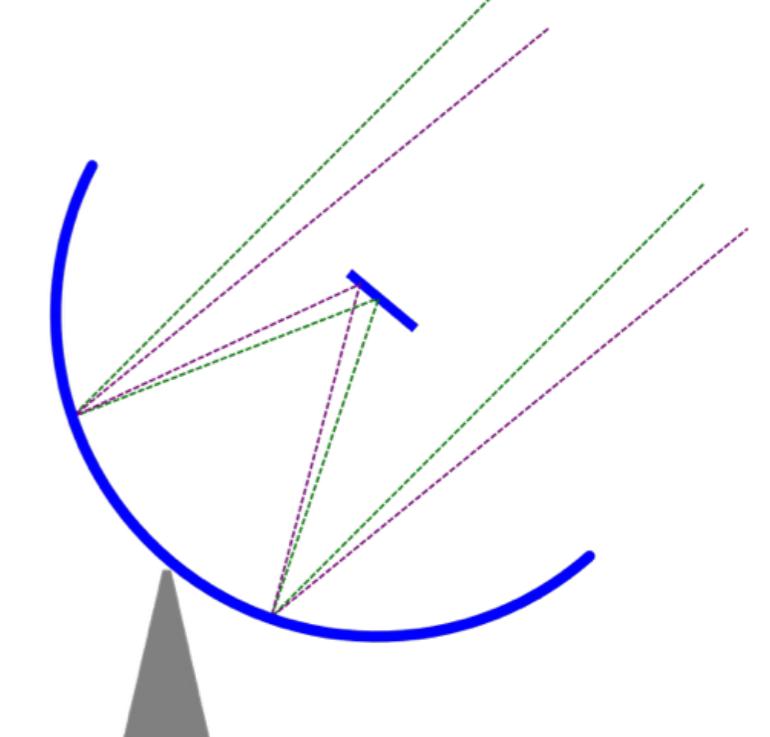
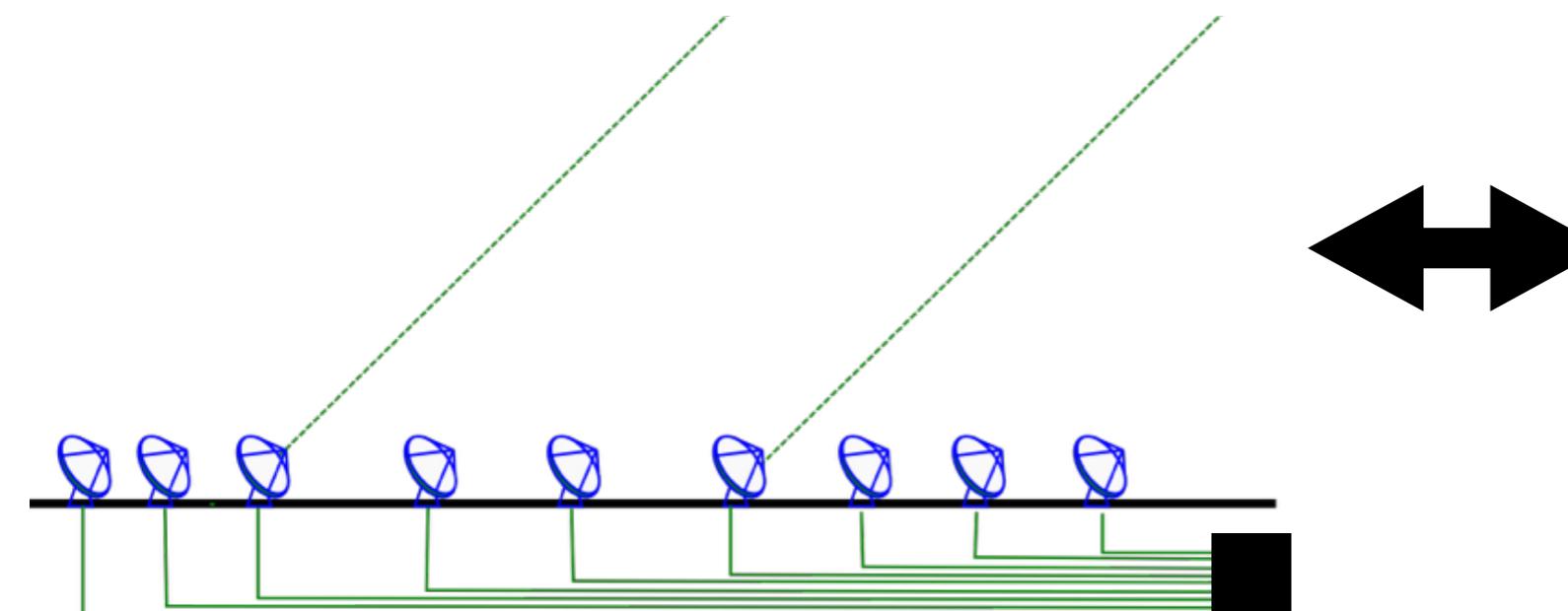
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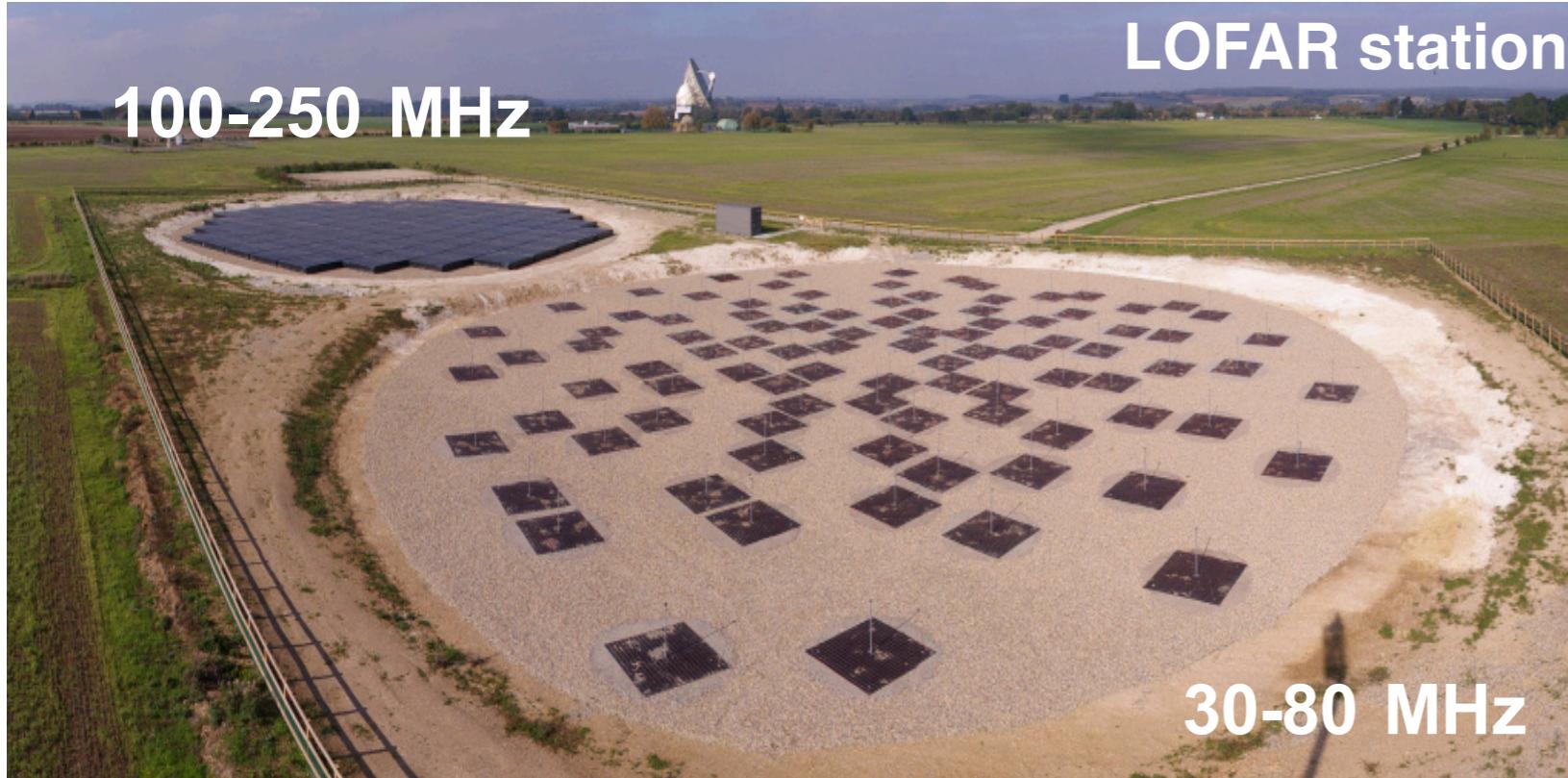


Calibration of a radio  
interferometer

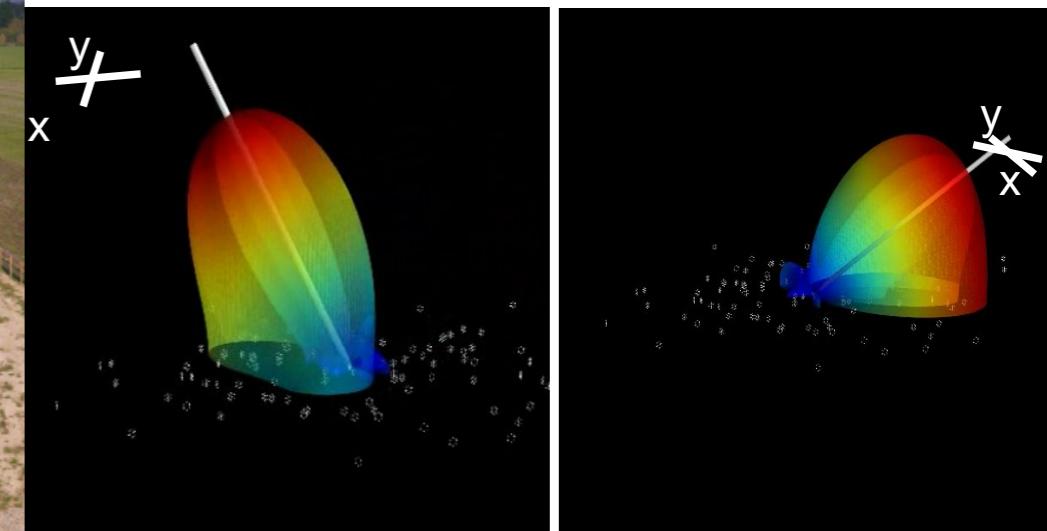
=  
Post-processing  
« adaptive optics »



# Some direction-dependent effects



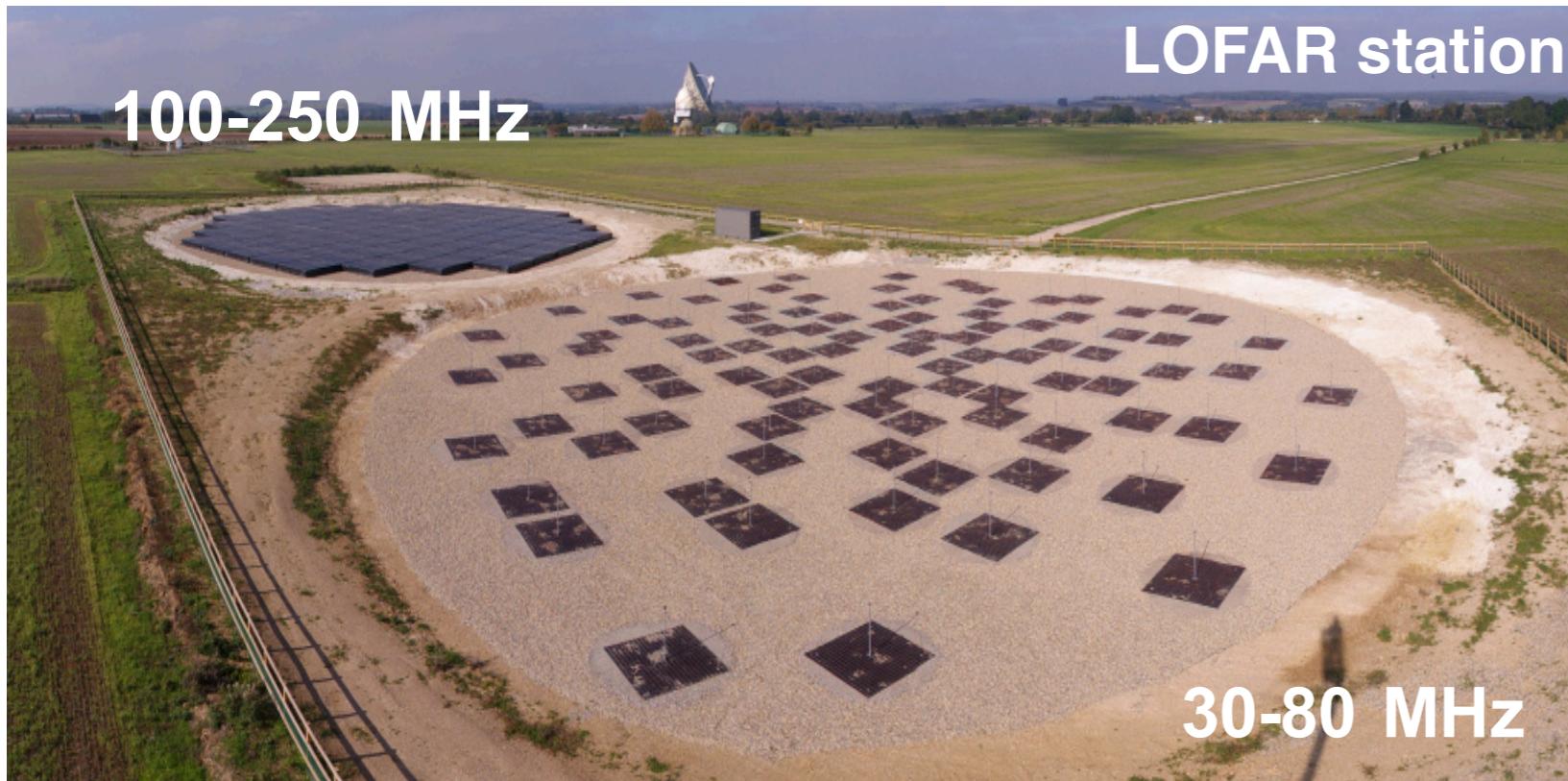
Antenna beam



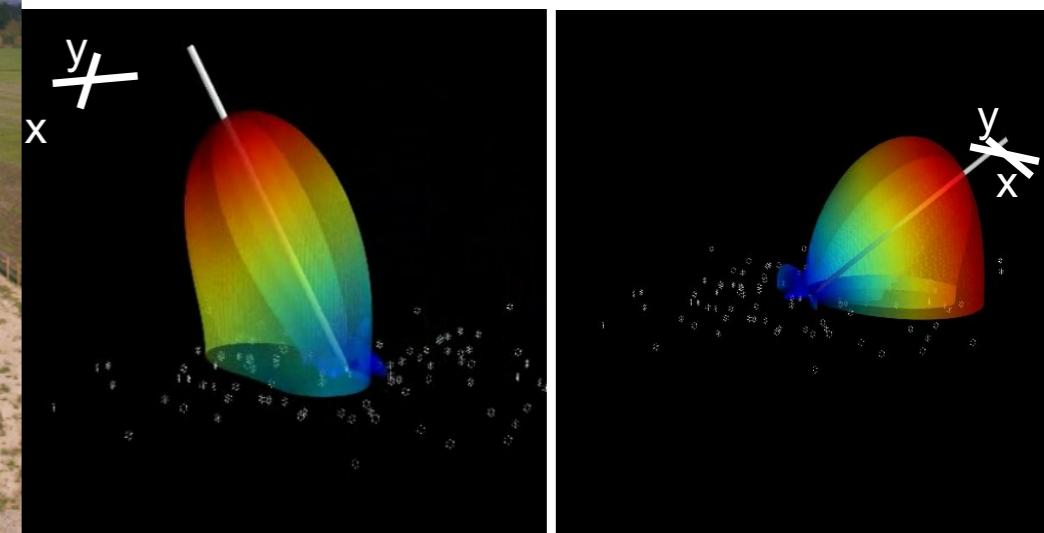
**Direction 1  
affects gain**

**Direction 2  
polarization**

# Some direction-dependent effects



LOFAR station

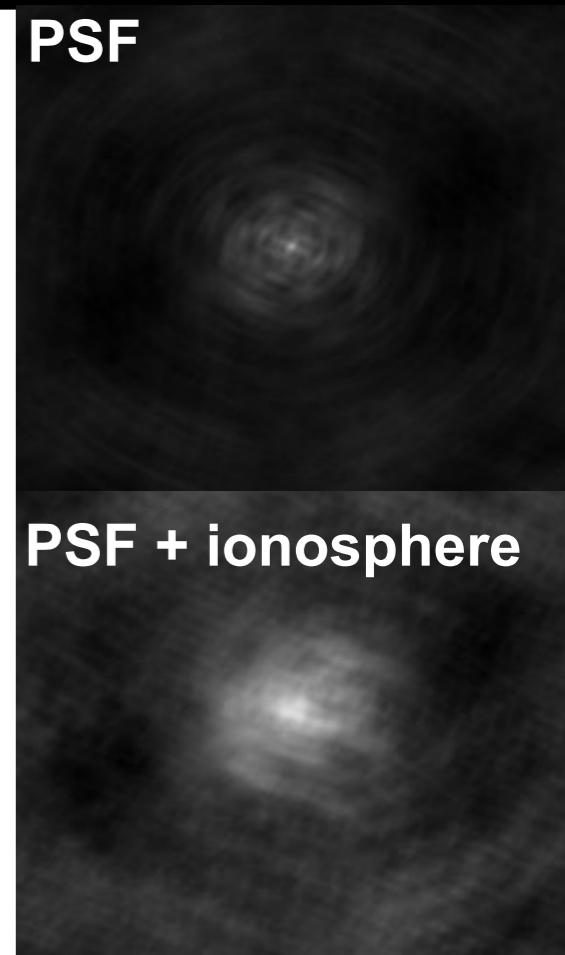
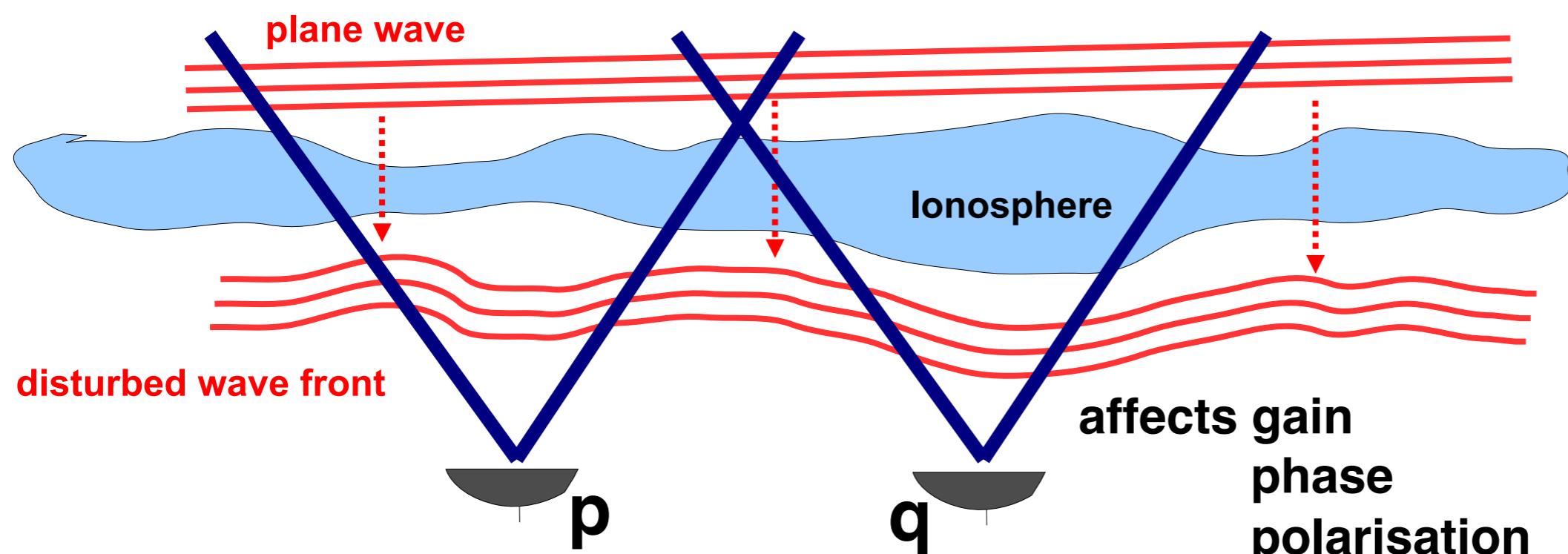


Direction 1  
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Direction 2  
polarization

Antenna beam

## Ionospheric/Atmospheric disturbance

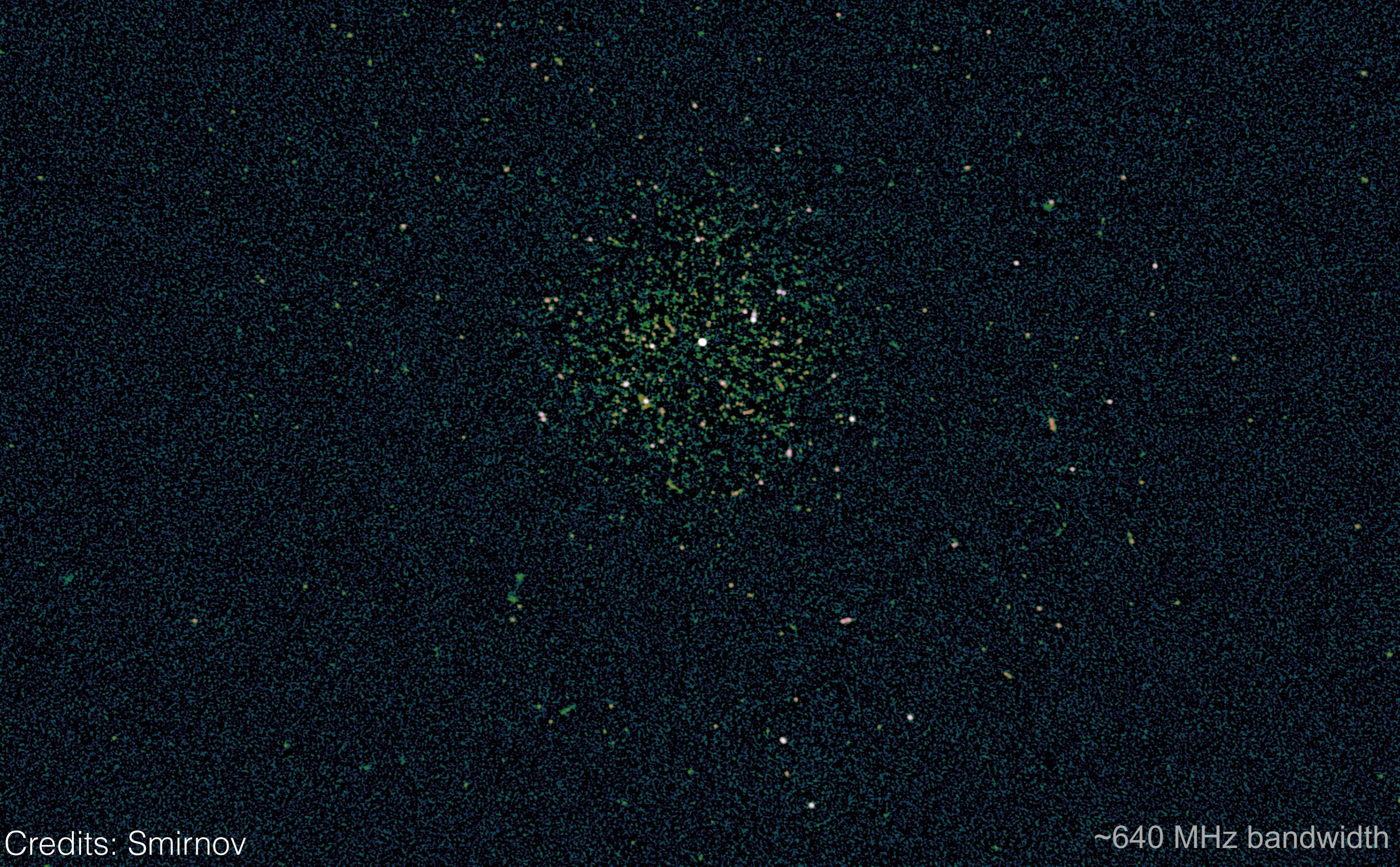


# 3C197

JVLA (2014 image)

1.4 GHz

22.82 Jy peak  
4.5 uJy noise  
5 million DR  
confusion limited



Credits: Smirnov

~640 MHz bandwidth

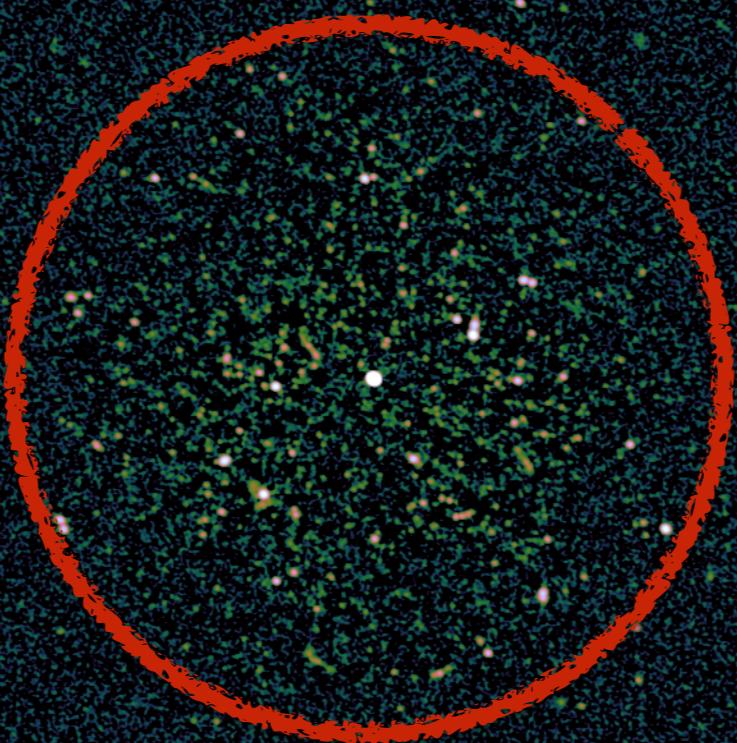
**3C197**

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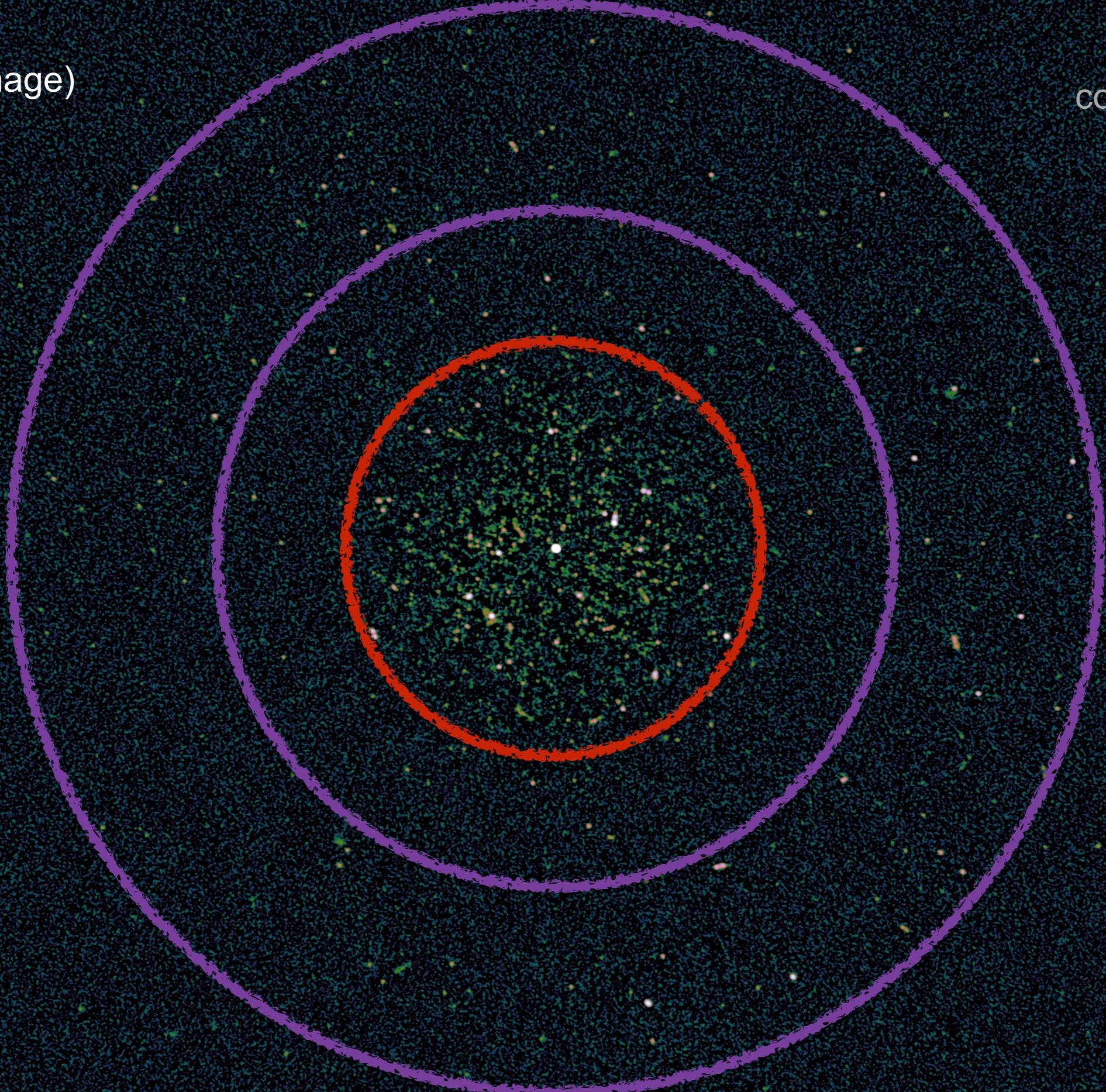
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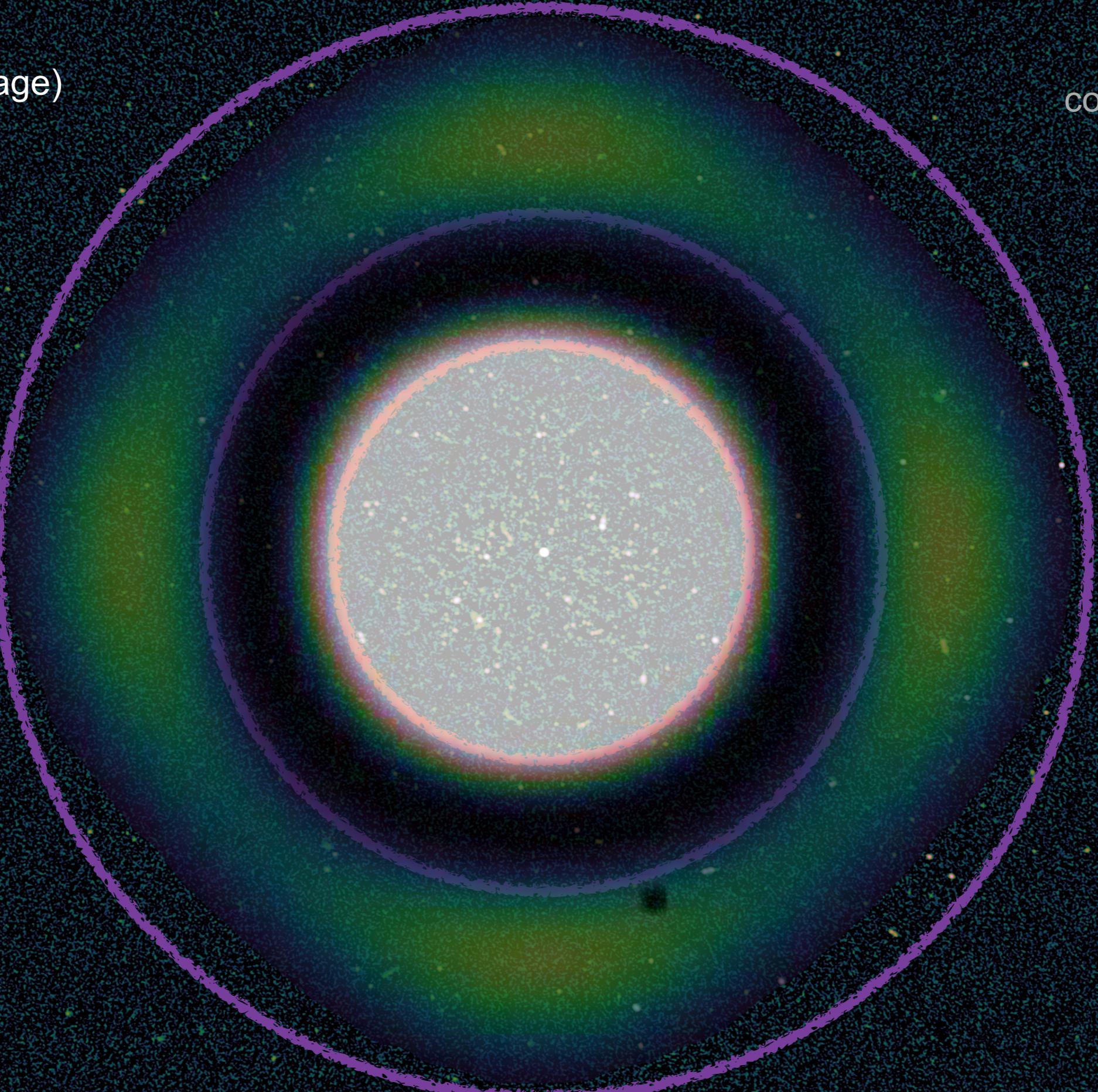
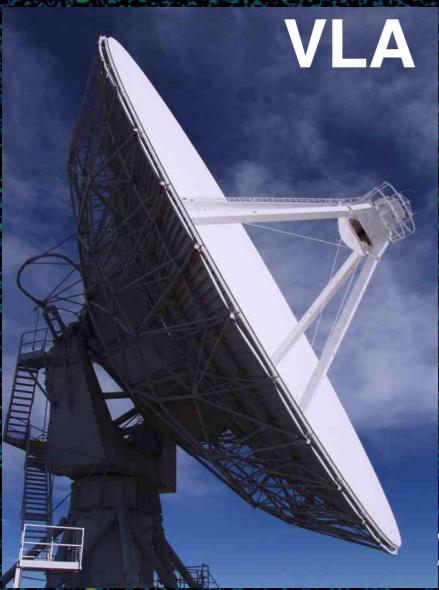
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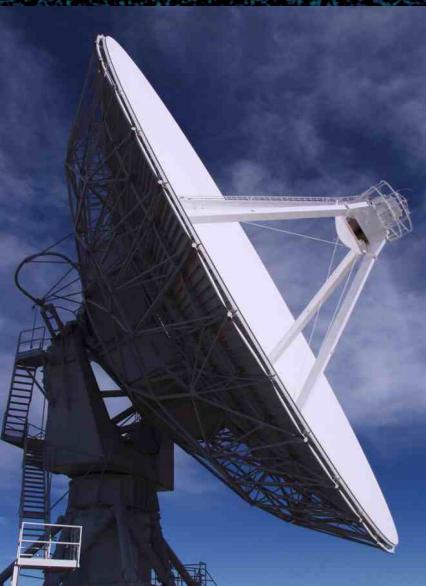
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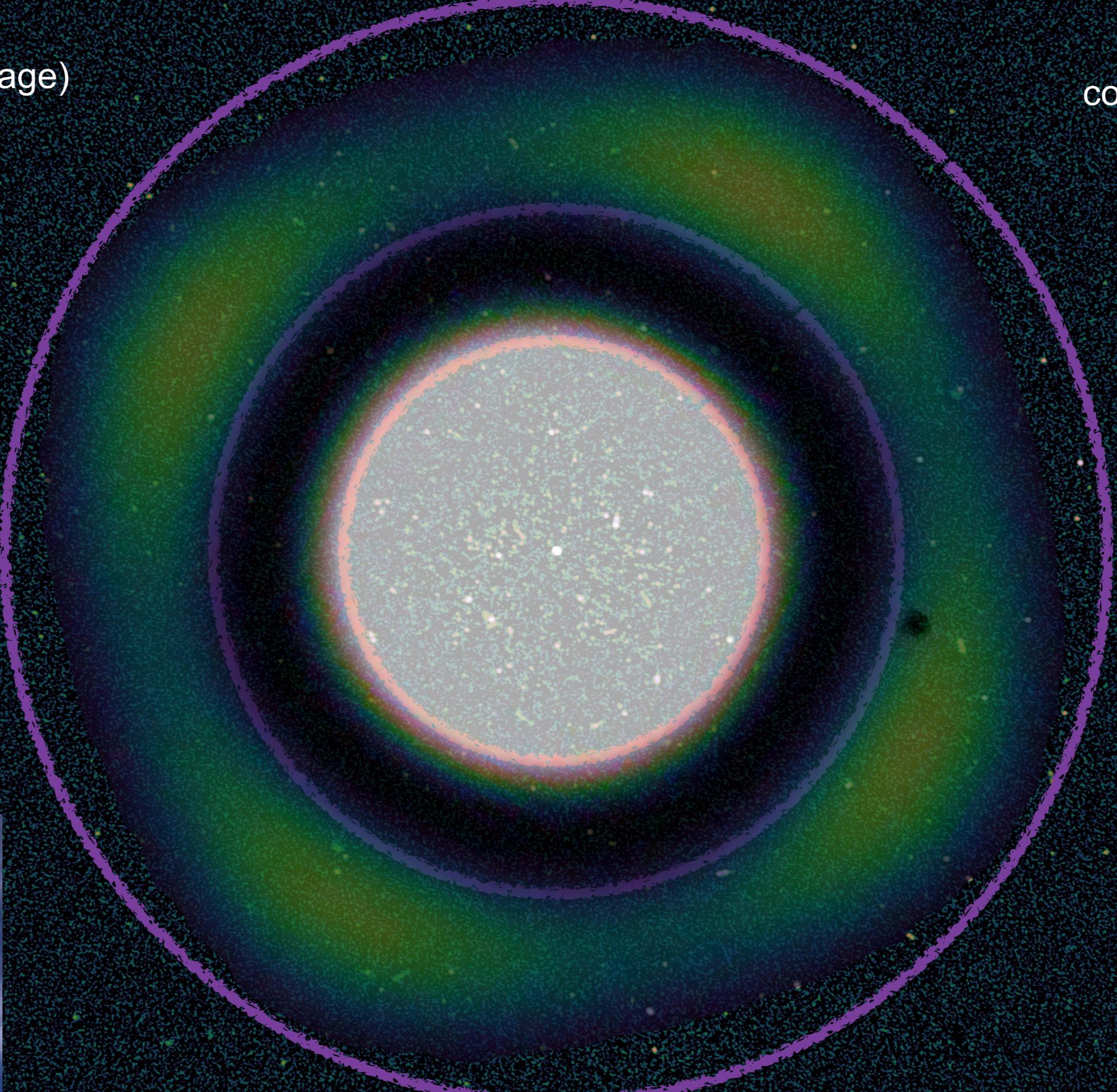
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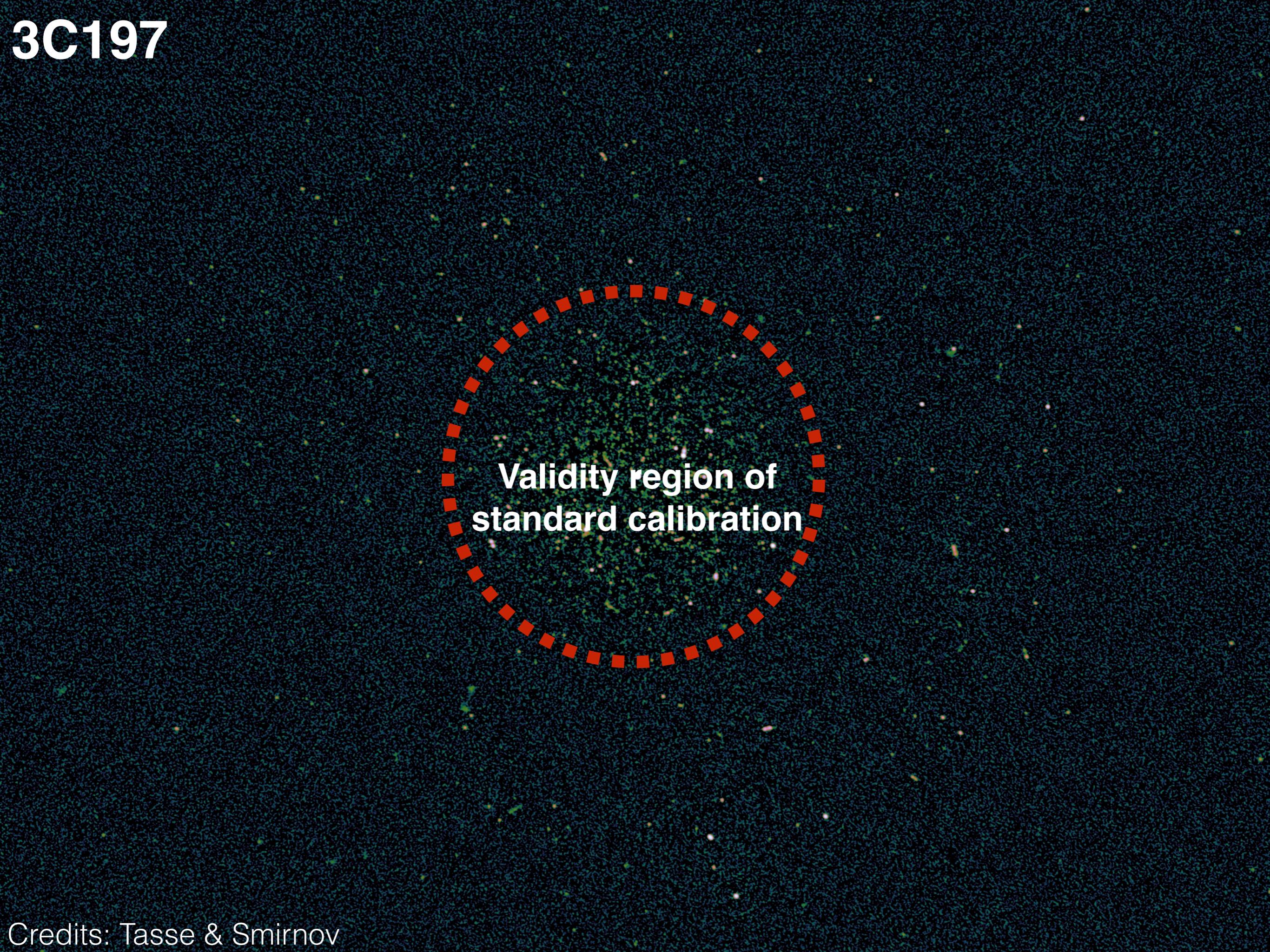
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Iheanetu, Girard et al., in prep

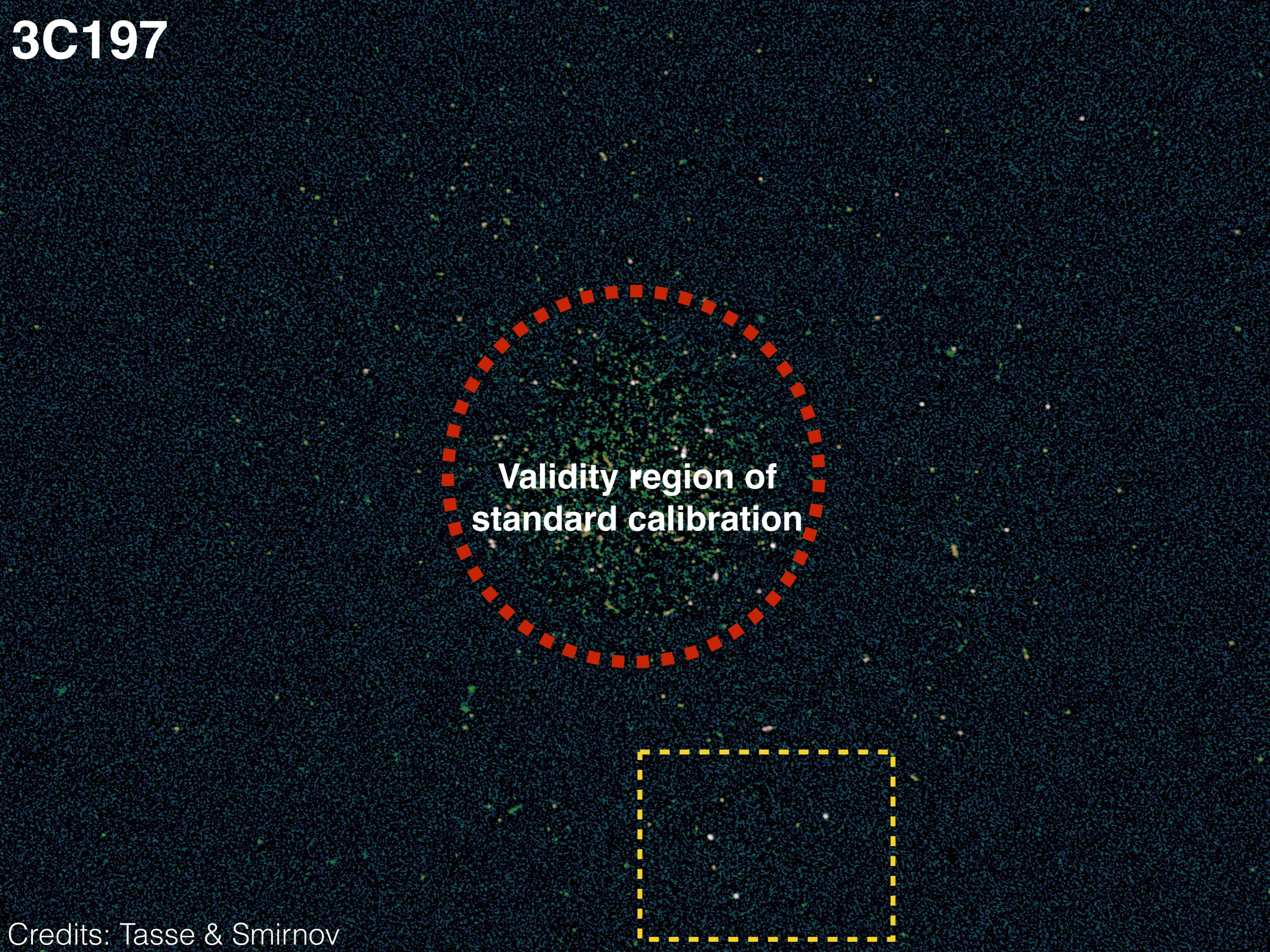
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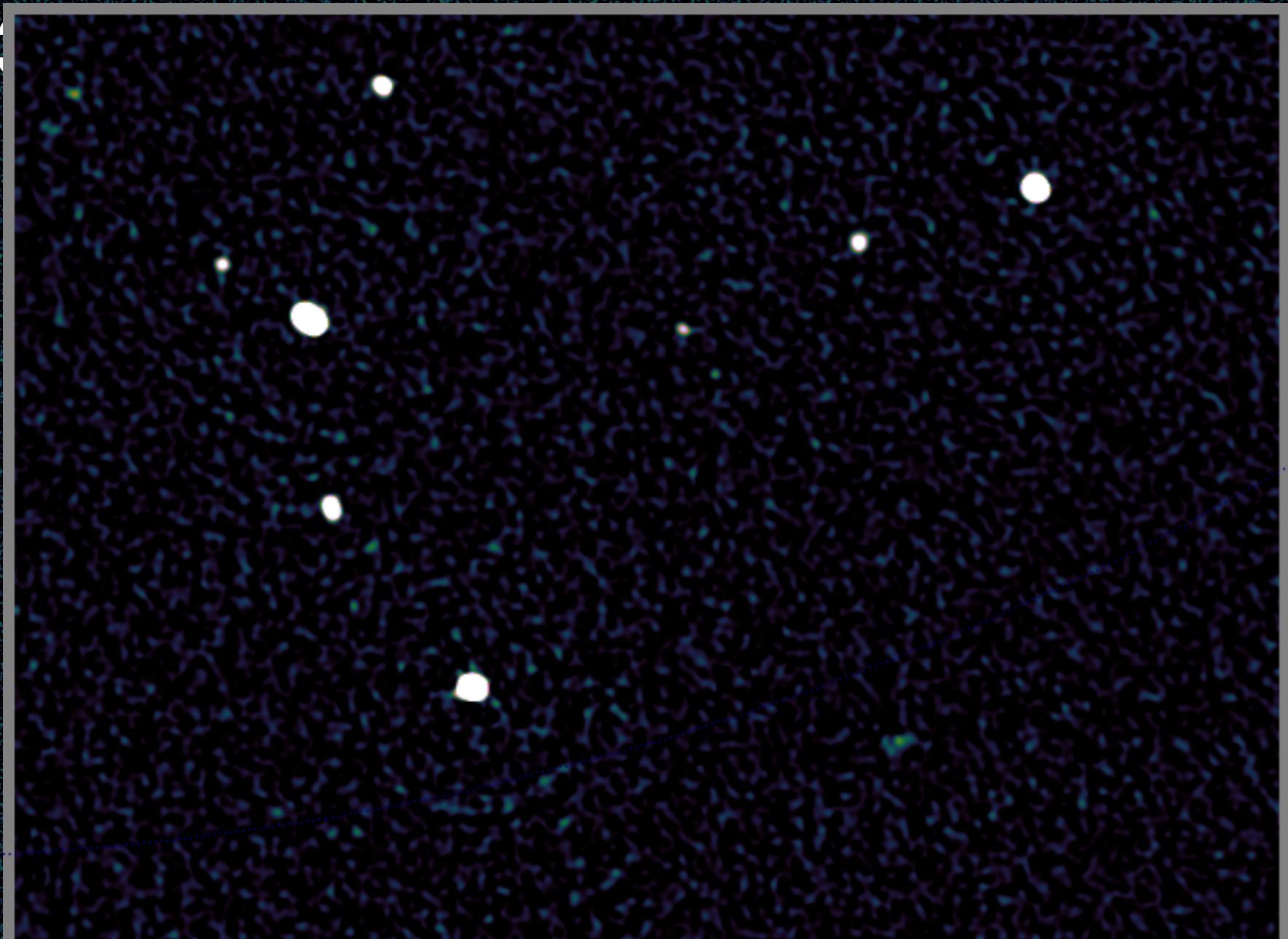


Validity region of  
standard calibration

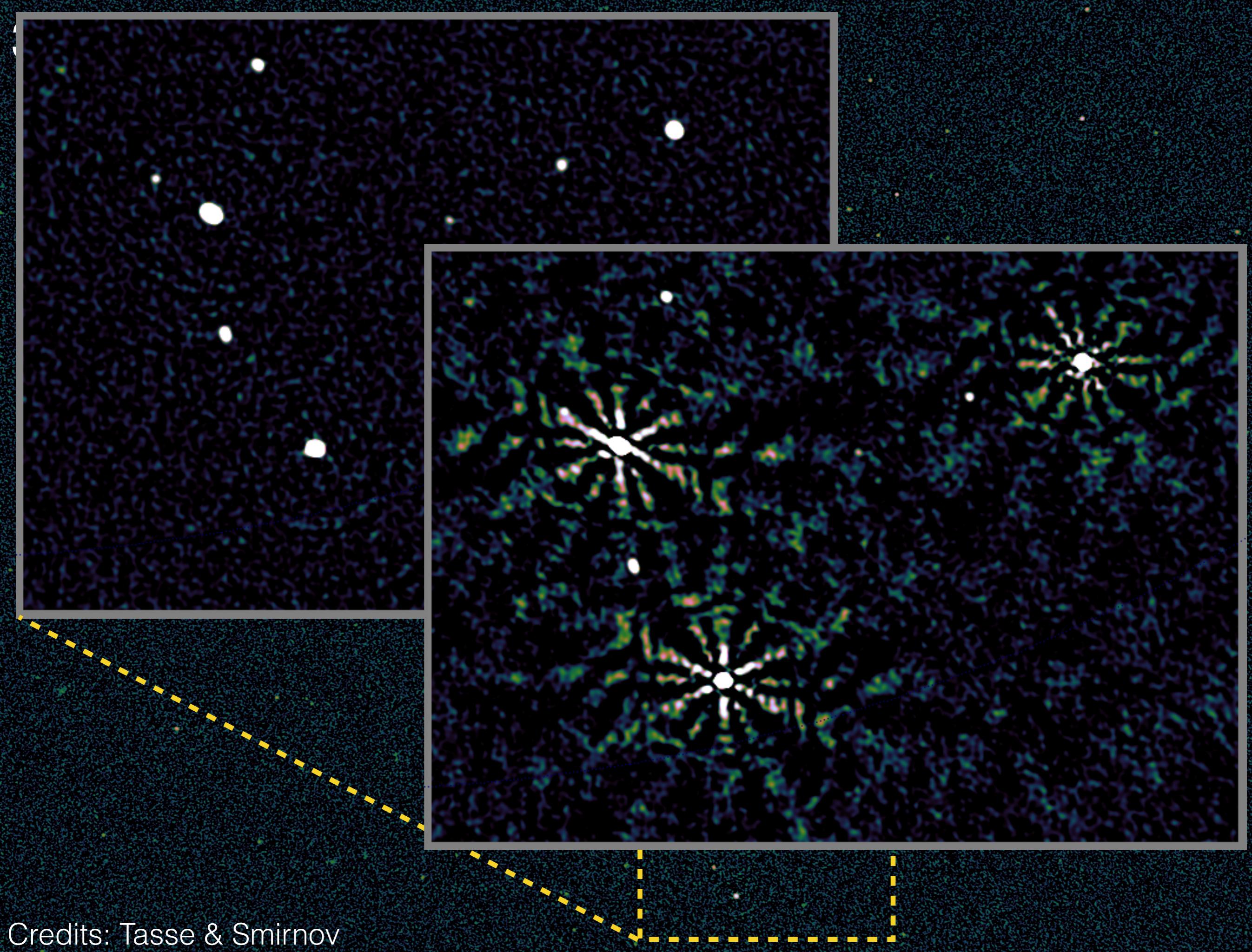
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Credits: Tasse & Smirnov



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## Improper calibration and deconvolution

- 1) Increases local and global noise
- 2) Fake variables sources
- 3) Fake detection of new sources

Hopefully, solutions exists : The RIME(Smirnov 2011)  
DDFacet (Tasse et al. 2017?)

# **Outline**

**New instrumentation, new problems  
(and new solutions !)**

**Sparsity and deconvolution**

**Application to transient imaging**

**Conclusions**



# Compressed Sensing



- \* E. Candès and T. Tao, “Near Optimal Signal Recovery From Random Projections: Universal Encoding Strategies?”, IEEE Trans. on Information Theory, 52, pp 5406–5425, 2006.
- \* D. Donoho, “Compressed Sensing”, IEEE Trans. on Information Theory, 52(4), pp. 1289–1306, April 2006.
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## A recent sampling theorem

**“Signals with exactly K components different from zero can be recovered perfectly from  $\sim K \log N$  incoherent measurements”**

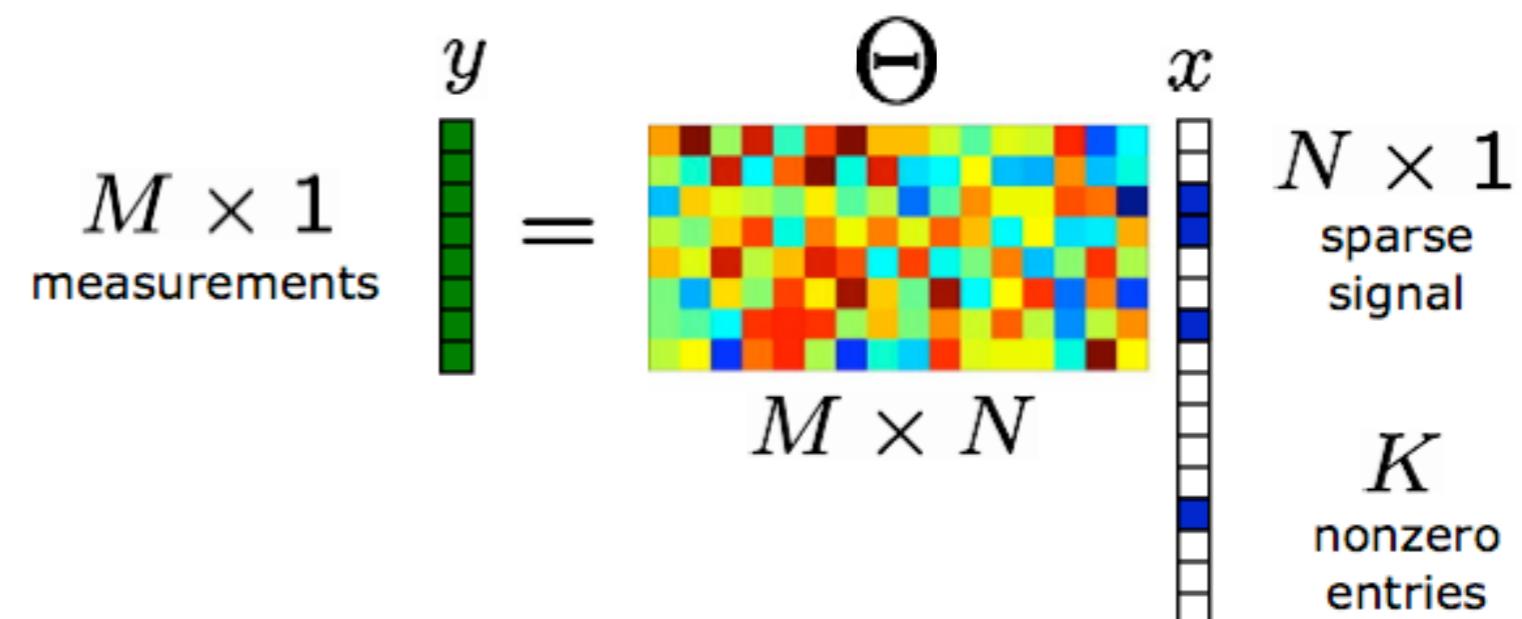
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$$y = \Theta x$$



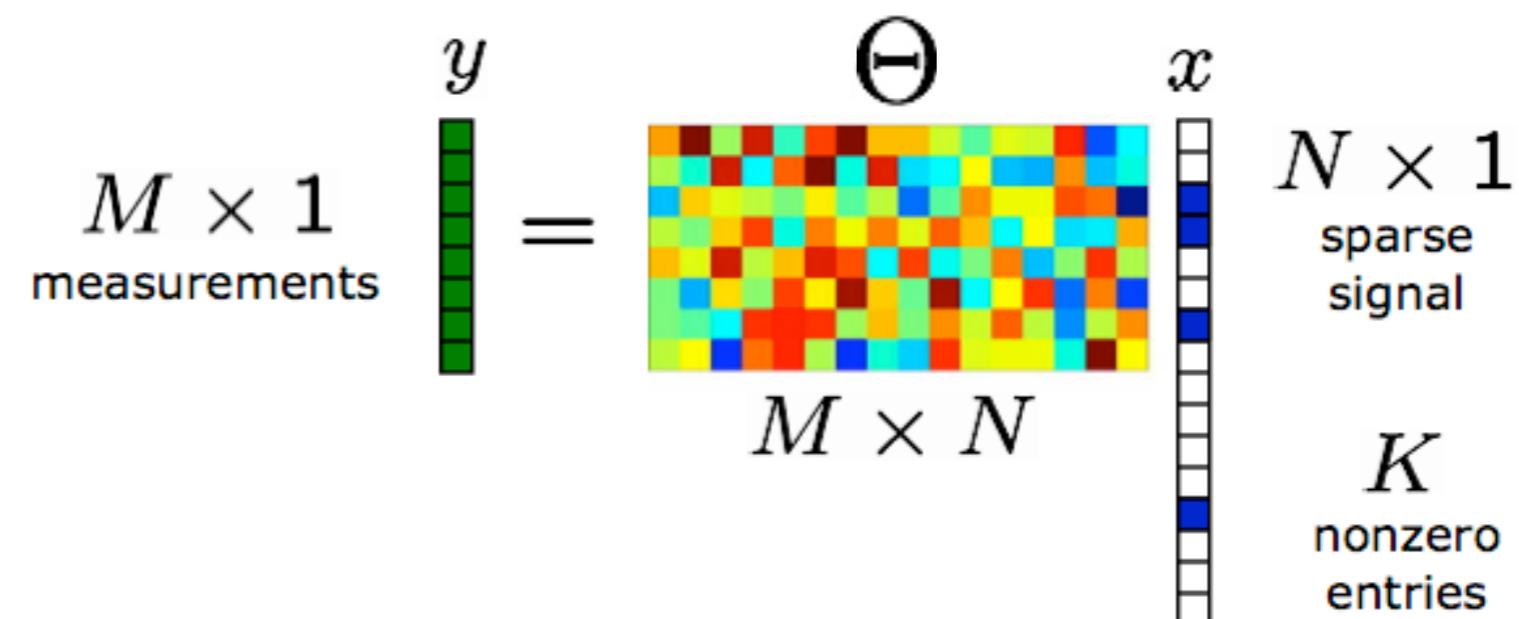
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Assumption on the signal  $x$ :

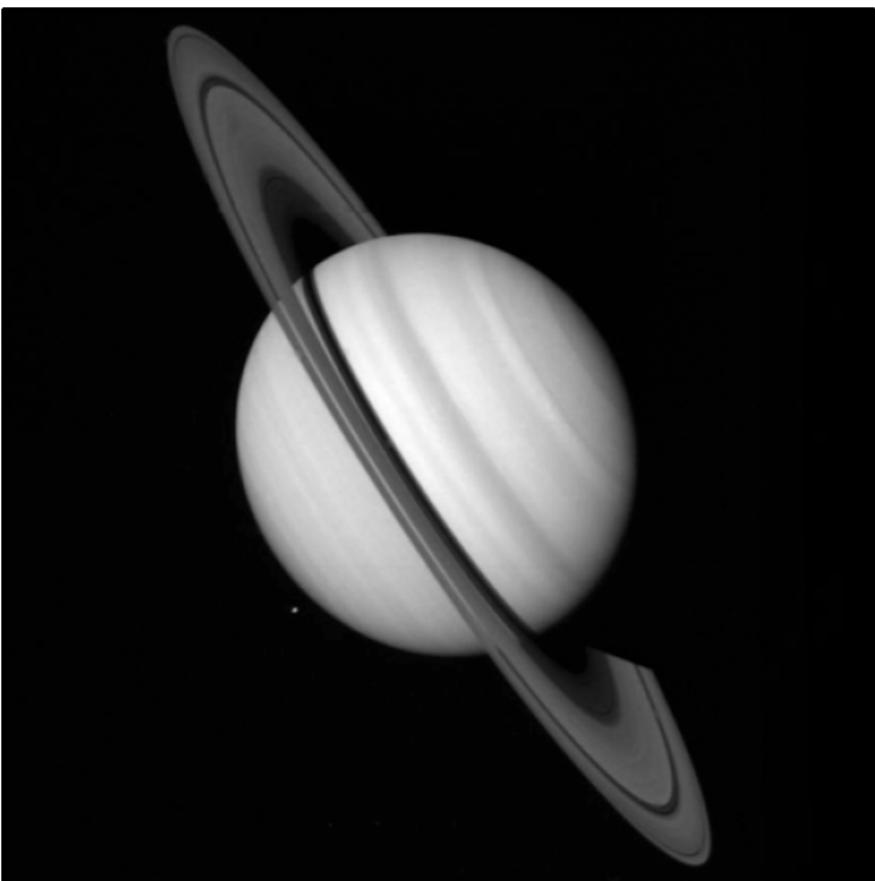
- **Underdetermined system**
- **Sparsity** of  $x$
- **Incoherence** ( $\Theta$  random)

}  $K < M \ll N$

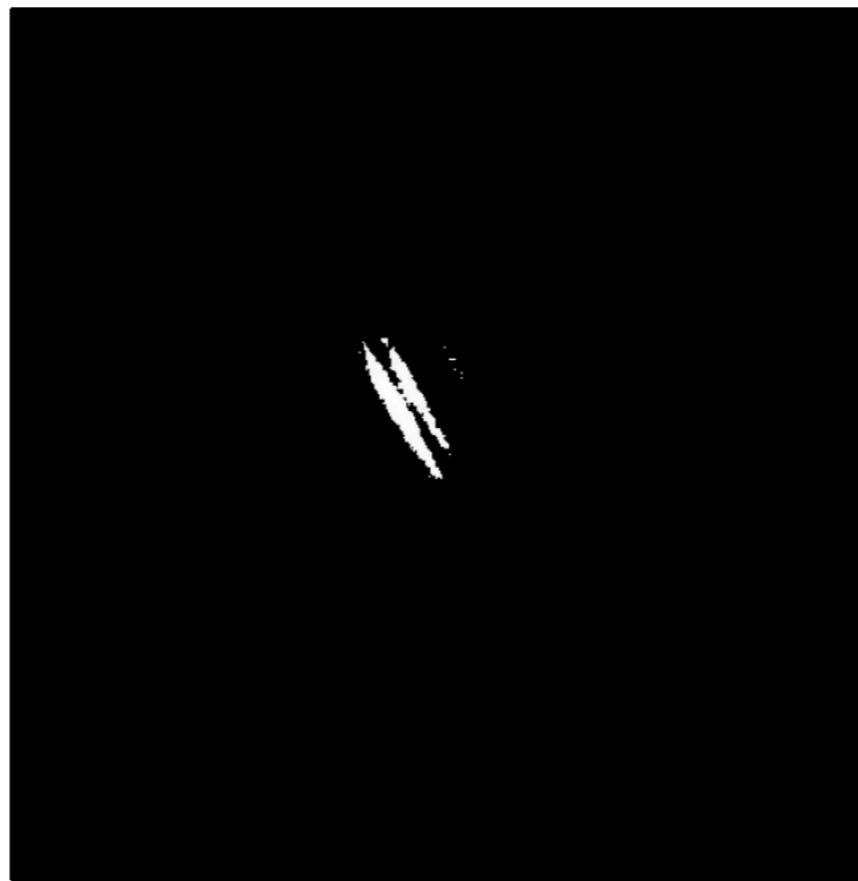
Reconstruction based on non-linear algorithms

$$\min_x \|x\|_1 \text{ s.t. } y = \Theta x$$

# Representation of a signal



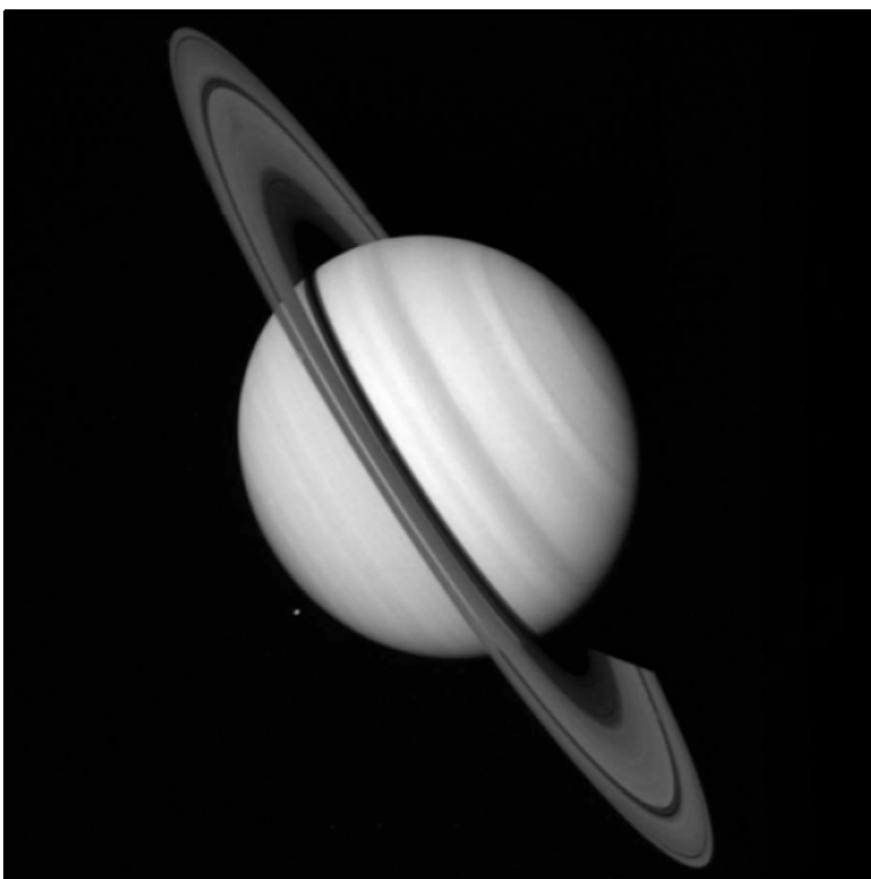
Original image



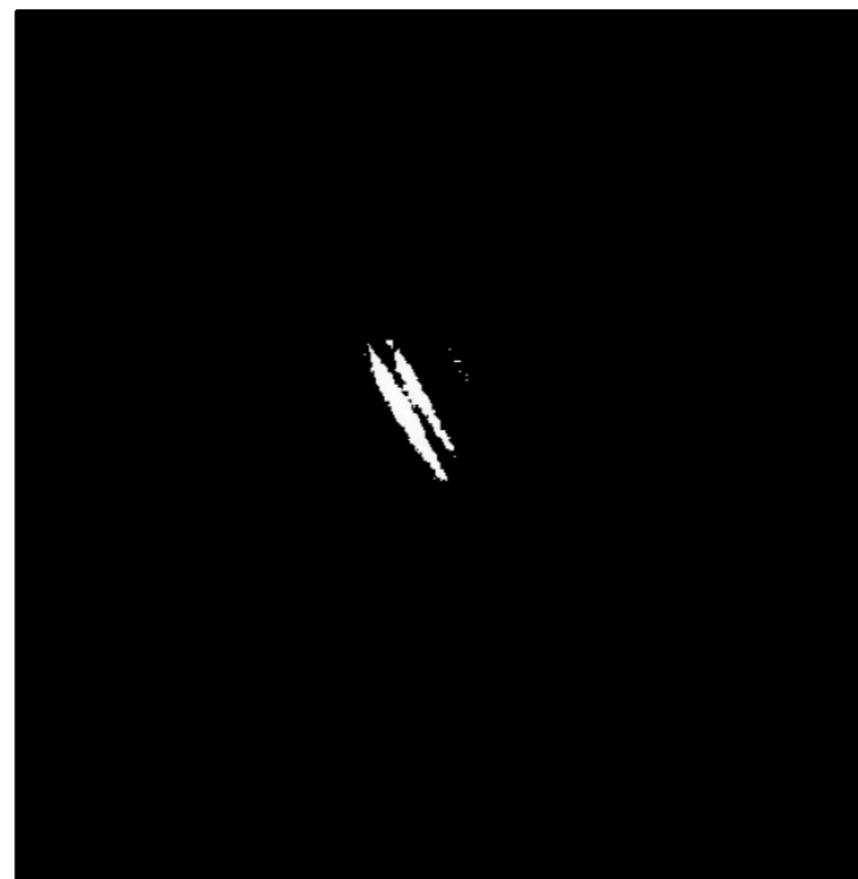
**Direct space**

5% of the highest coeff

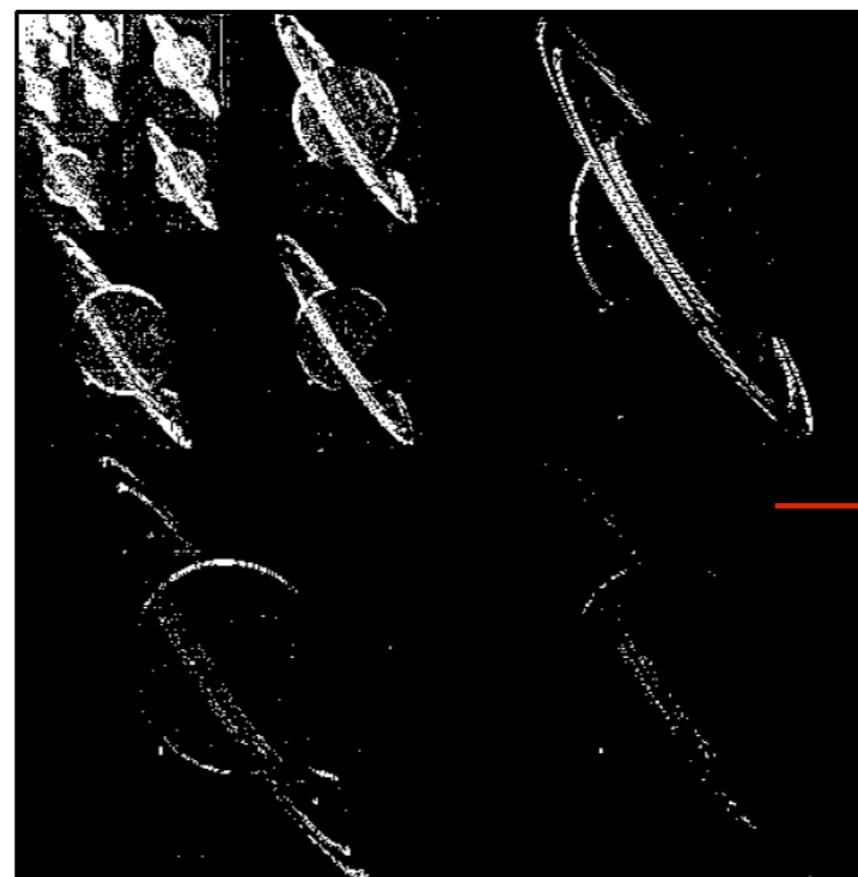
# Representation of a signal



Original image



**Direct space**  
5% of the highest coeff



**Wavelet space**  
0,5% of the highest coeff

Approximate image



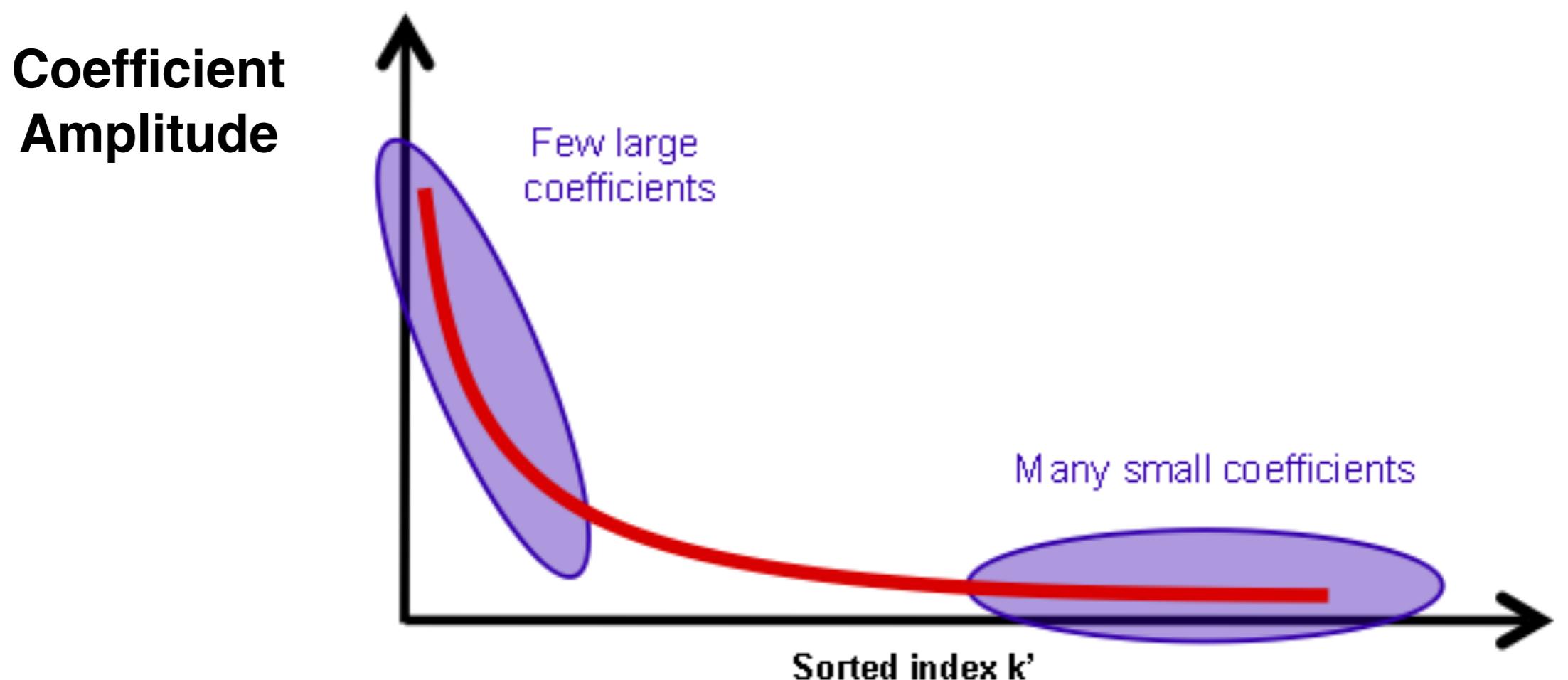
# Sparsity

Sparse signal = represented by a small number of coefficients in a « good » dictionary

Dictionary  $\Phi = \{\phi_1, \dots, \phi_K\}$

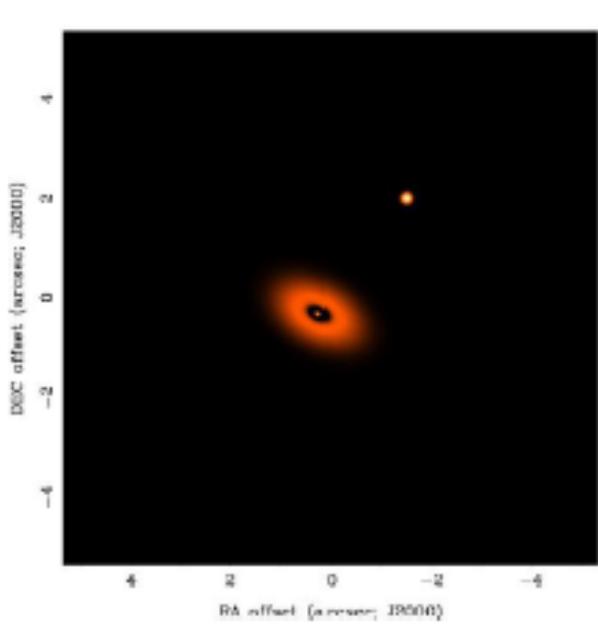
What makes a good dictionary ?

- Fast computation
- Signal analysis through the statistical properties of the coefficients
- **Sparsity of coefficients**



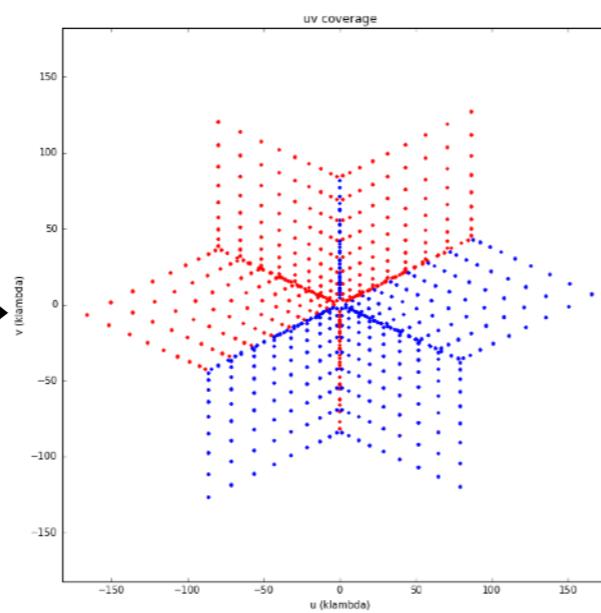
# Aperture imaging and deconvolution

Interferometry

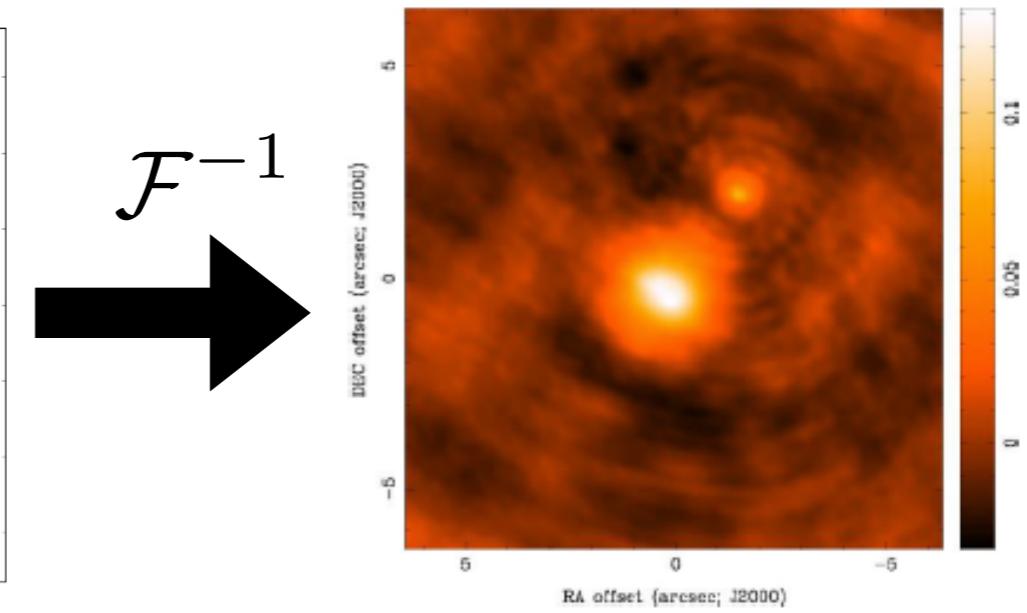


True sky

Imaging from a sparse set of Fourier samples



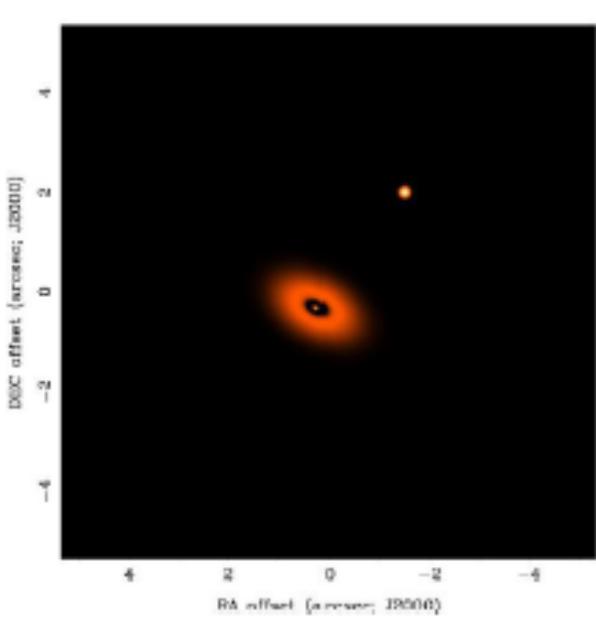
Sampling of the Fourier Plane



Observed sky

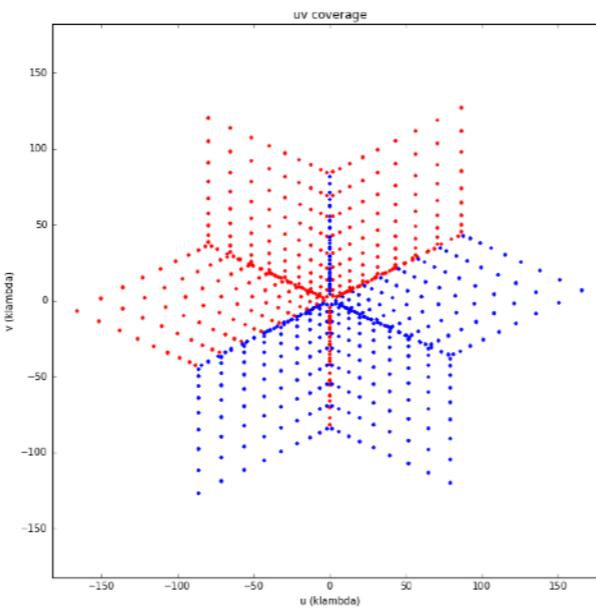
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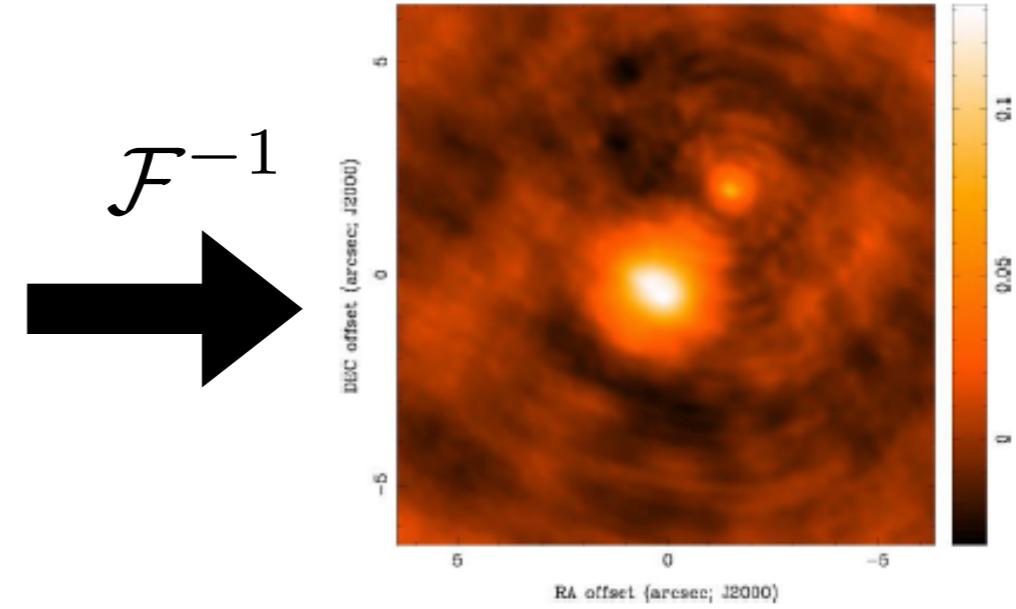


True sky

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Observed sky

Inverse problem

$$Y = HX + N$$

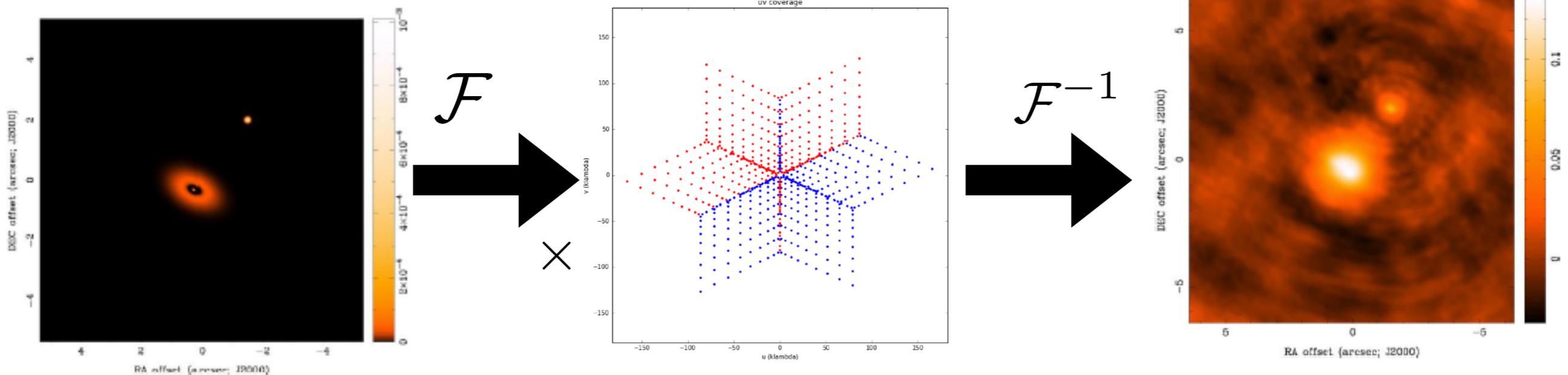
Fourier samples      Instrumental response      True sky      Noise

→ CLEAN (1974)

# Aperture imaging and deconvolution

# Interferometry

# Imaging from a sparse set of Fourier samples



# True sky

## Sampling of the Fourier Plane

# Observed sky

# Inverse problem

$$Y = HX + N \longrightarrow \text{CLEAN (1974)}$$

Fourier samples  
Instrumental response  
True sky  
Noise

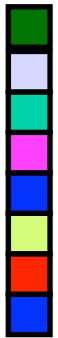
Combining the mathematical framework of **Compressed Sensing** and **sparsity**

$$P : \min_{\alpha} \|Y - H\Phi\alpha\|_2^2 + \lambda \|\alpha\|_1$$

↑      ↑  
**Data term**      **Sparsity constraint**

# Radio interferometry & Compressed Sensing

$y$

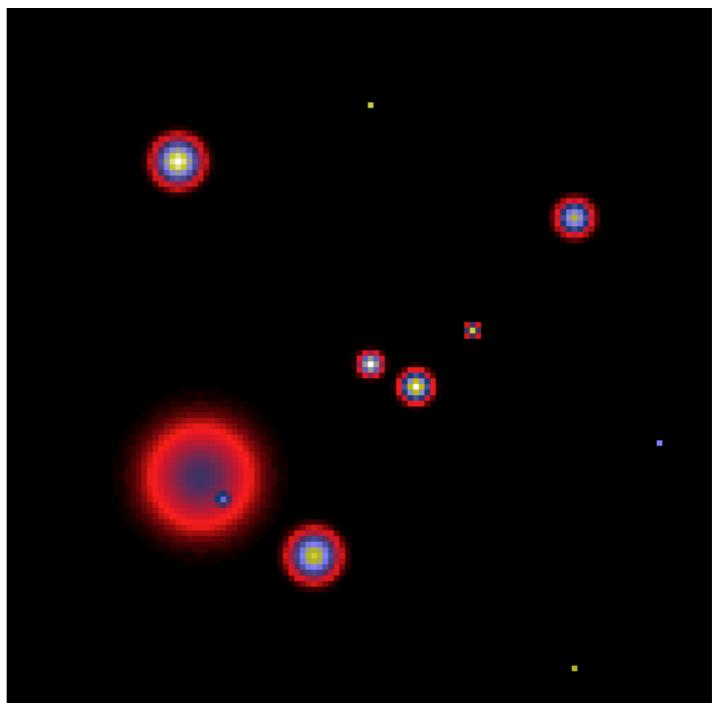


=

**Visibilities**



VLA



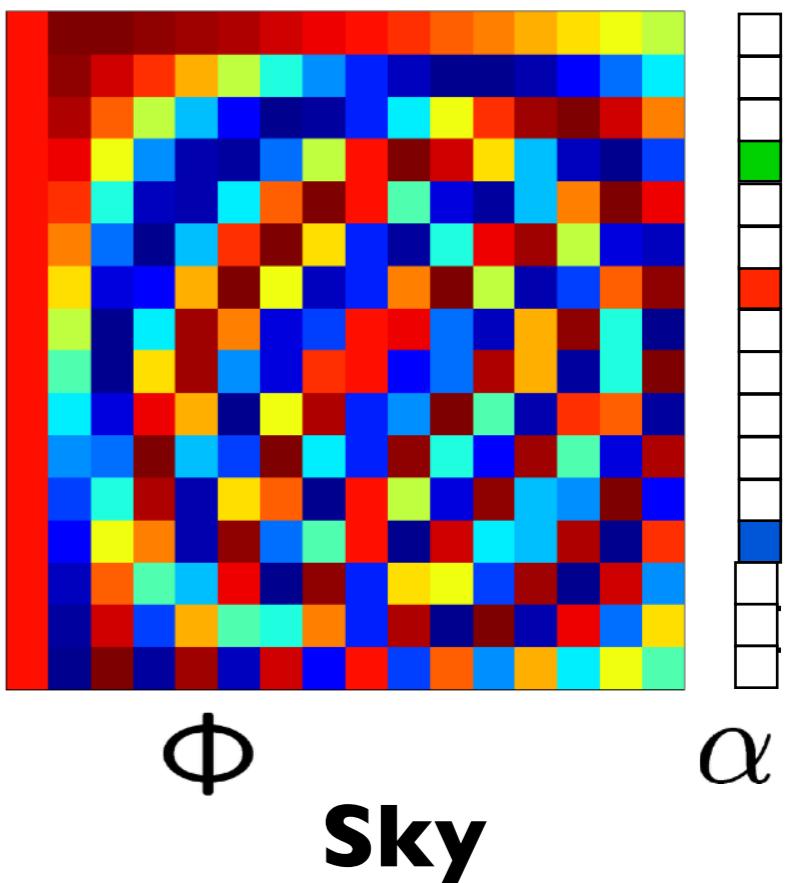
**Sky**

$X$

# Radio interferometry & Compressed Sensing



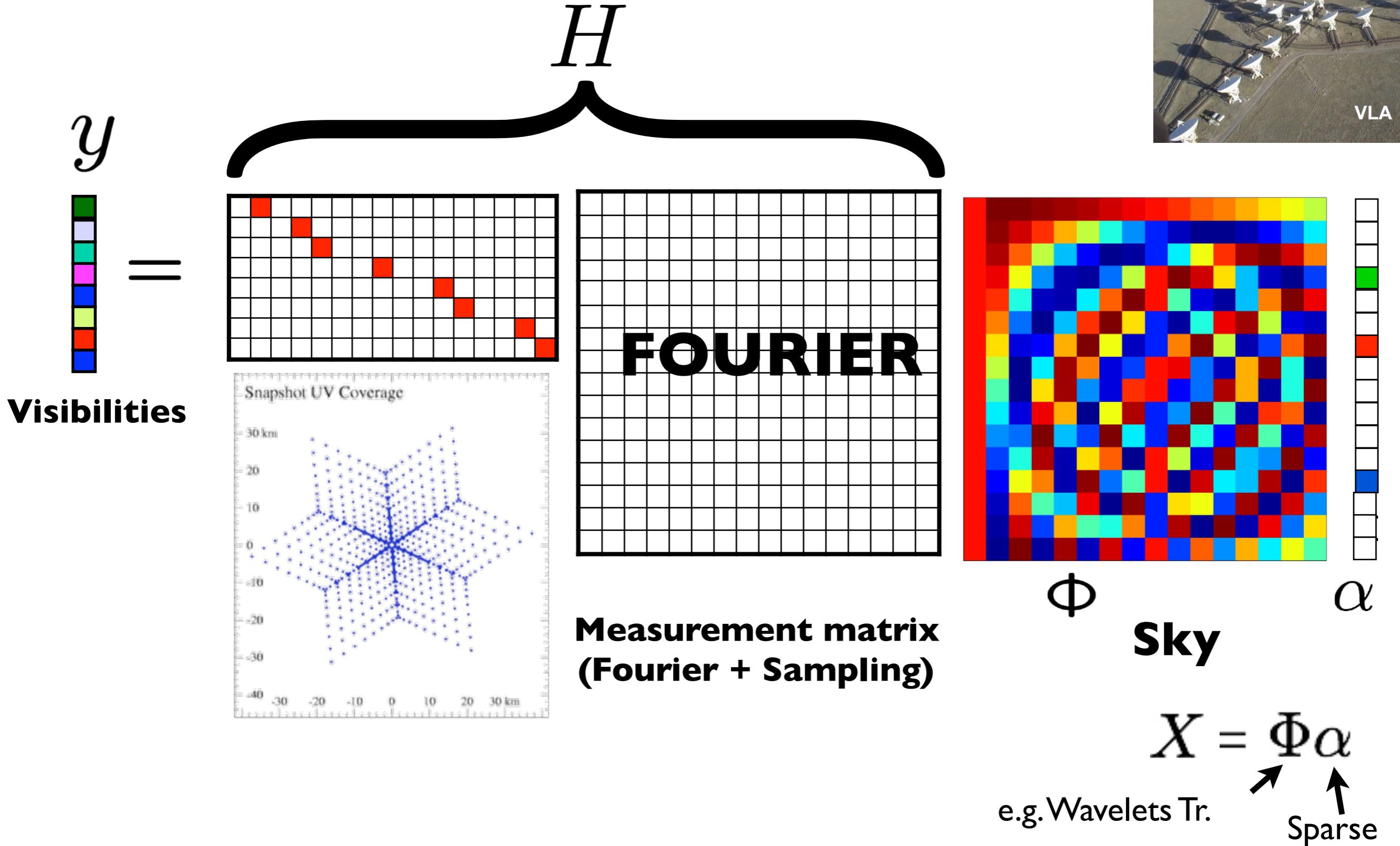
$$y = \text{Visibilities}$$



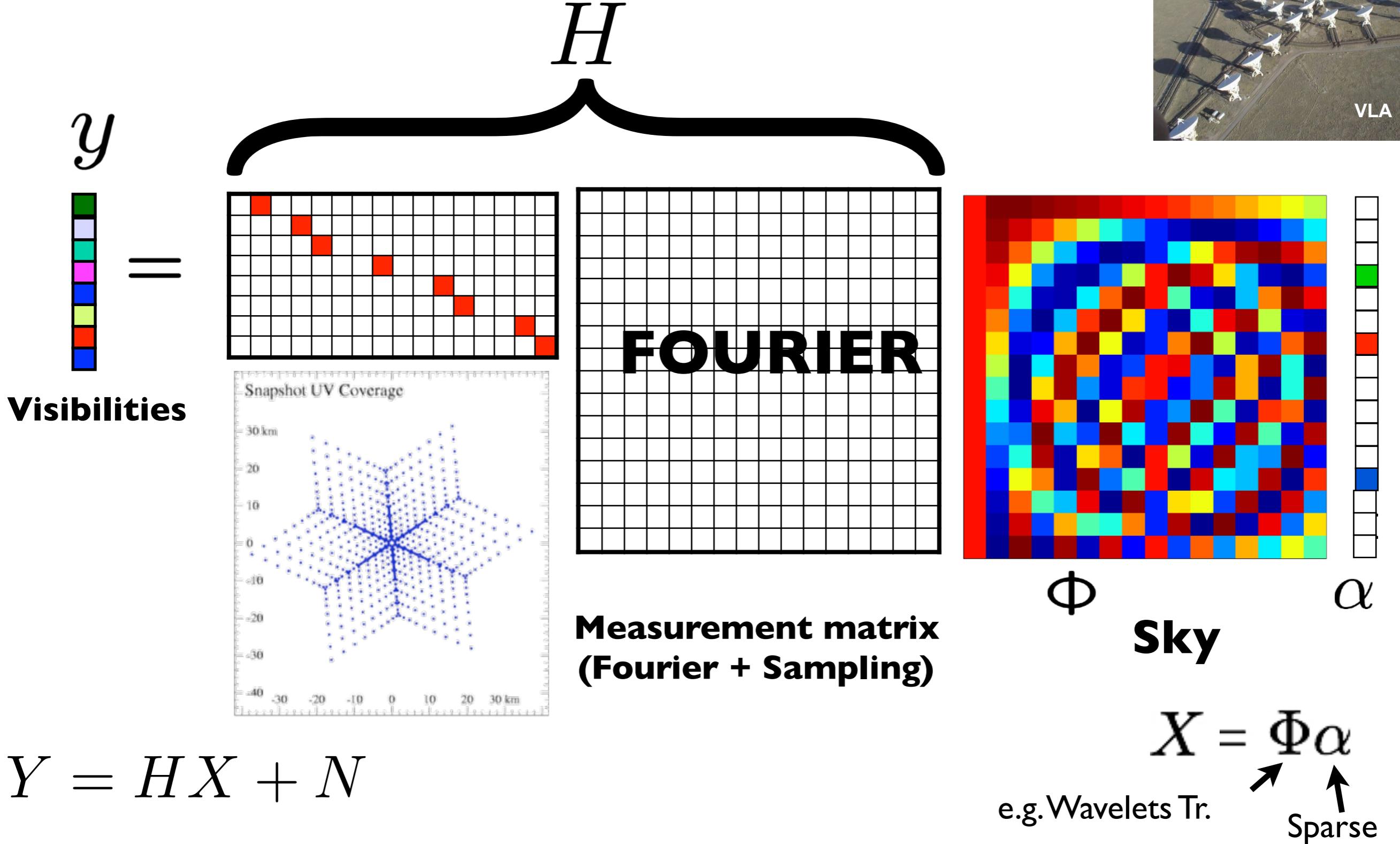
$$X = \Phi \alpha$$

e.g. Wavelets Tr.  
Sparse

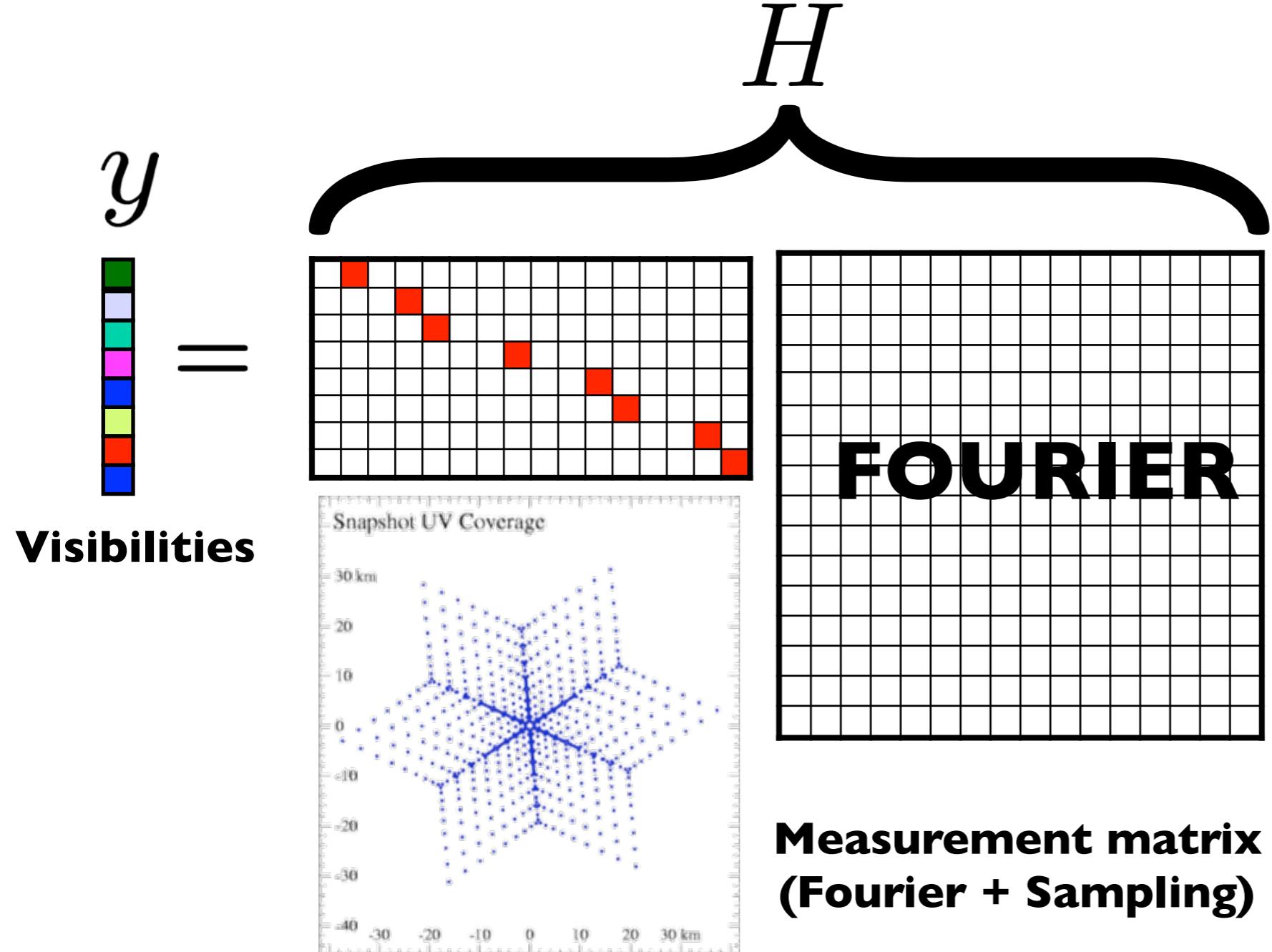
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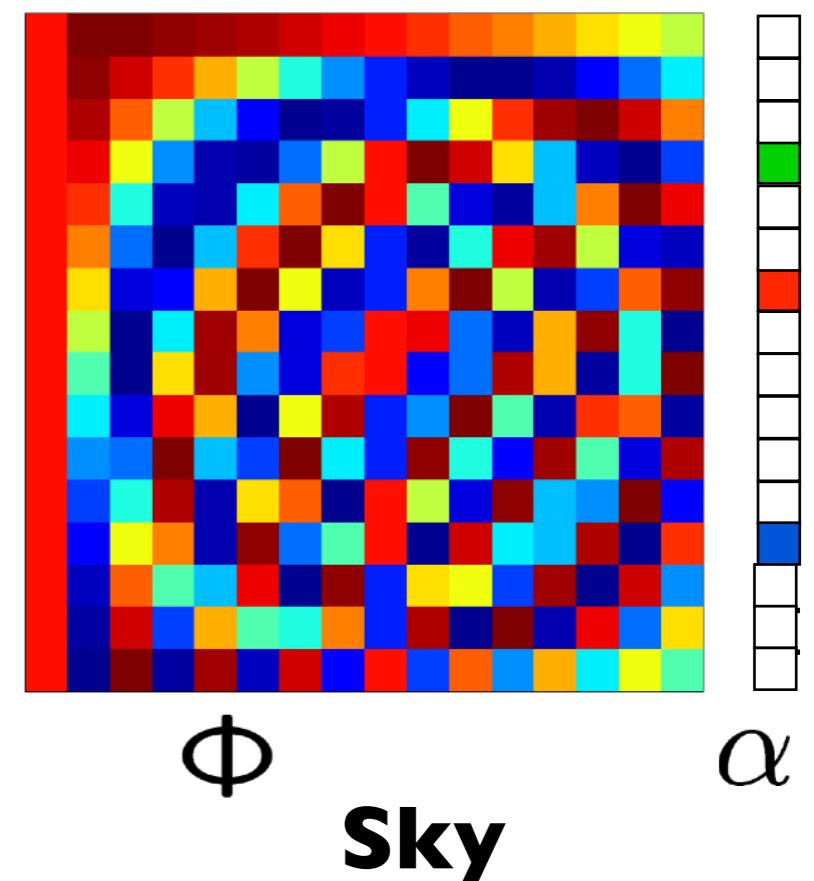
# Radio interferometry & Compressed Sensing



$$Y = HX + N$$

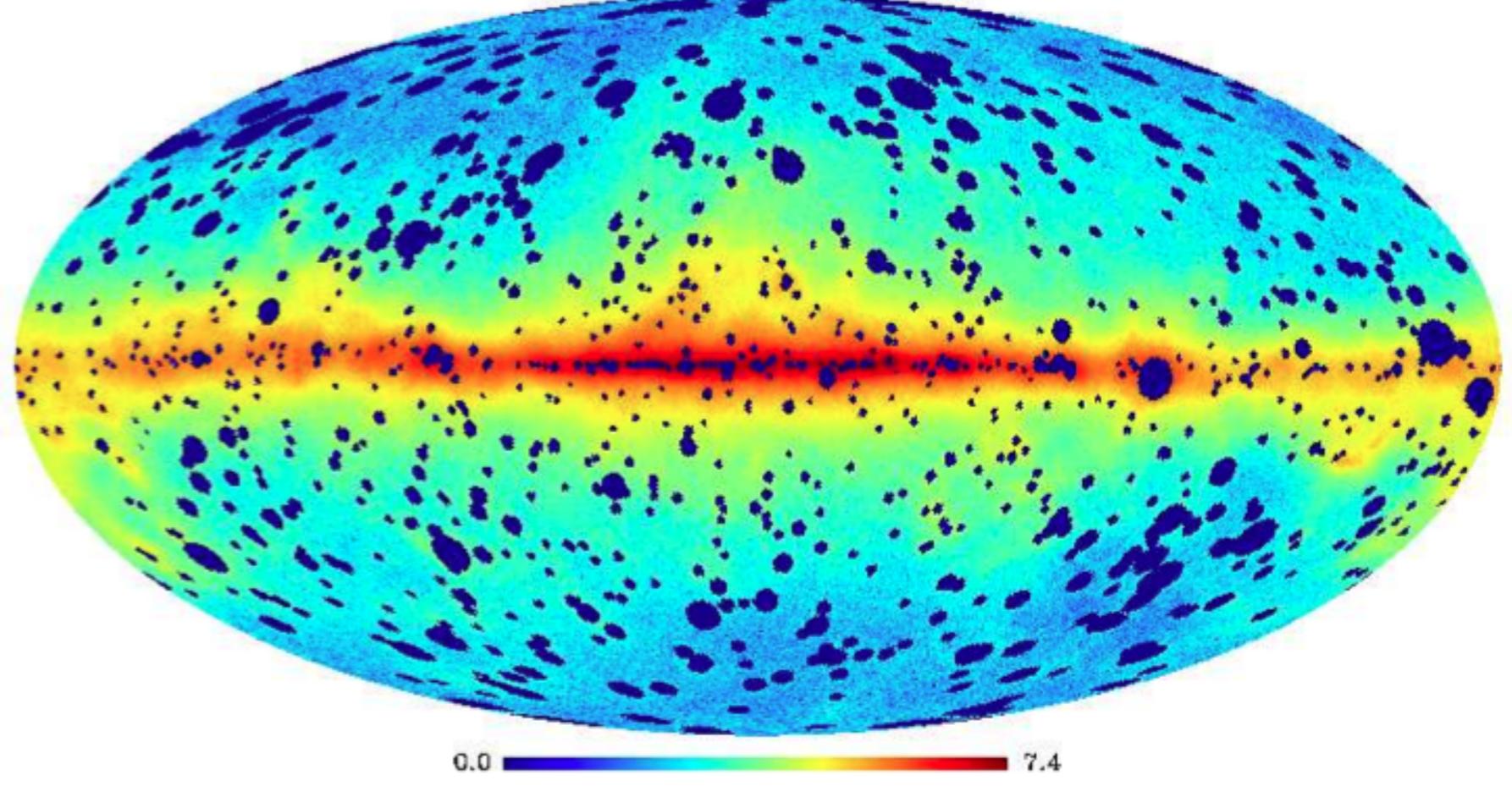
$$\min_{\alpha} \|\alpha\|_p^p \quad \text{subject to} \quad \|Y - H\Phi\alpha\|^2 \leq \epsilon$$

$$\min_{\alpha} \|V - \mathbf{A}\Phi\alpha\|_2^2 + \sum_j \lambda_j |\alpha_j|.$$

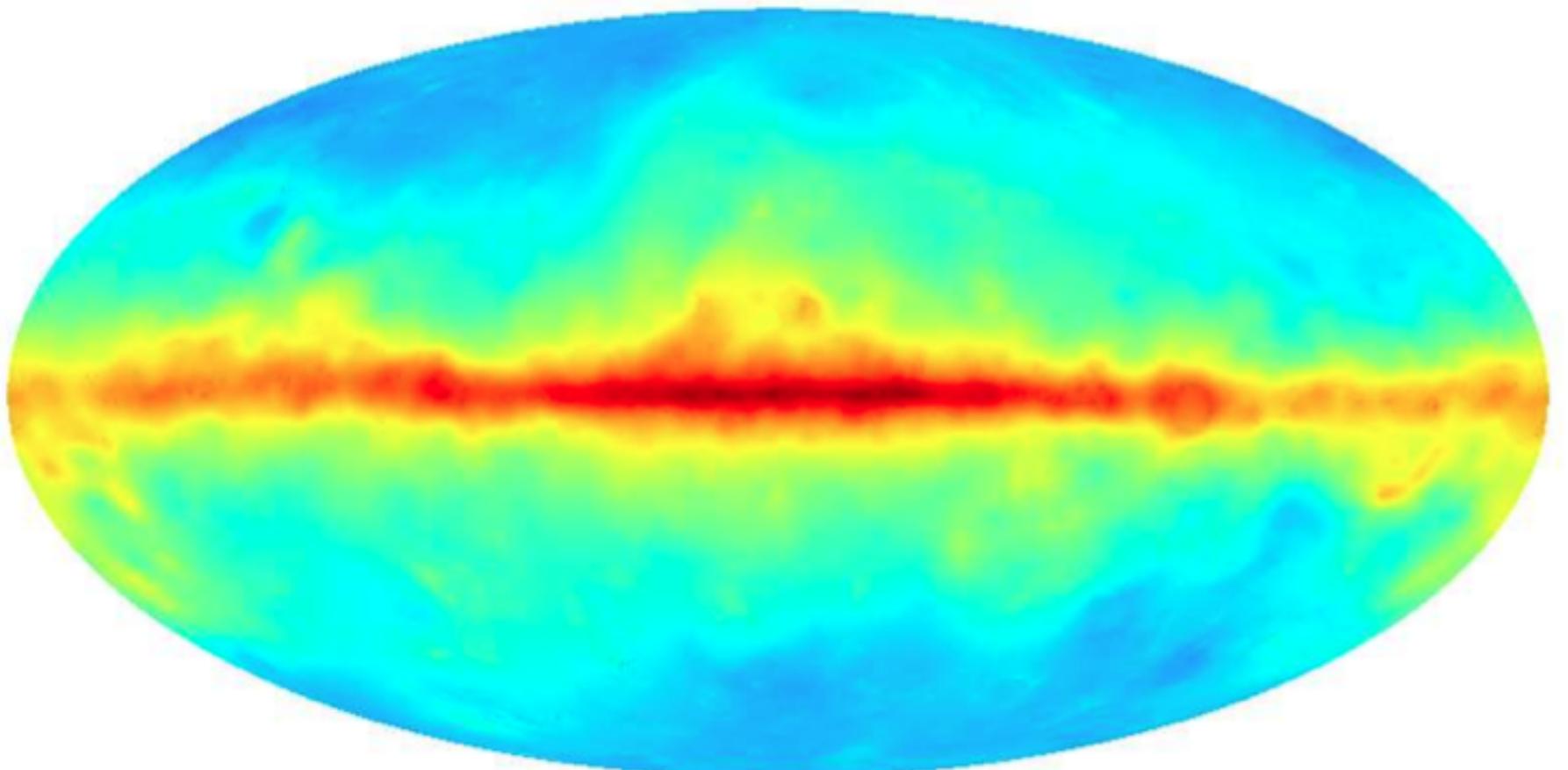


# Applications: In-painting

Simulated  
Fermi data



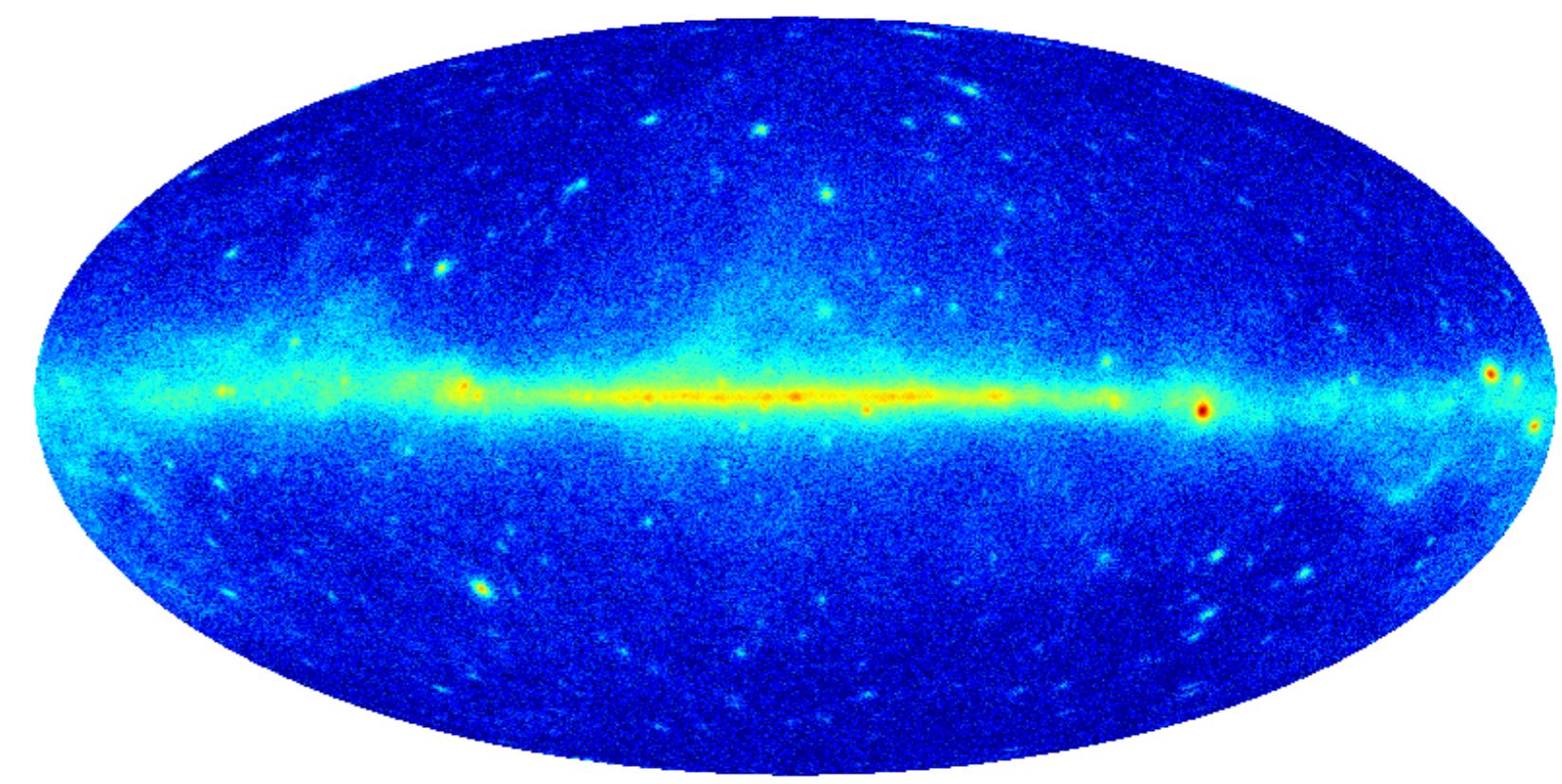
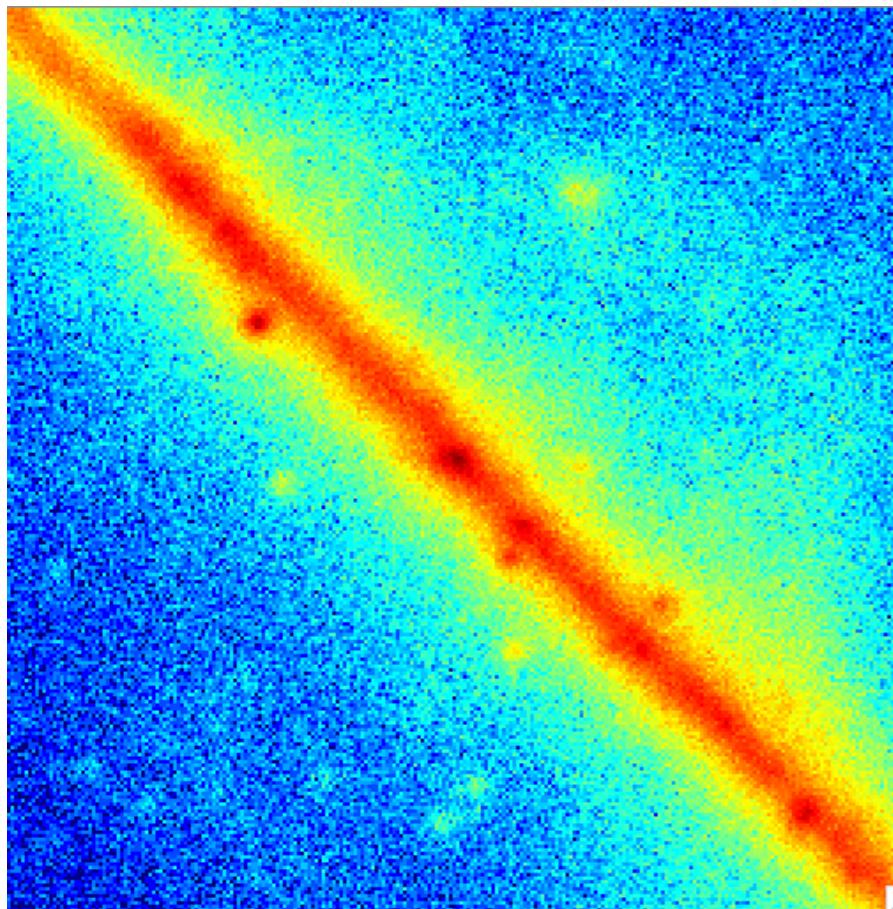
Simulated Fermi data denoised and inpainted by MS-VSTS+IUWT



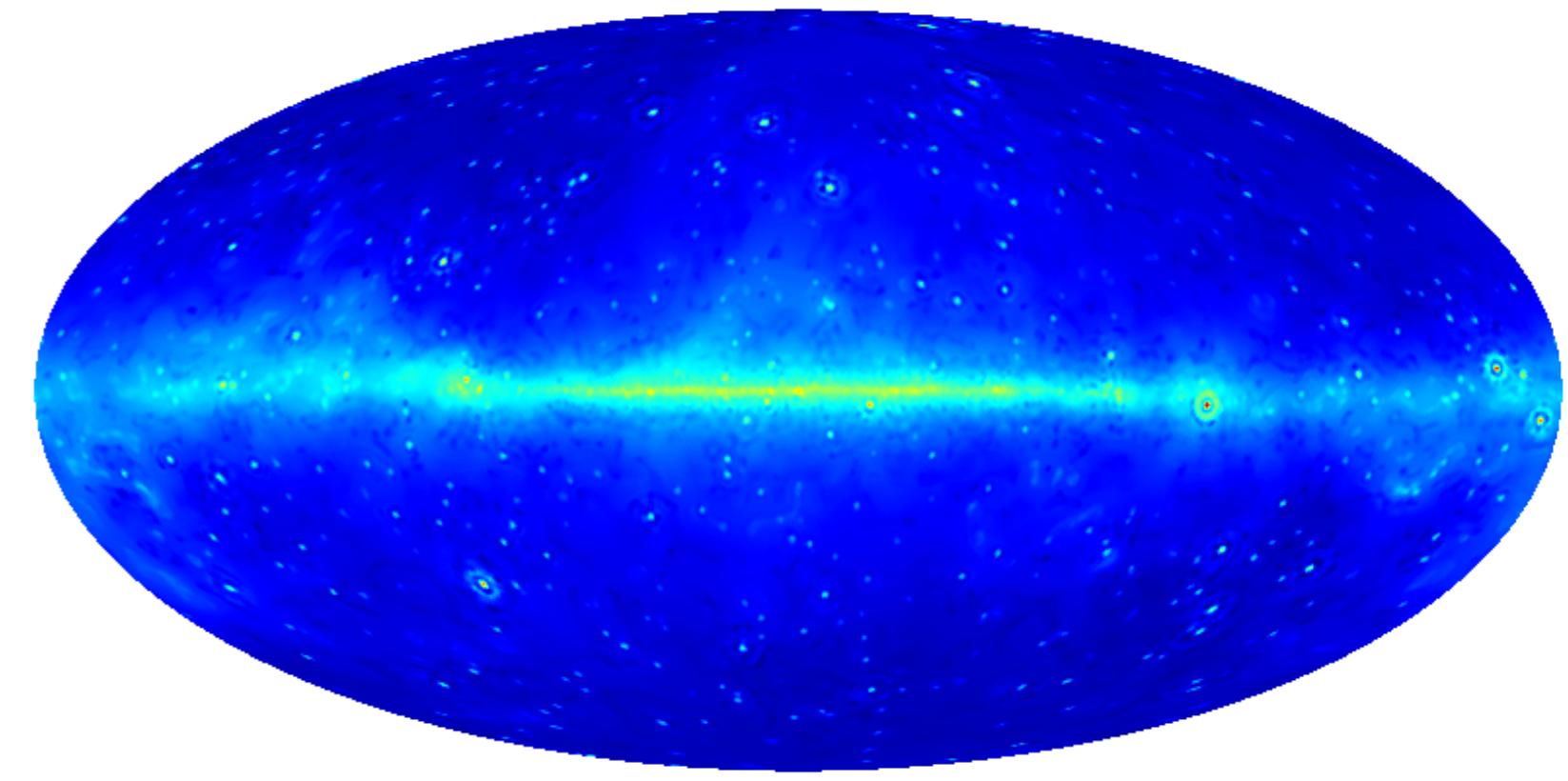
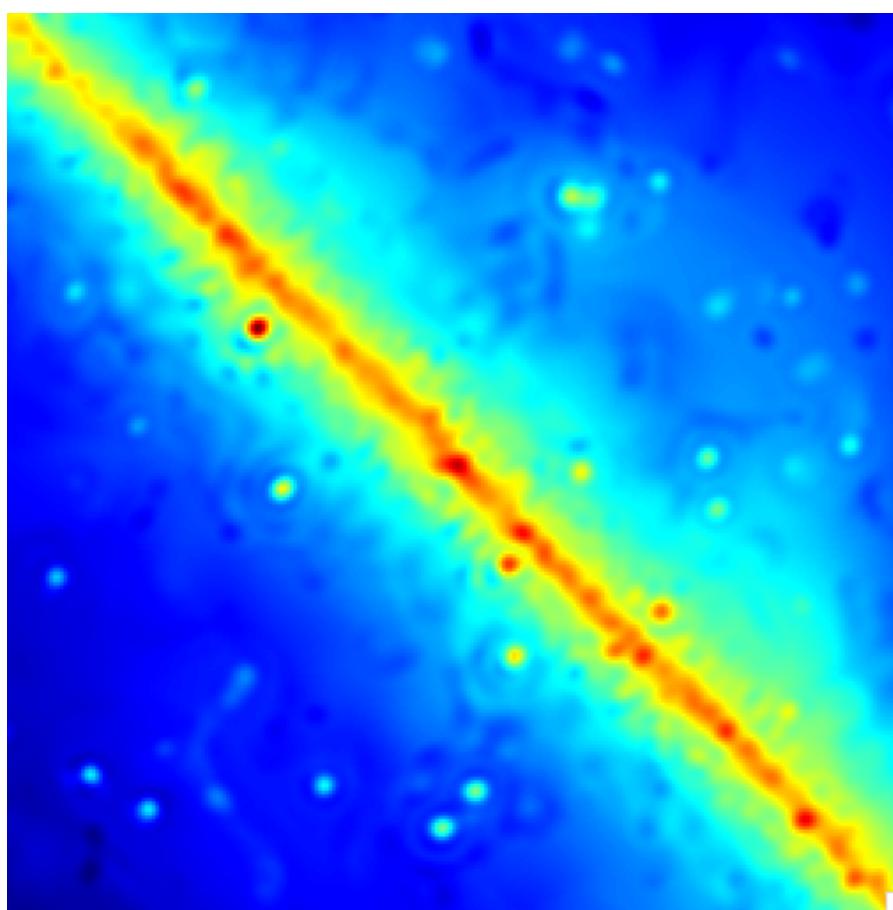
Credits: AIM/LCS

# Applications: Denoising / Deconvolution

Simulated Fermi Poisson Data – Energy band = 360 MeV – 589 MeV



Simulated Fermi Poisson Data – Energy band = 360 MeV – 589 MeV

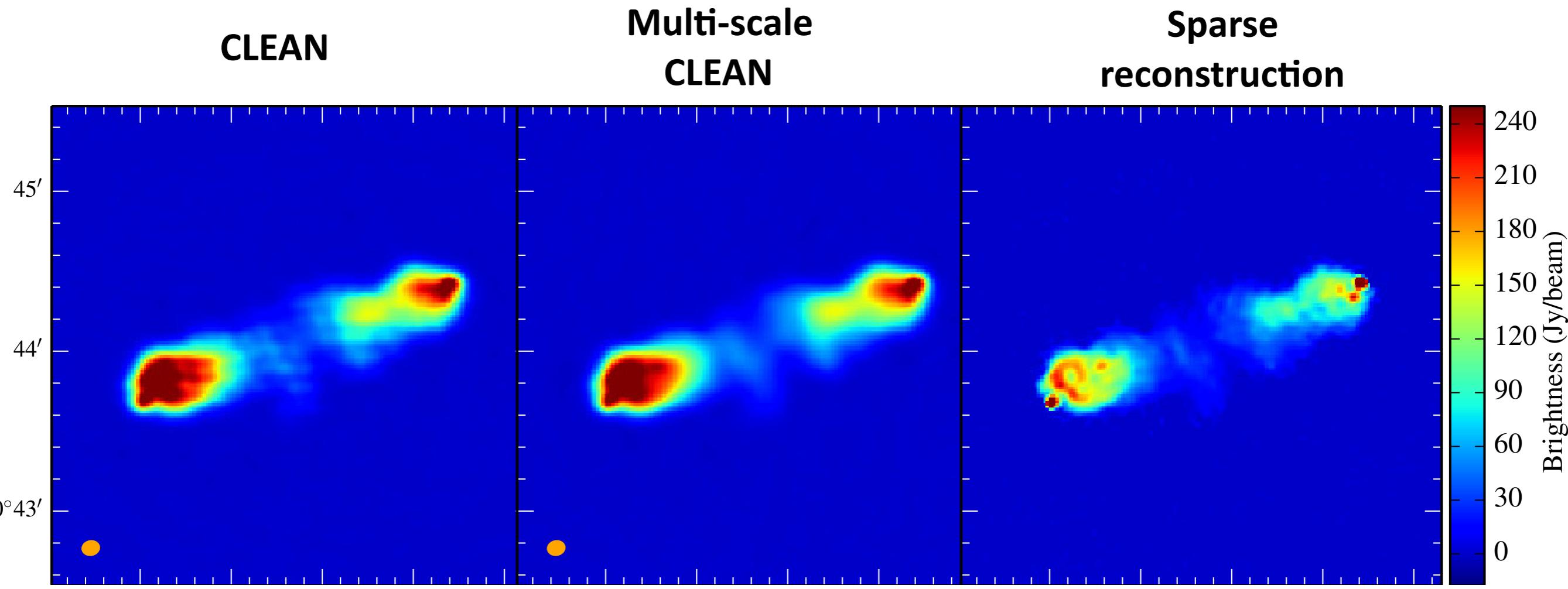


0.02S      3.8 Log ( )

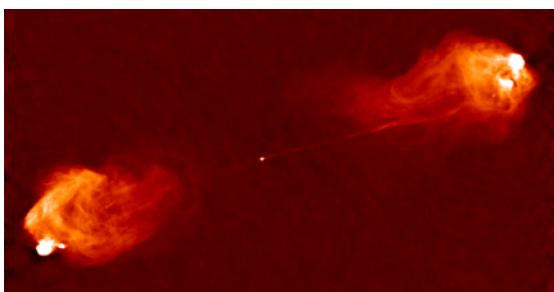
Credits: AIM/LCS

# Sparse reconstruction

LOFAR (150 MHz) Cygnus A image reconstruction

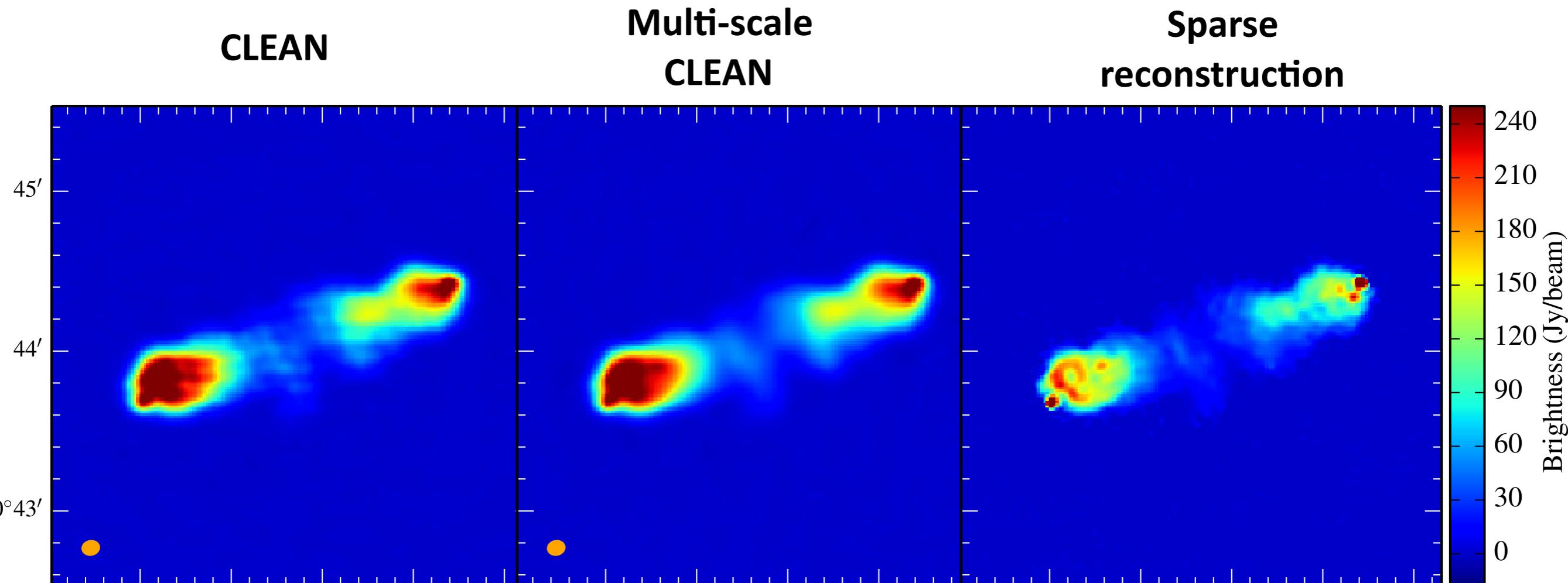


Factor of 4 improvement in angular resolution

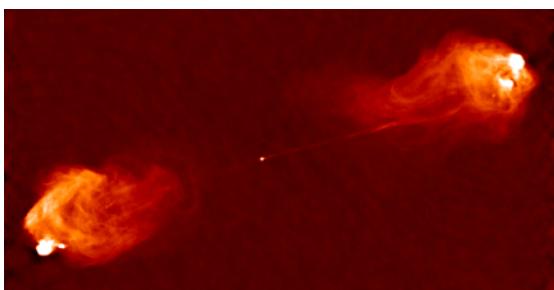


# Sparse reconstruction

## LOFAR (150 MHz) Cygnus A image reconstruction

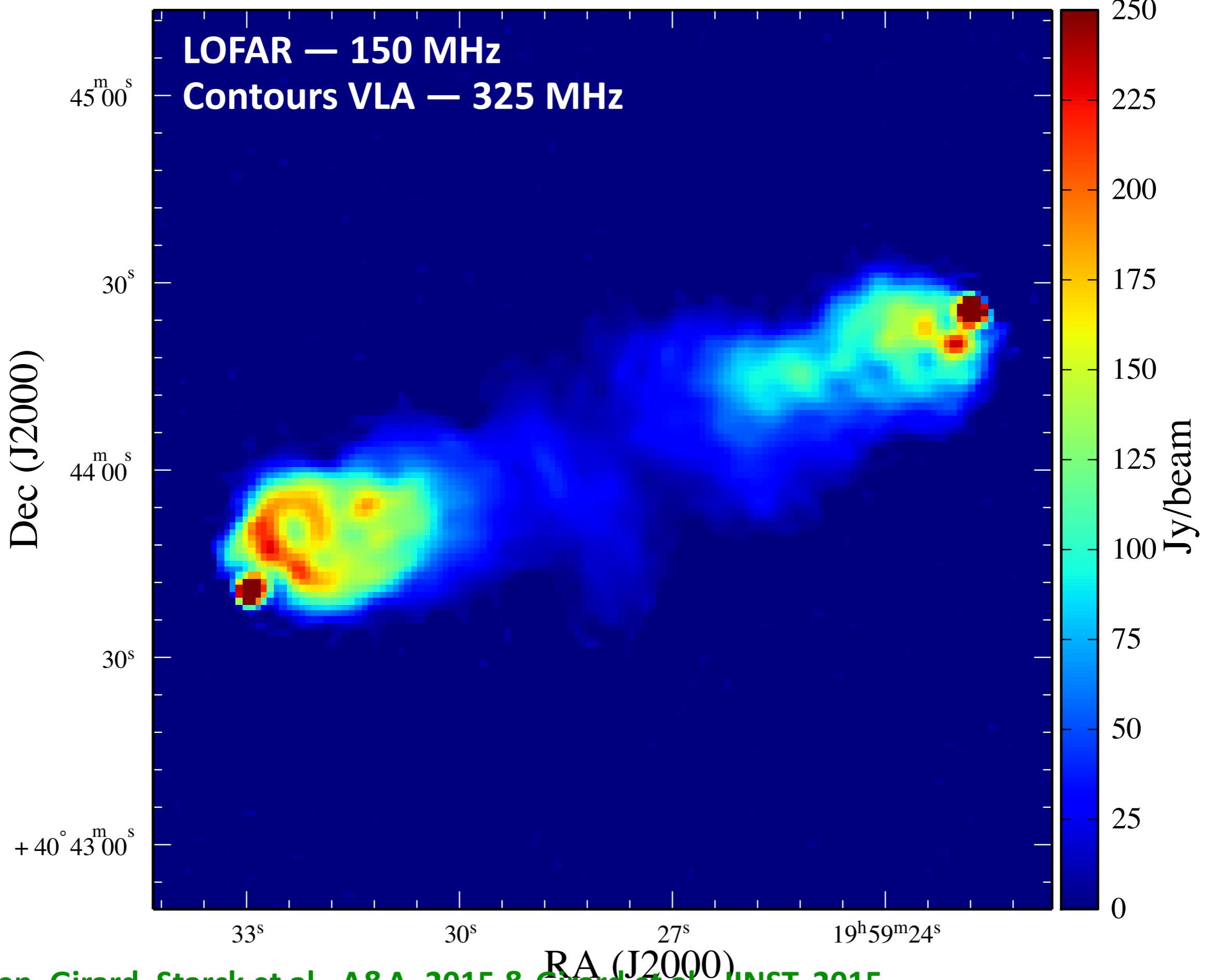


Factor of 4 improvement in angular resolution

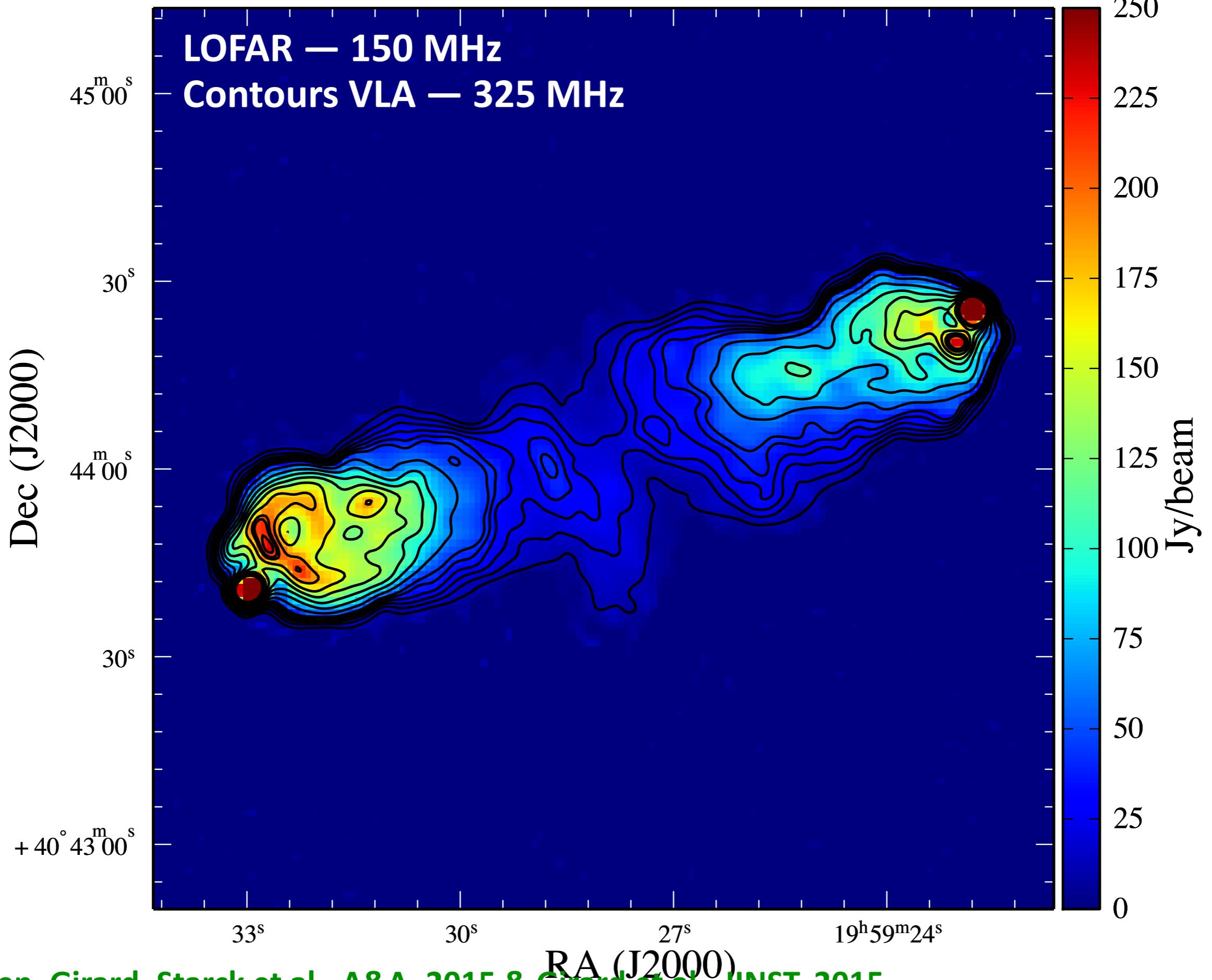


but are the reconstructed structures real ?

# Sparse reconstruction



# Sparse reconstruction



# Other « Compressed Sensing » / Convex optimization / Sparse imagers on the market (non exhaustive)

## « Signal processing world »

### - Iterative Soft-Thresholding

*Beck & Teboulle (2009)*



## « Radio interferometry world »

Li et al., (2011c)

Wenger et al. (2010,2013)

Hardy (2013)

Garsden, Girard (2015)

### - OMP (Orthogonal Matching Pursuit)

*Davis et al. (2007)*



Fannjiang (2013)

Van belle (MS thesis 2016)

+ CLEAN (MP) ...

R. Armstrong (in prep)

### - Douglas-Rachford splitting

*Combettes & Pesquet (2007)*



Wiaux et al. (2009b)

McEwen & Wiaux (2011)

Carrillo et al. (2012)

### - SDMM/ADMM

Simultaneous-Direction Method of Multipliers

Alternating

*Afonso et al., Setzer et al. Attouch & Soueycatt, 2009*

*Combettes & Pesquet (2011)*



in Carrillo et al. (2013)

### - SARA/PURIFY Carrillo et al., 2015

### - MORESANE

Dabbech et al. 2015

### - Point-RESOLVE

Junklewitz, 2013

### - SASIR

Garsden et al. 2015, Girard et al. 2015

Girard, Jiang et al., in rev.

# **Outline**

**New instrumentation, new problems  
(and new solutions !)**

**Sparsity and deconvolution**

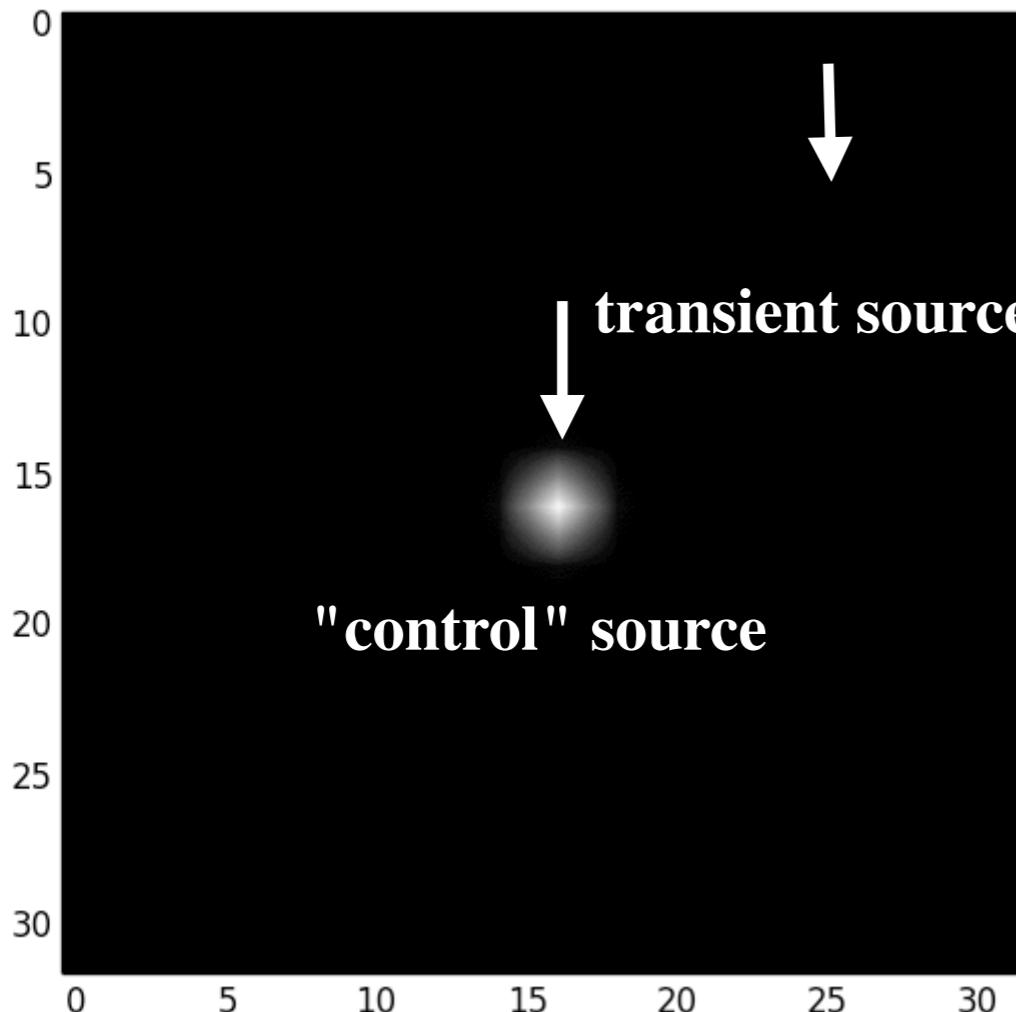
**Application to transient imaging**

**Conclusions**

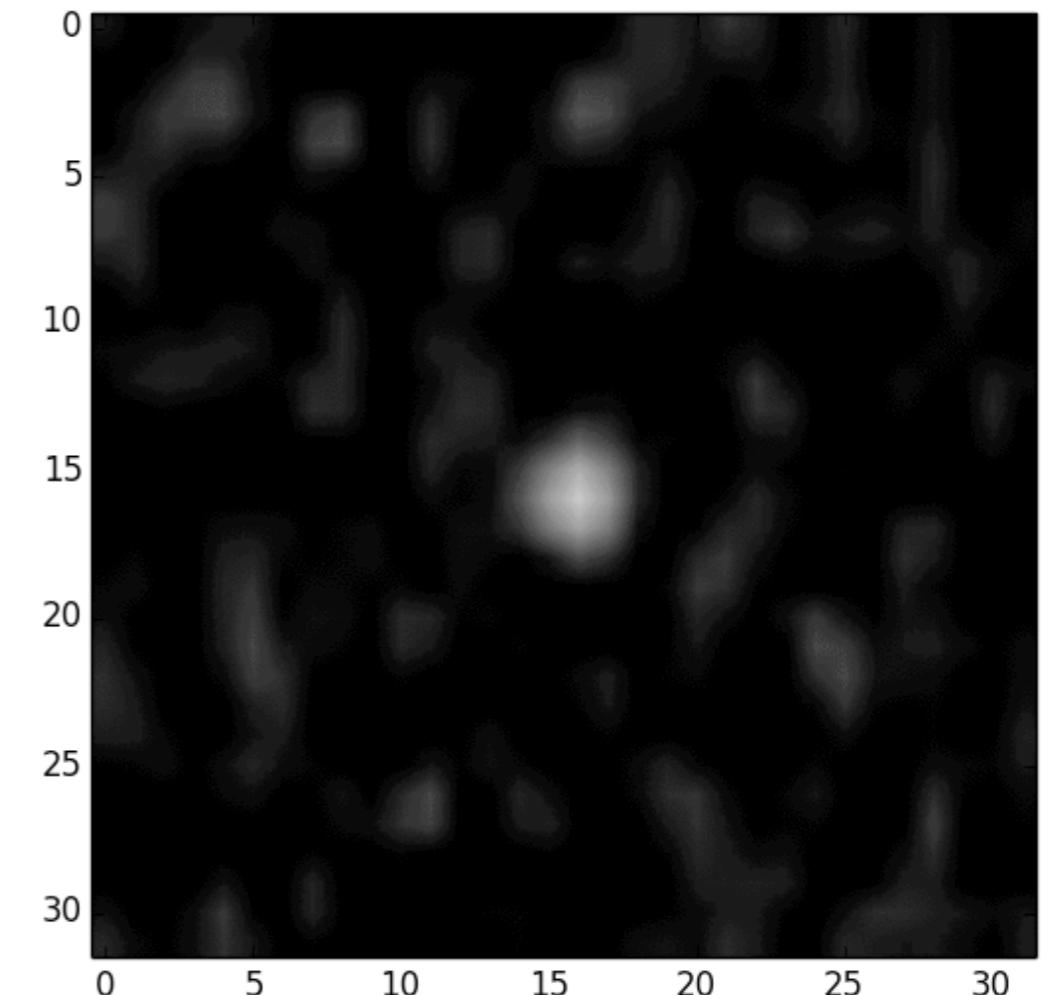
# Application to transient imaging

2D+time

Sky model



Dirty map



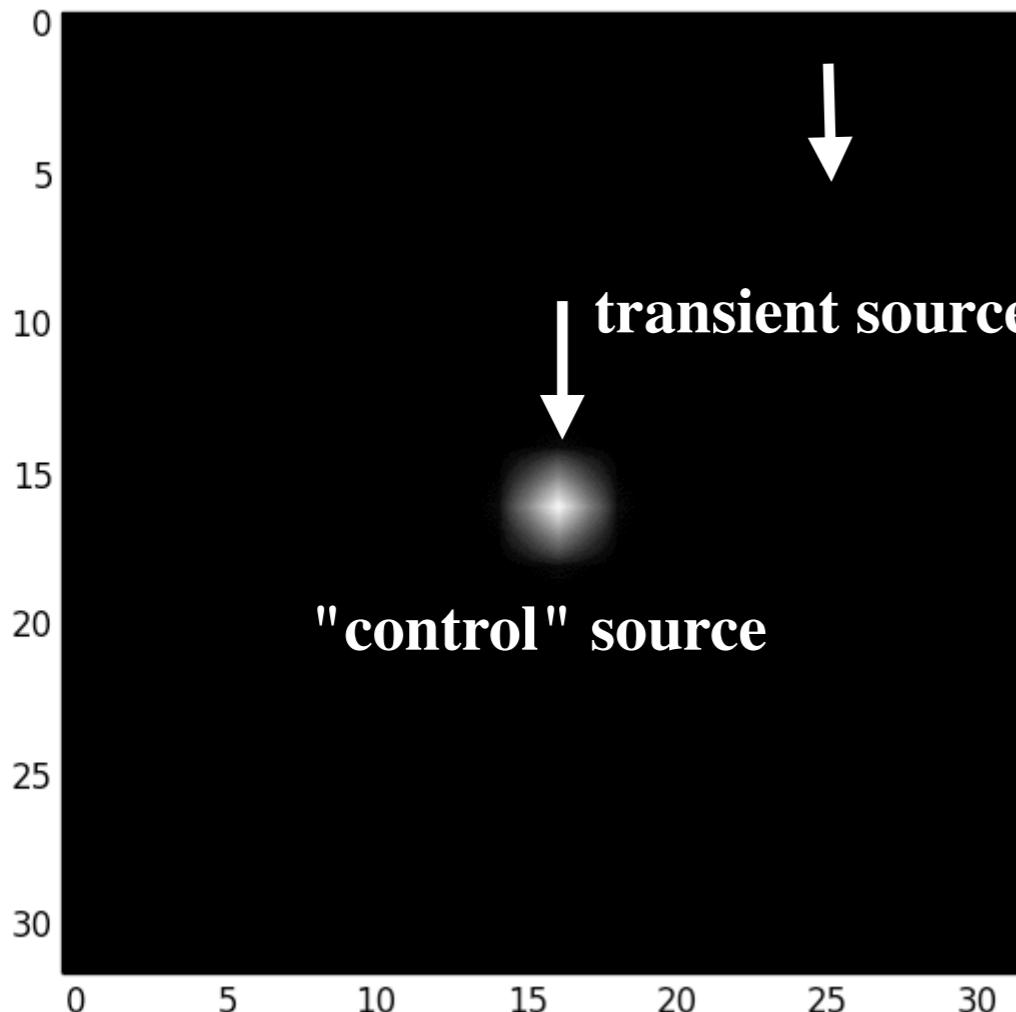
- "control" steady source
- transient gaussian source

A new source appeared but side lobes as well !  
Need a time-agile deconvolution algorithm

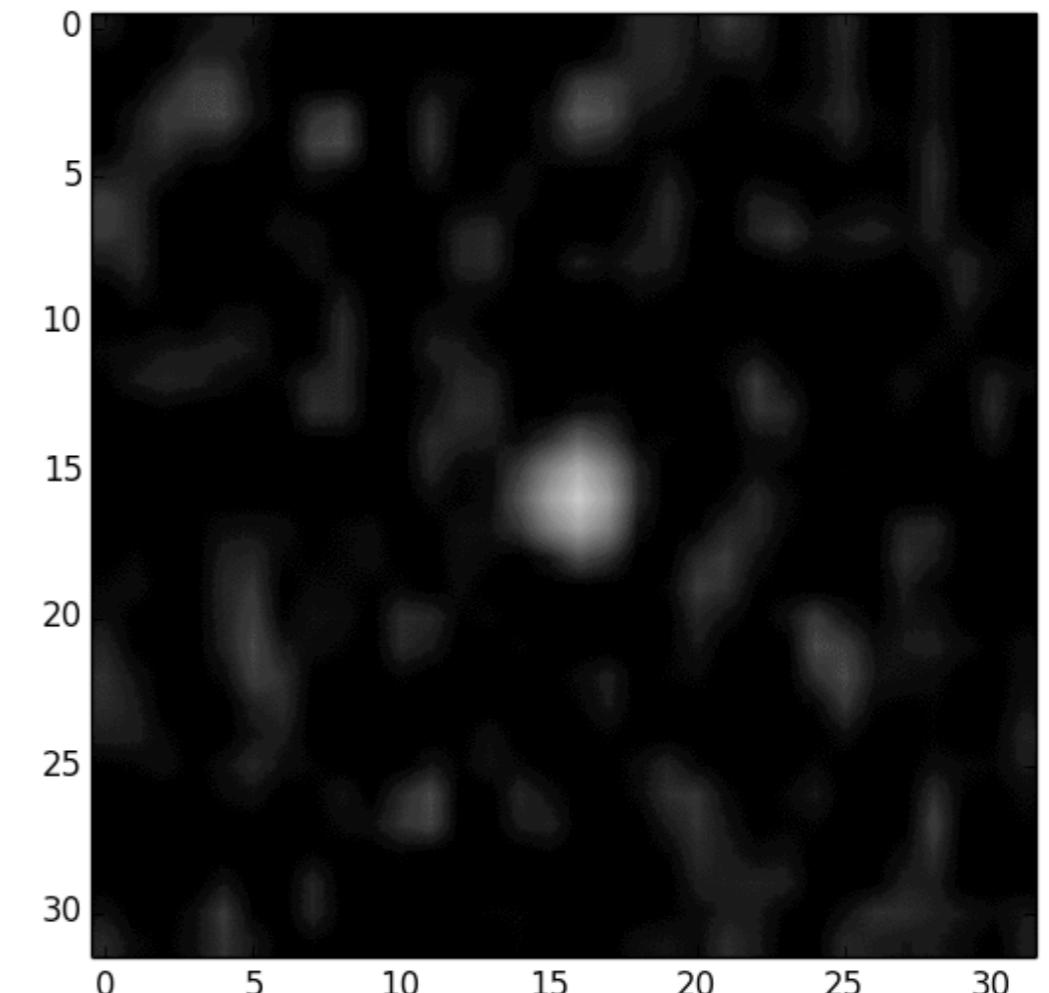
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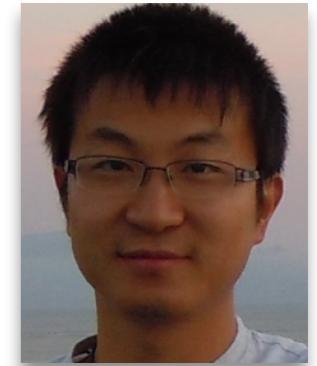
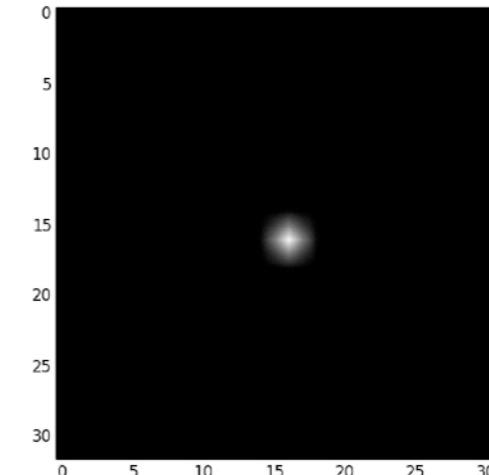
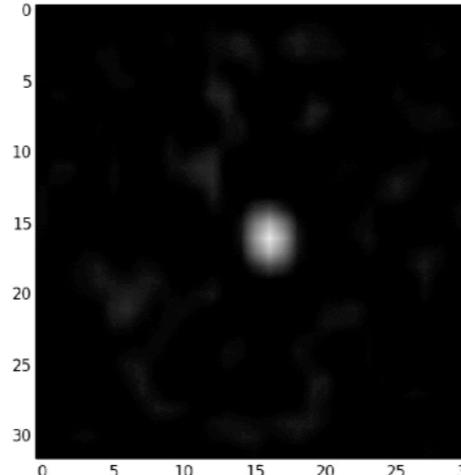
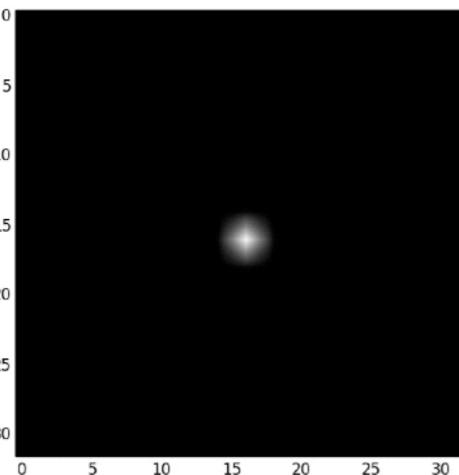
2D+time

Sky model

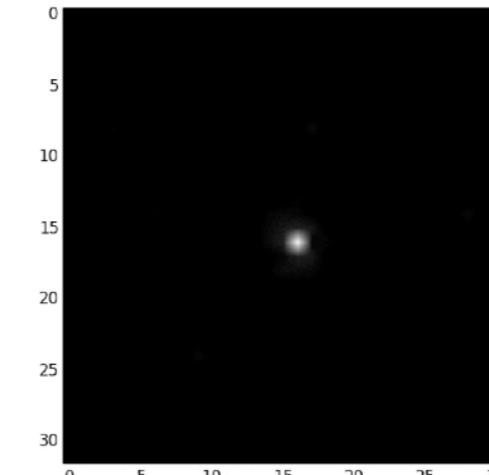
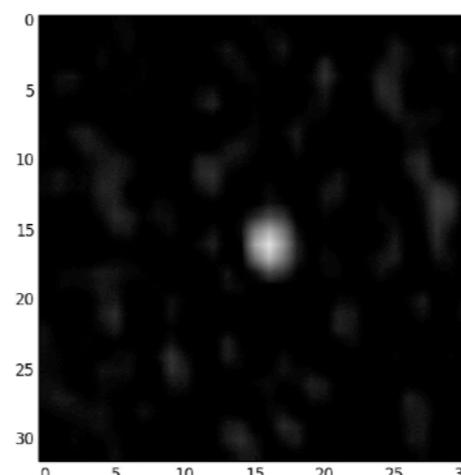
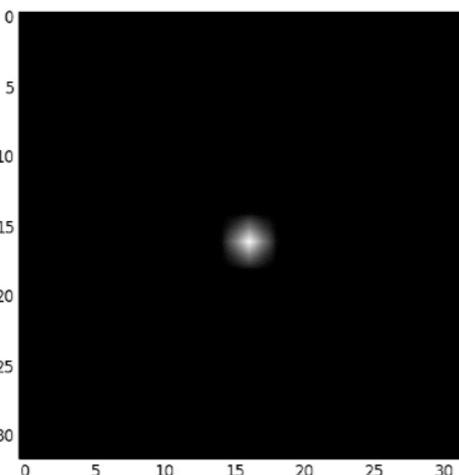
Dirty map

Reconstruction

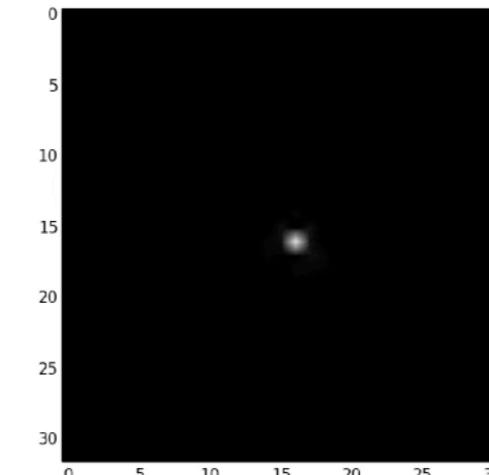
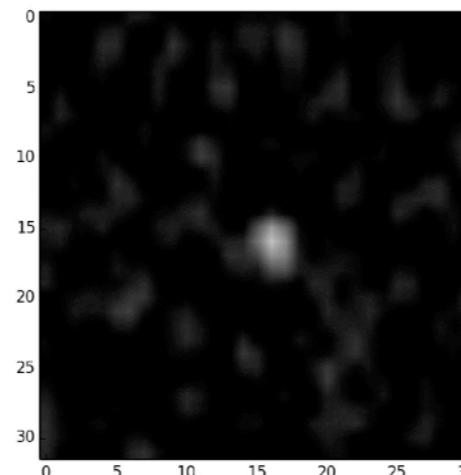
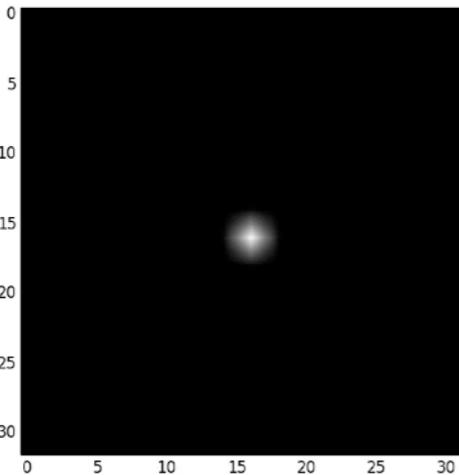
$\sigma=0.5$



$\sigma=1.0$



$\sigma=1.5$



# Application to transient imaging

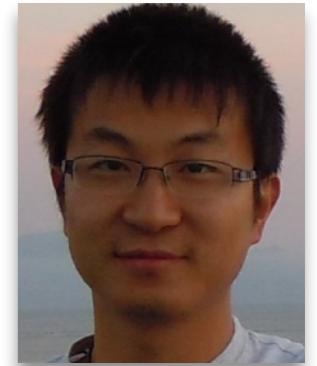
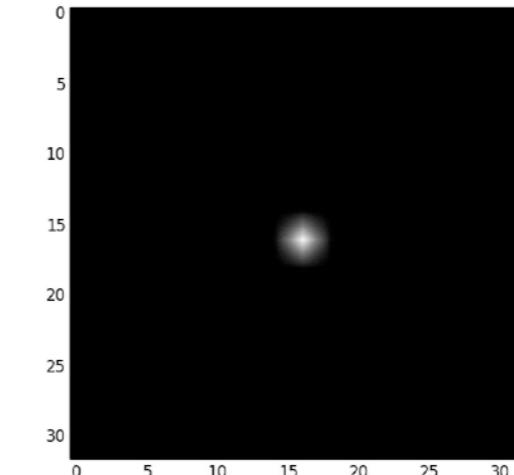
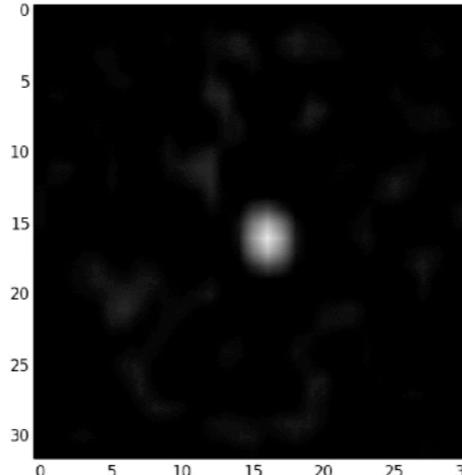
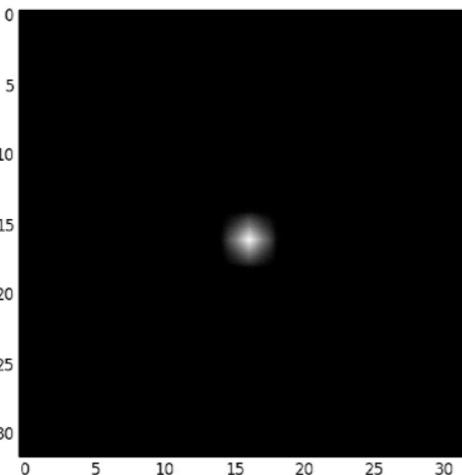
2D+time

Sky model

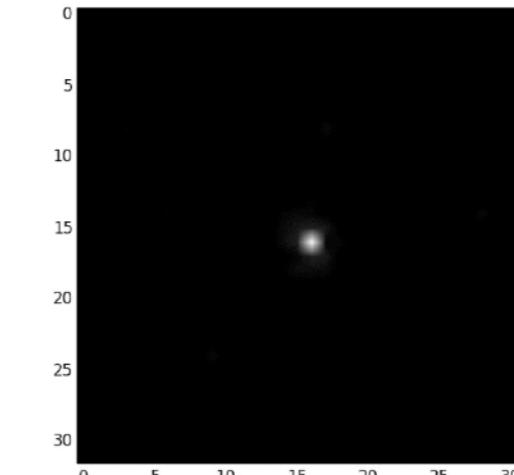
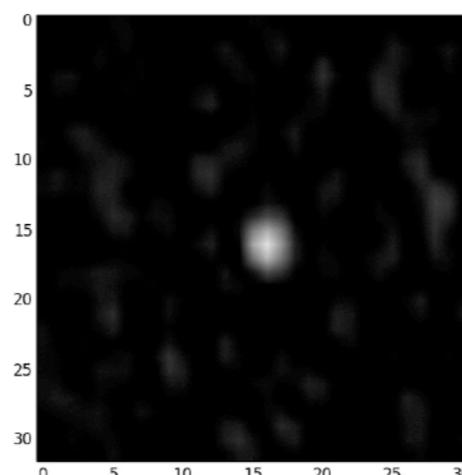
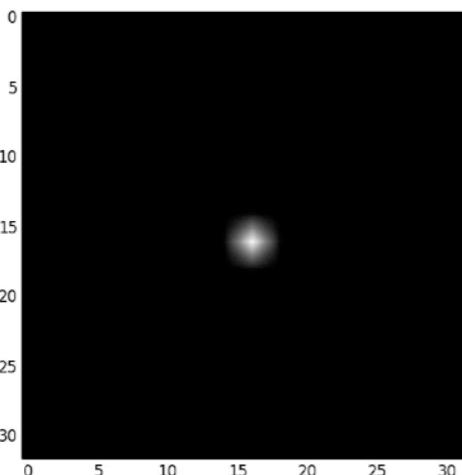
Dirty map

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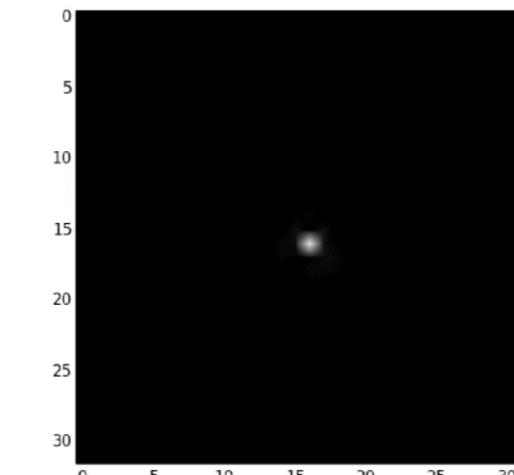
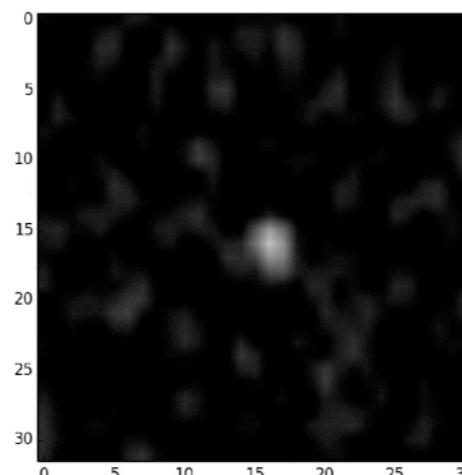
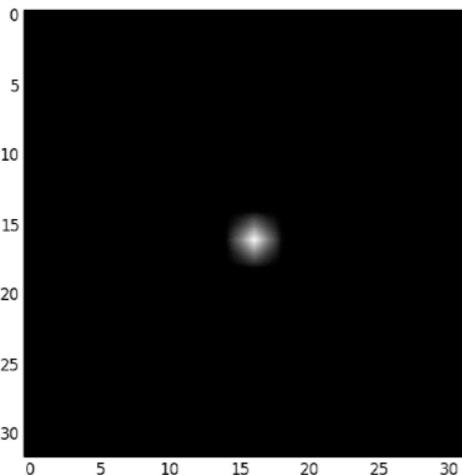
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$\sigma=1.0$



$\sigma=1.5$



# Numerical experiments on simulated data

**Detection robustness towards**

- noise level  $\sigma$  *detection problem*
- time integration  $N_t$  *dilution problem*

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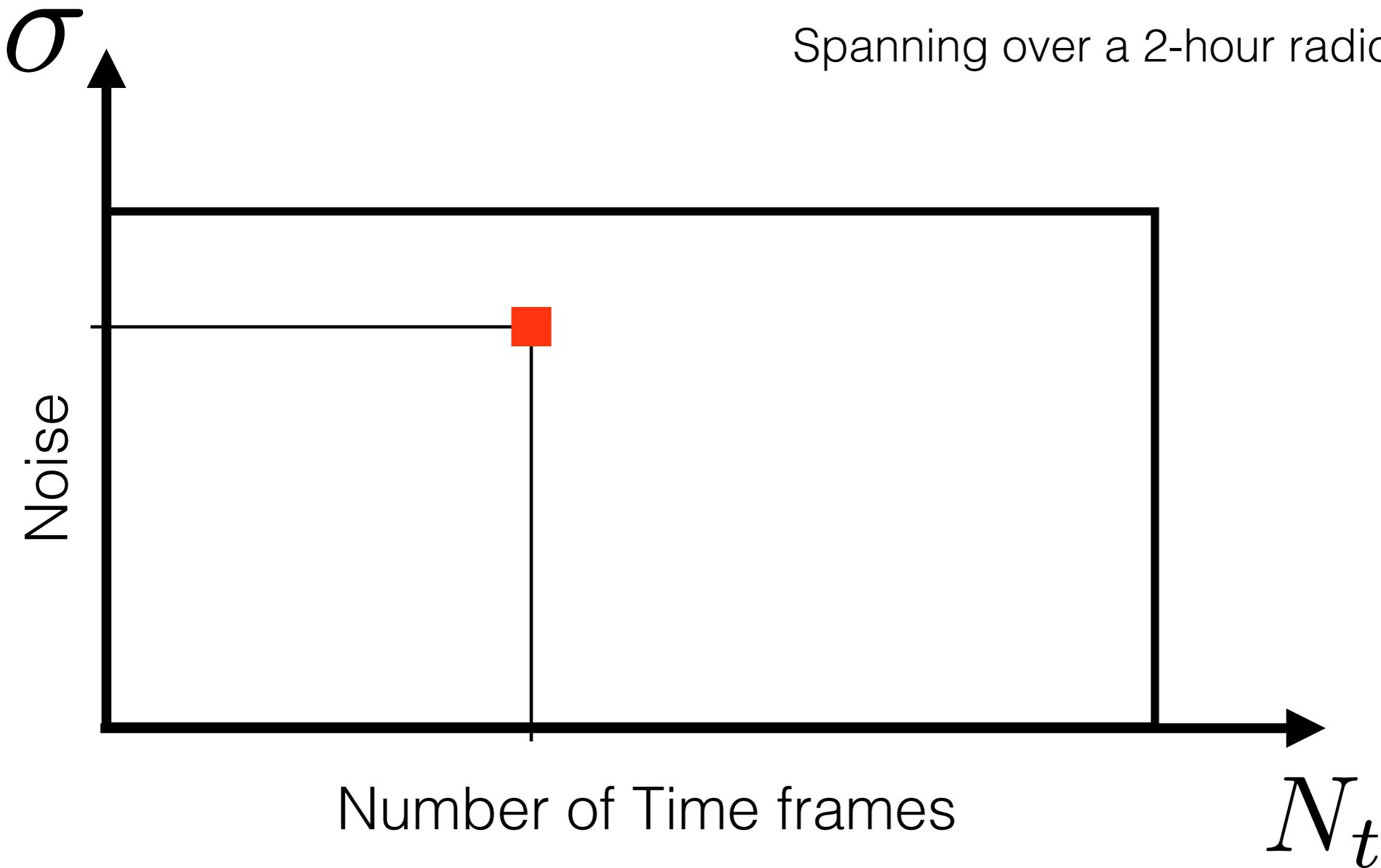
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## → Probing the parameter space $\sigma$ [ 0. - 2.0 ] arb. units

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$N_t$  [ 2 - 250 ] time frame

# Spanning over a 2-hour radio simulation



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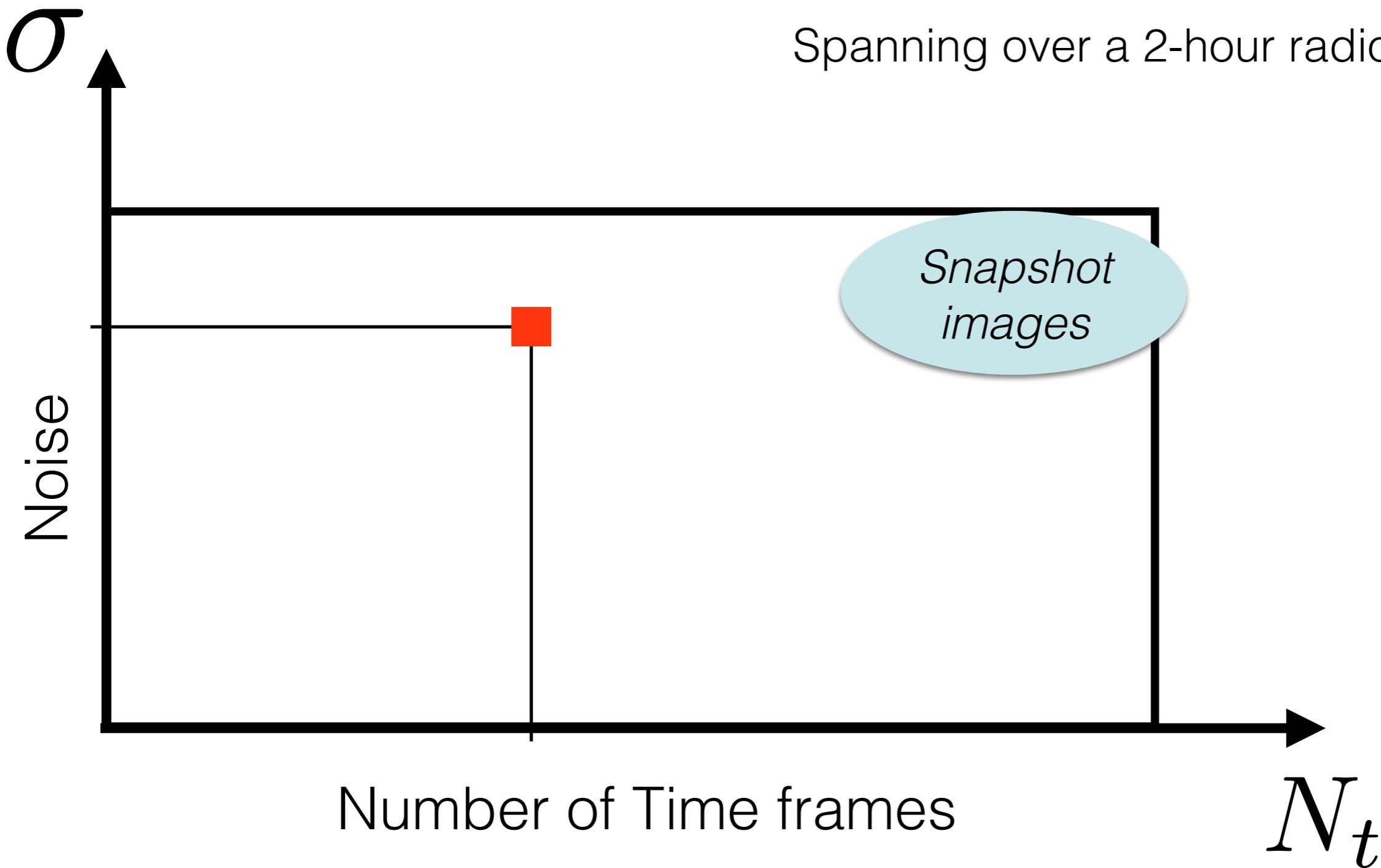
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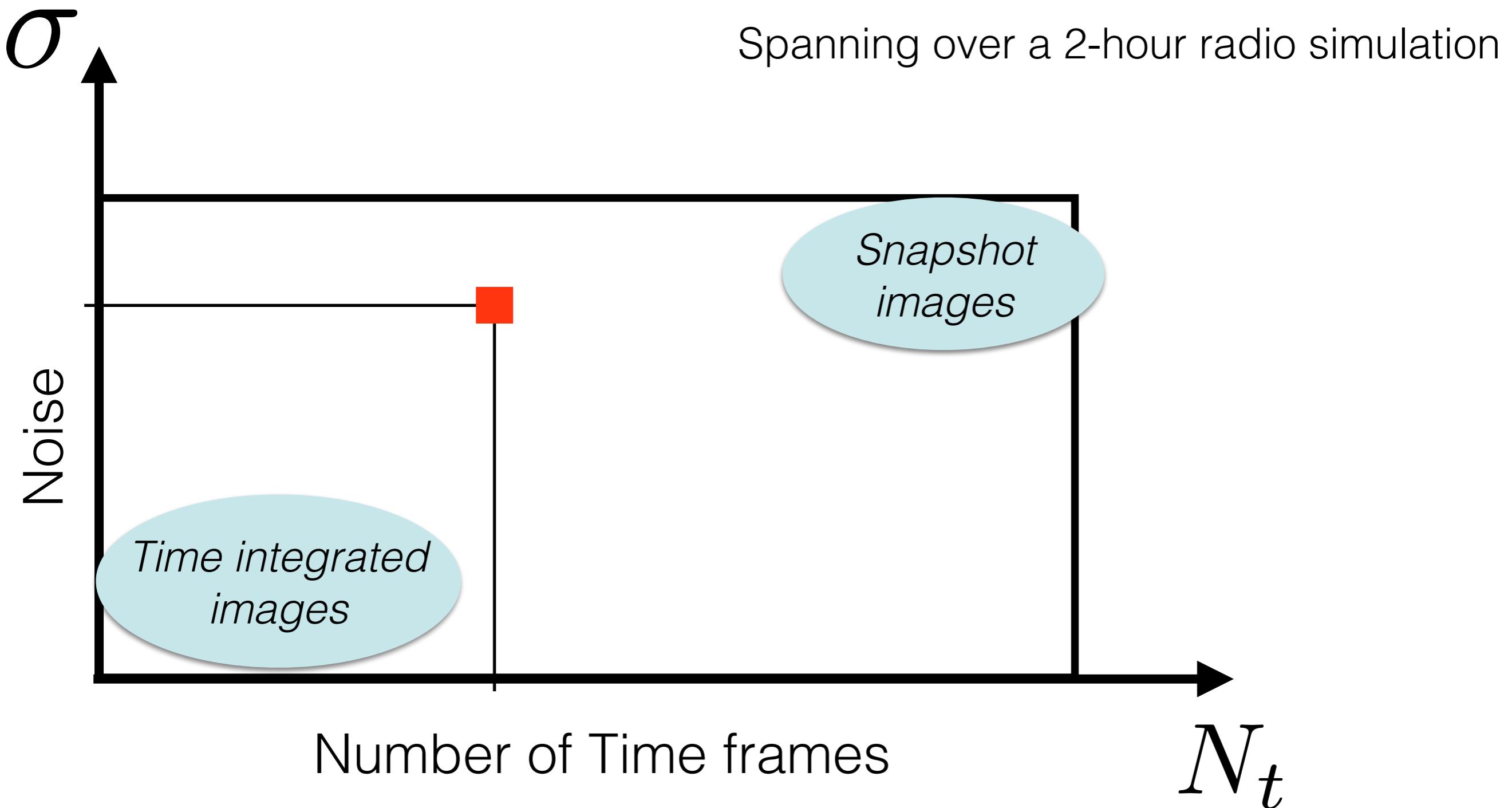
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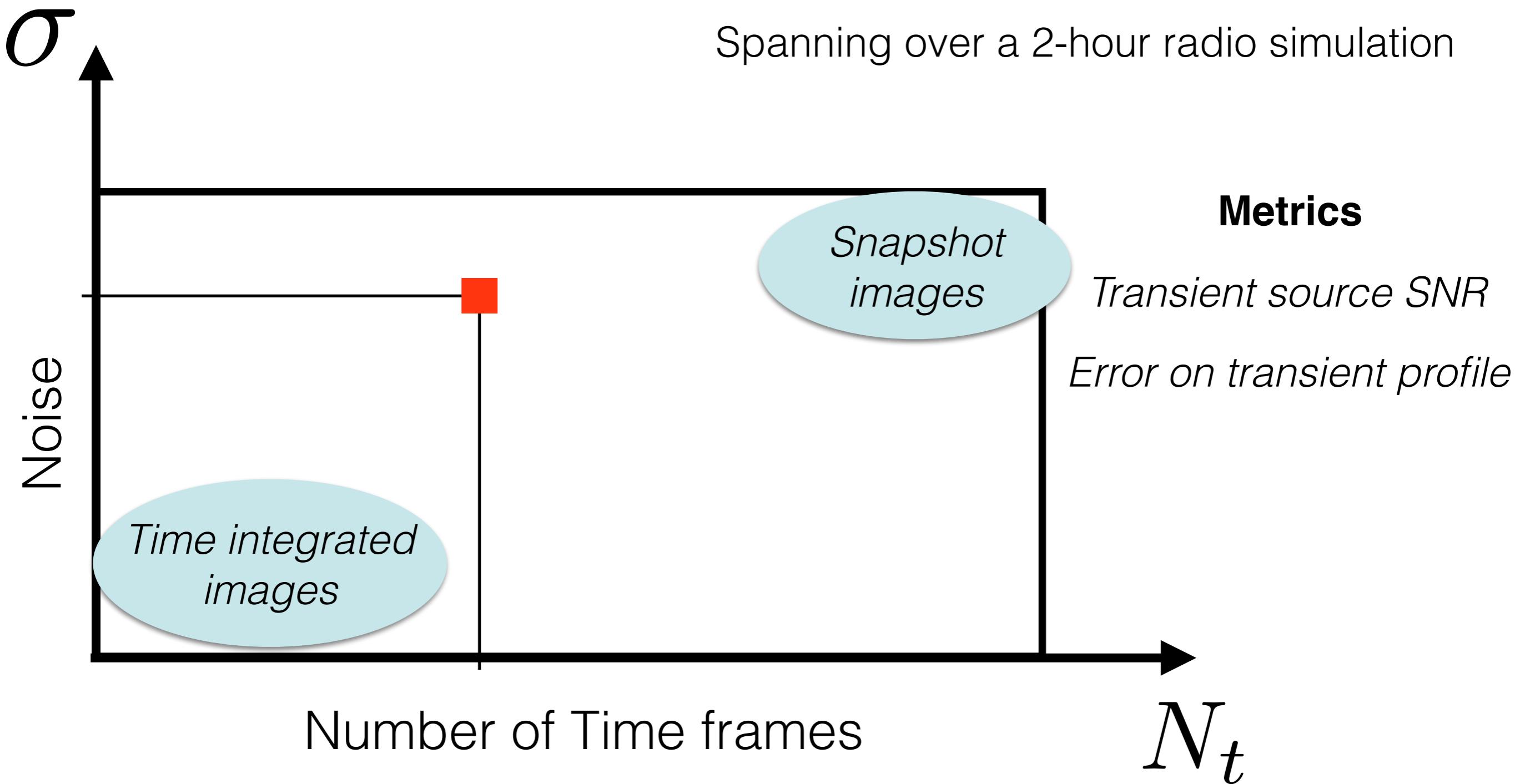
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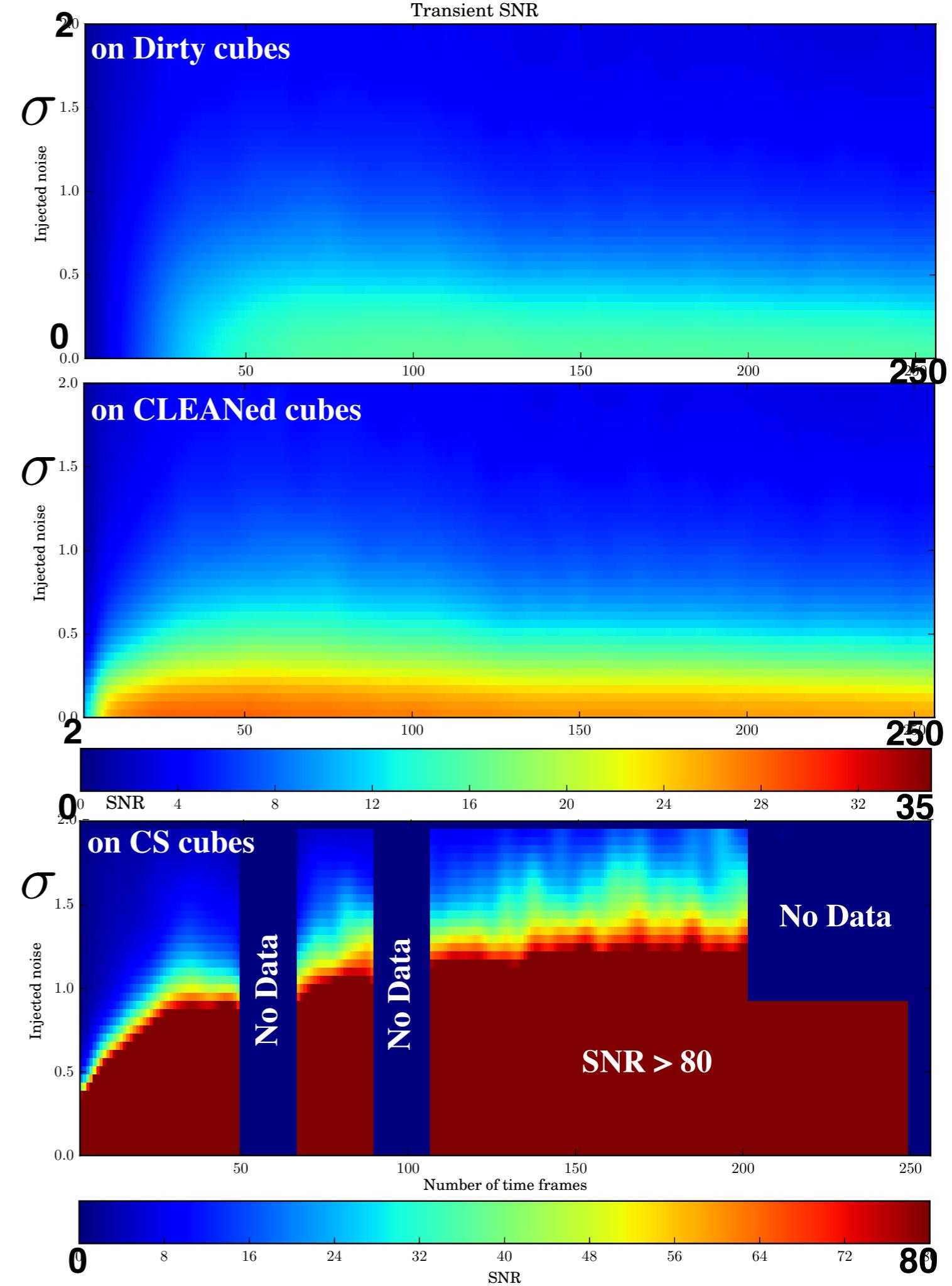
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# Spanning over a 2-hour radio simulation



# Test #1: SNR - Results



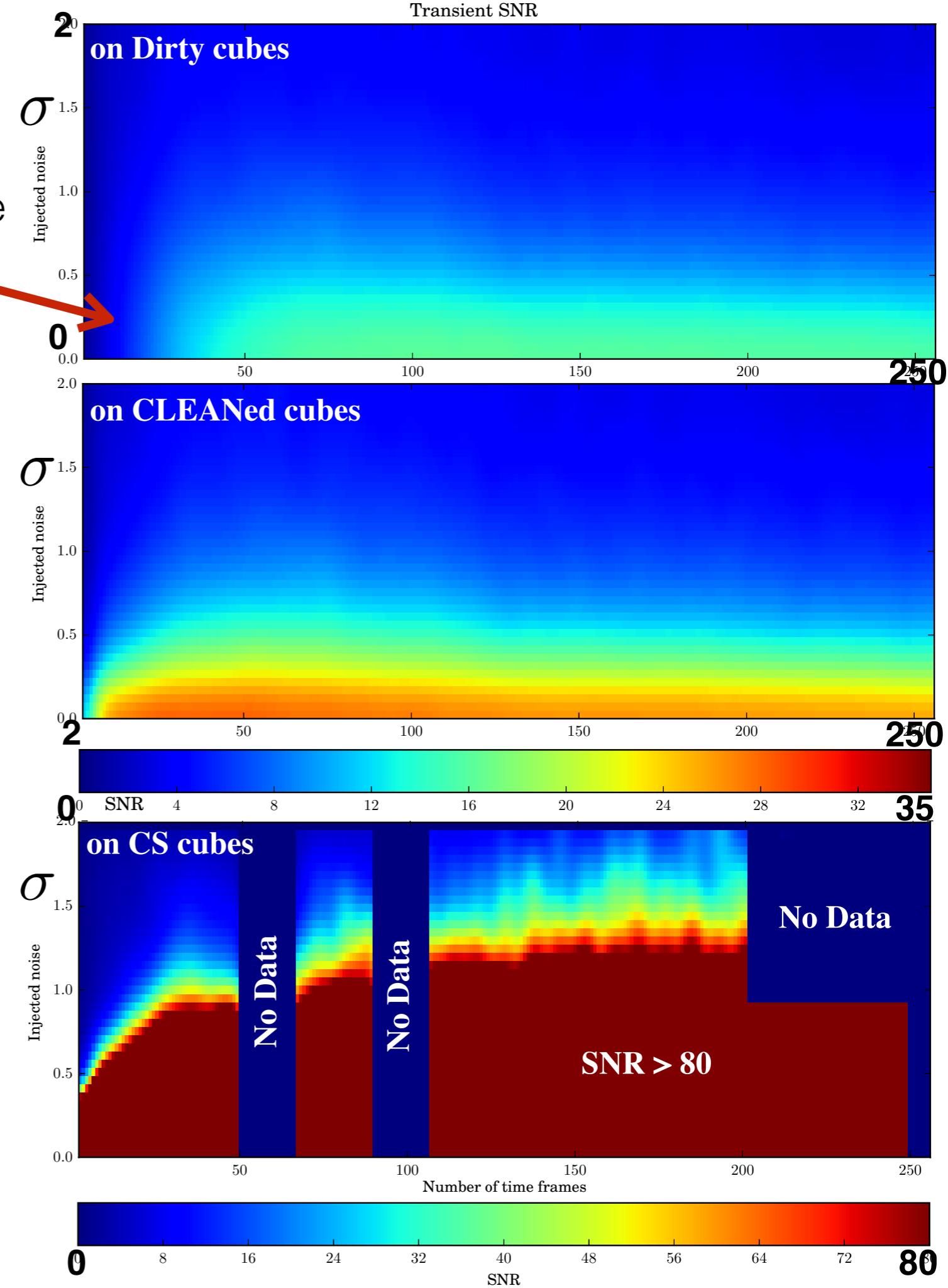
# Test #1: SNR - Results

## Dirty cubes

Marginal improvement of SNR with time

SNR turn-over below  $N_t = 40$

→ temporal dilution



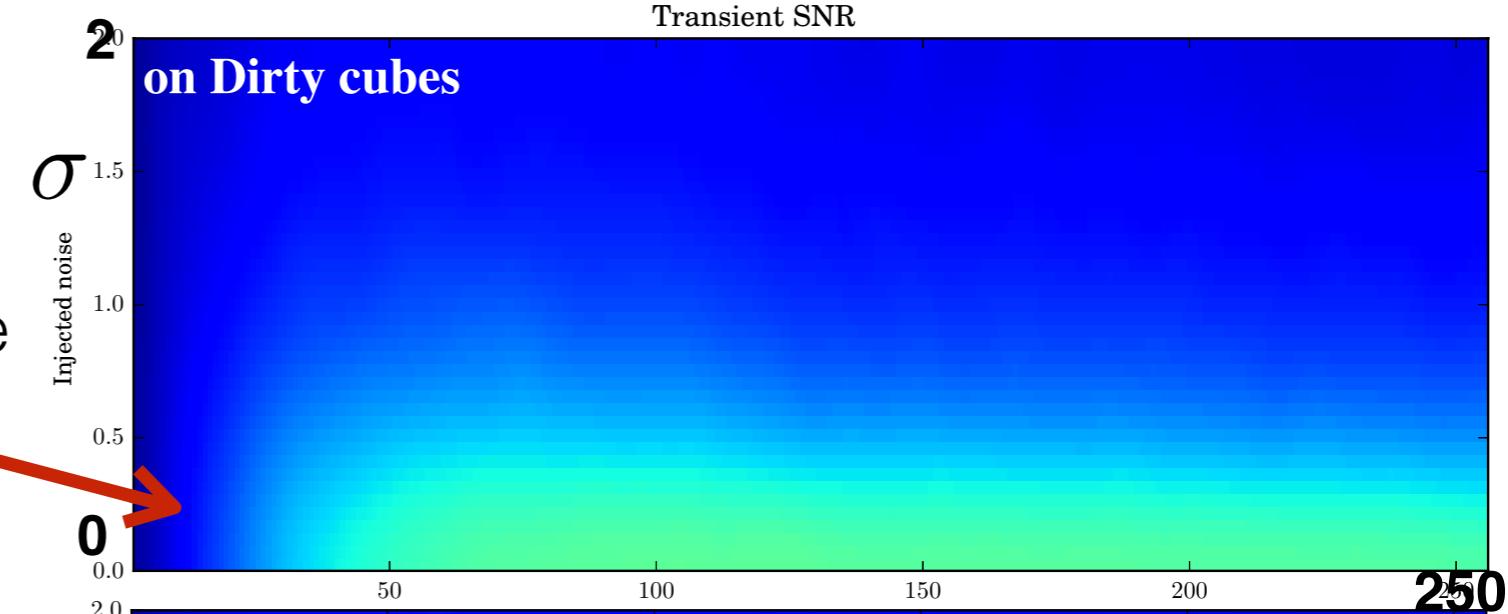
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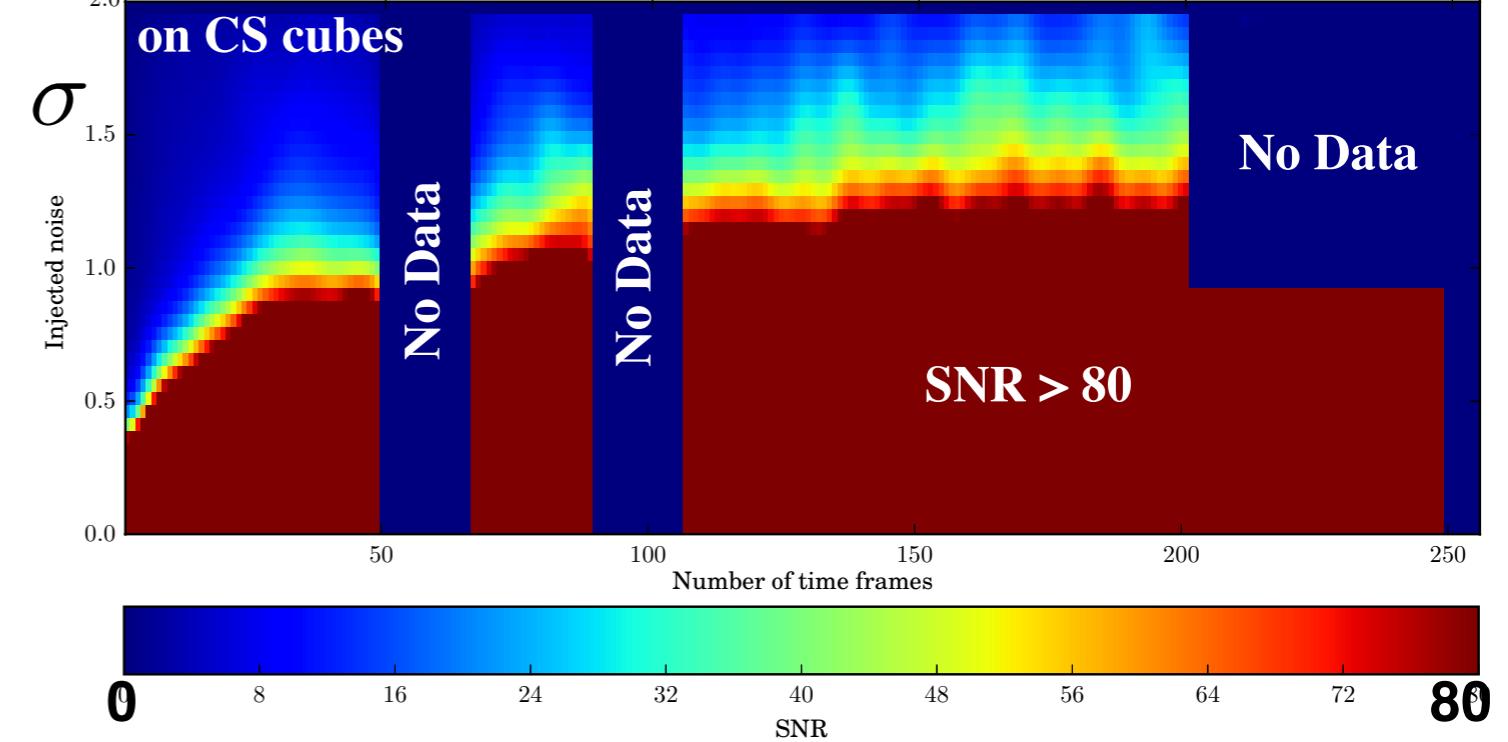
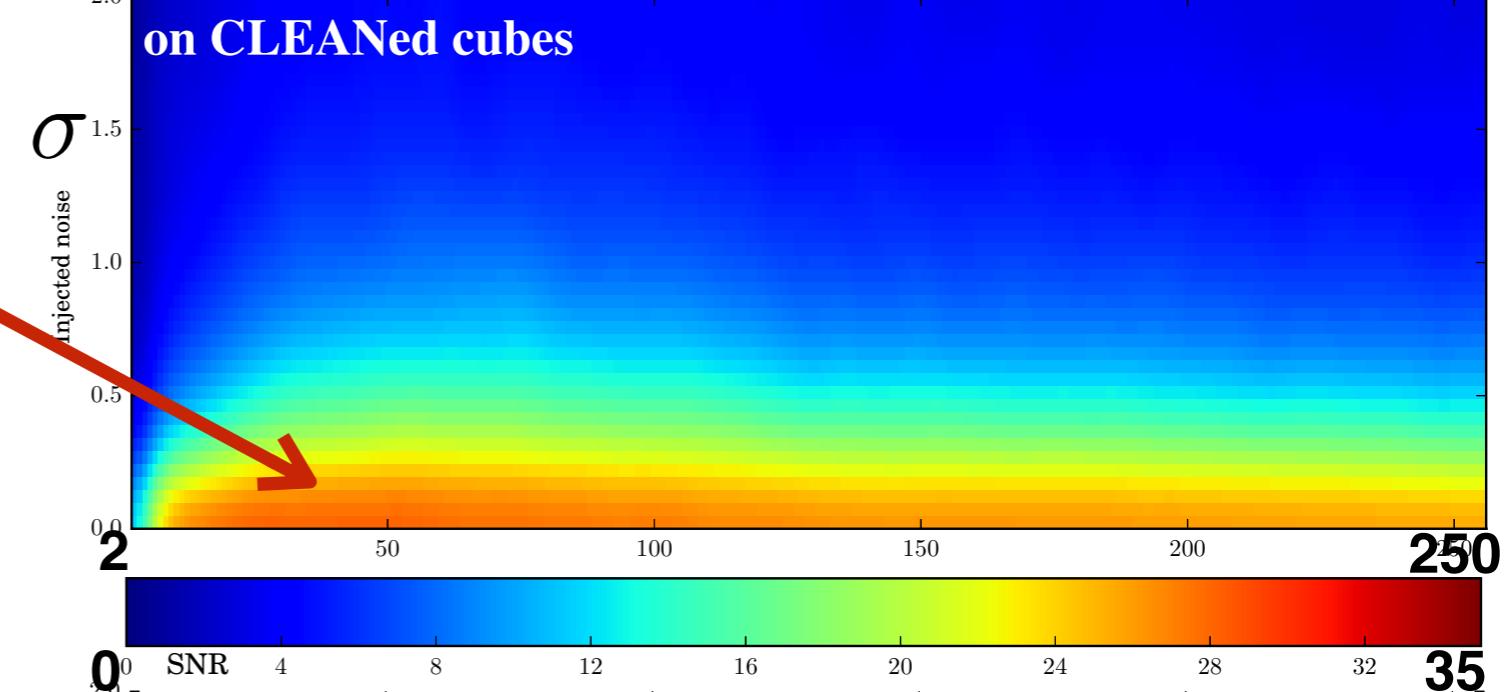
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## CLEANed cubes

Higher SNR at low noise

Temporal dilution effect reduced



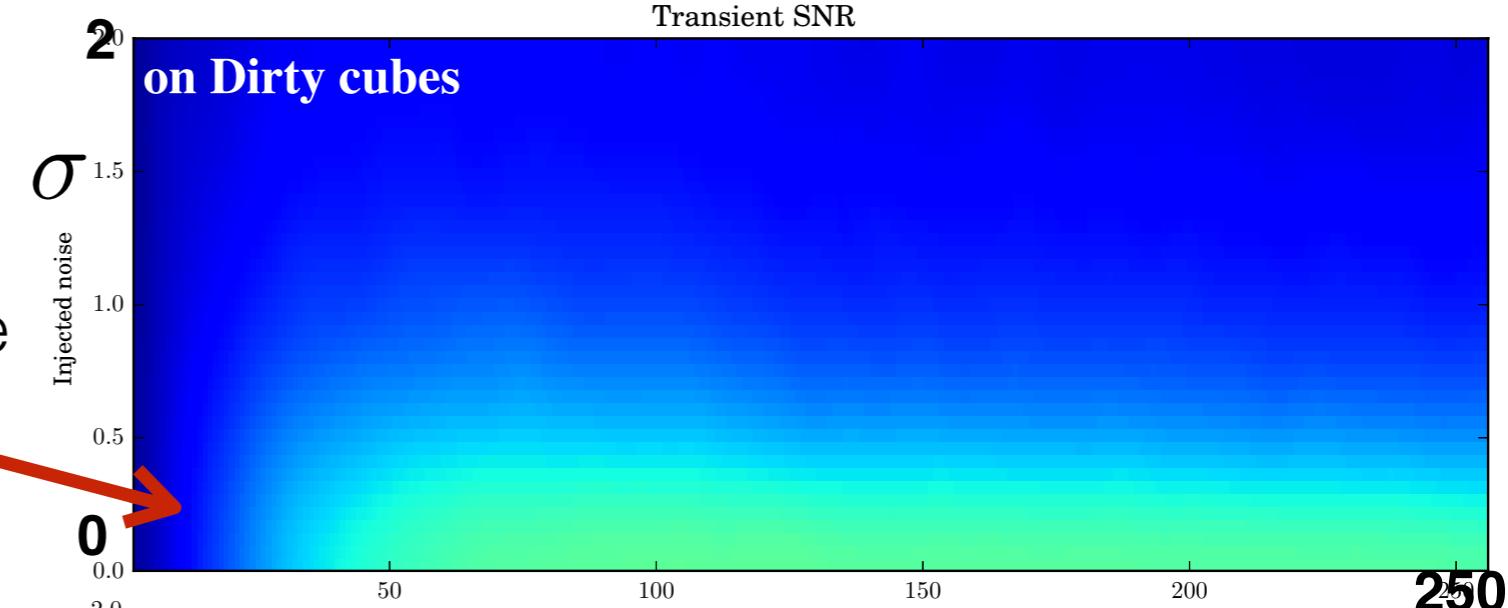
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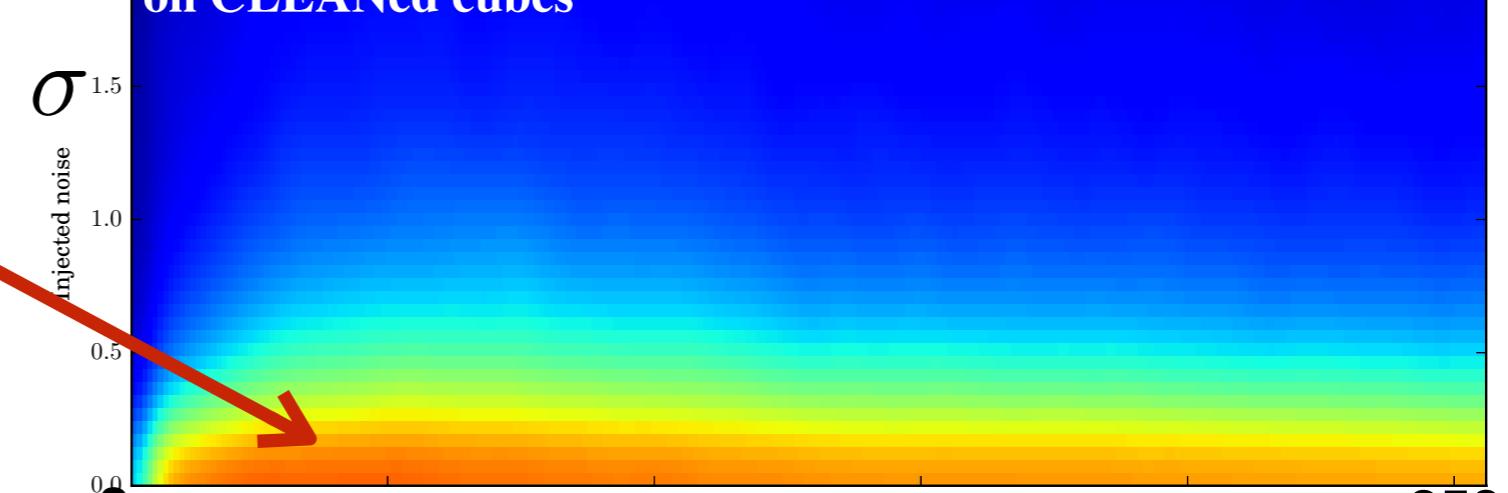
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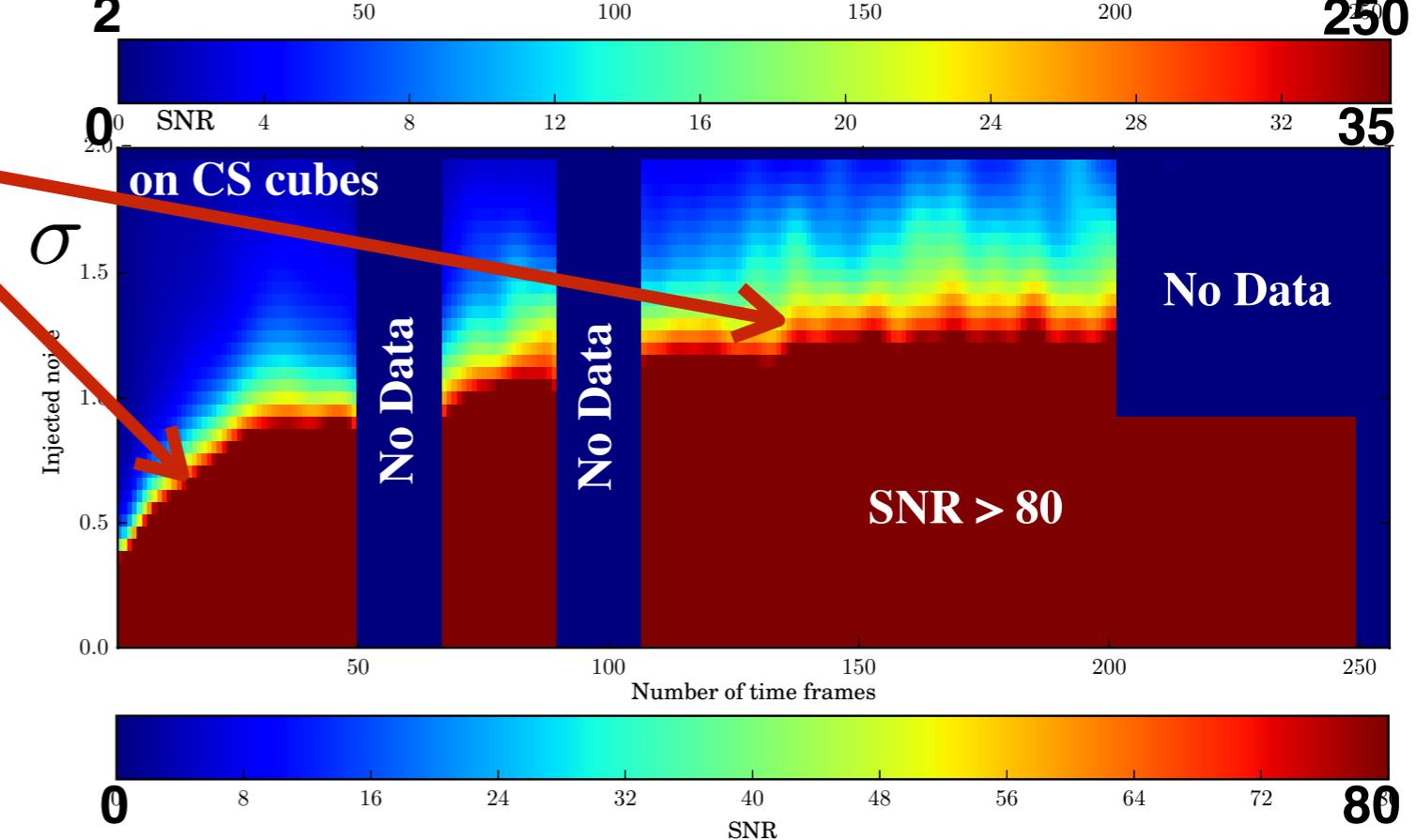
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## 2D1D Sparse cube

Higher SNR at low & high noise

Slow decrease of SNR due to dilution



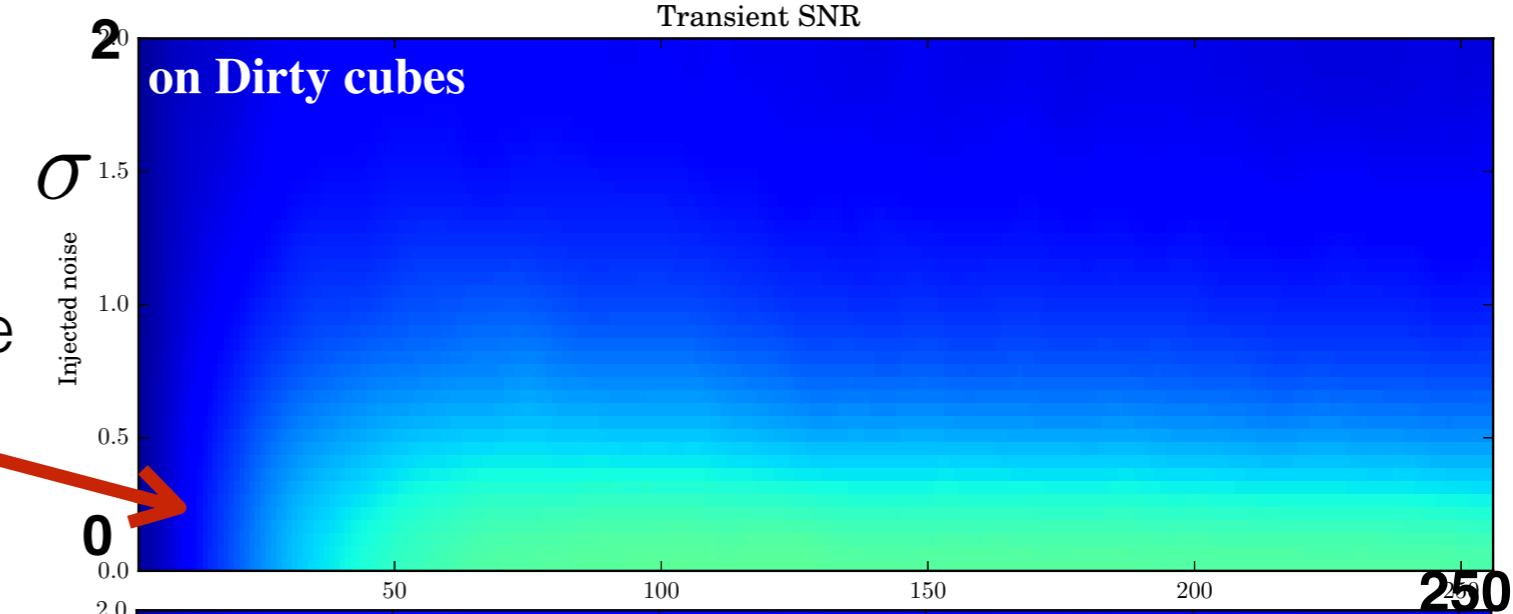
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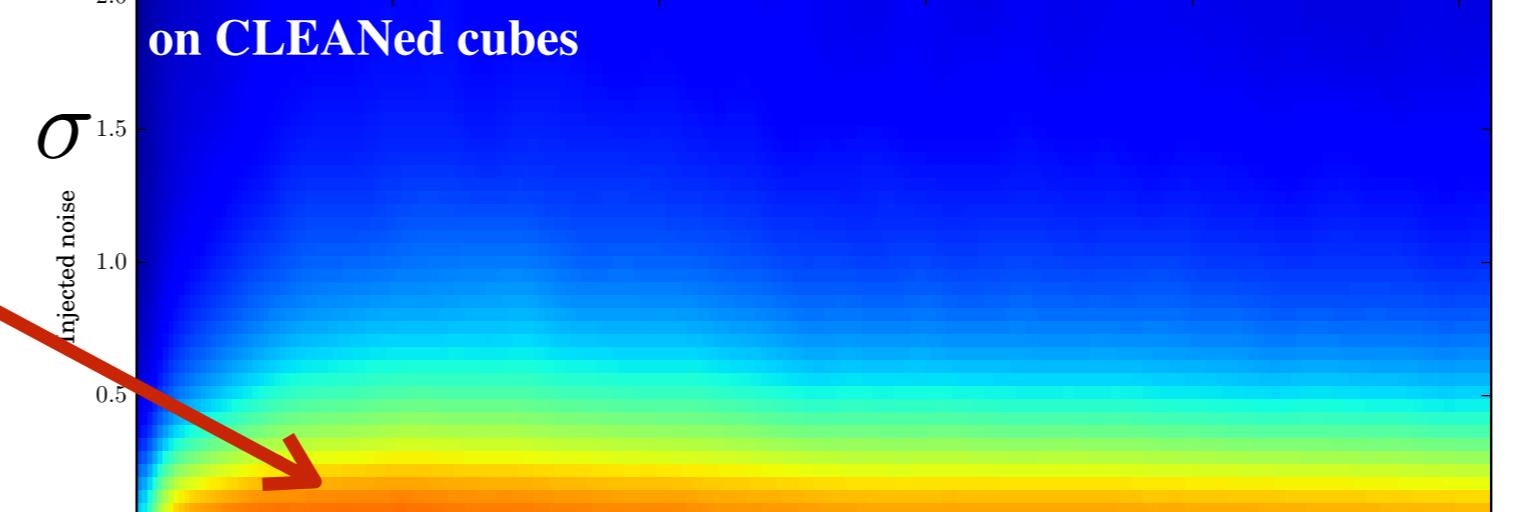
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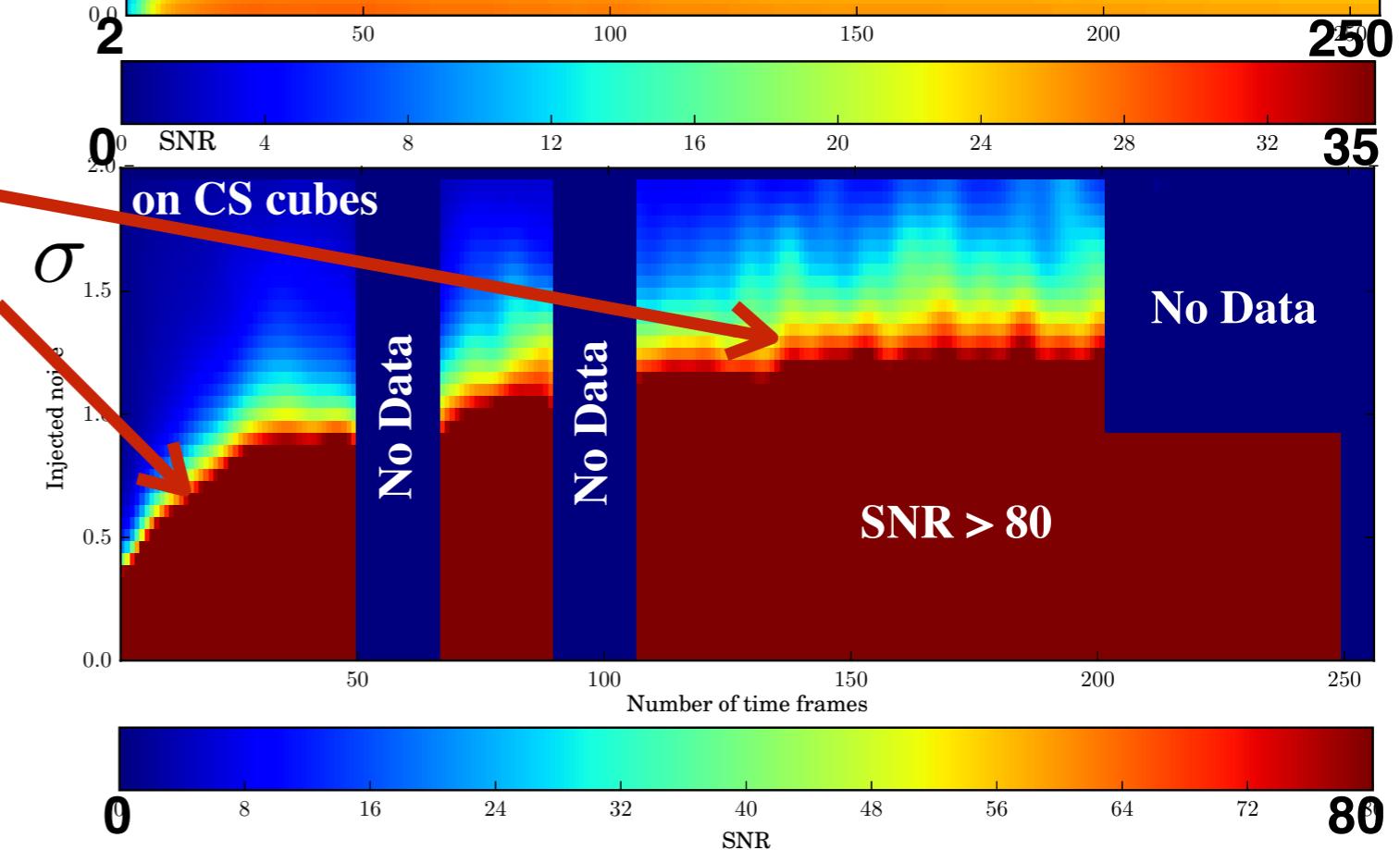
Temporal dilution effect reduced



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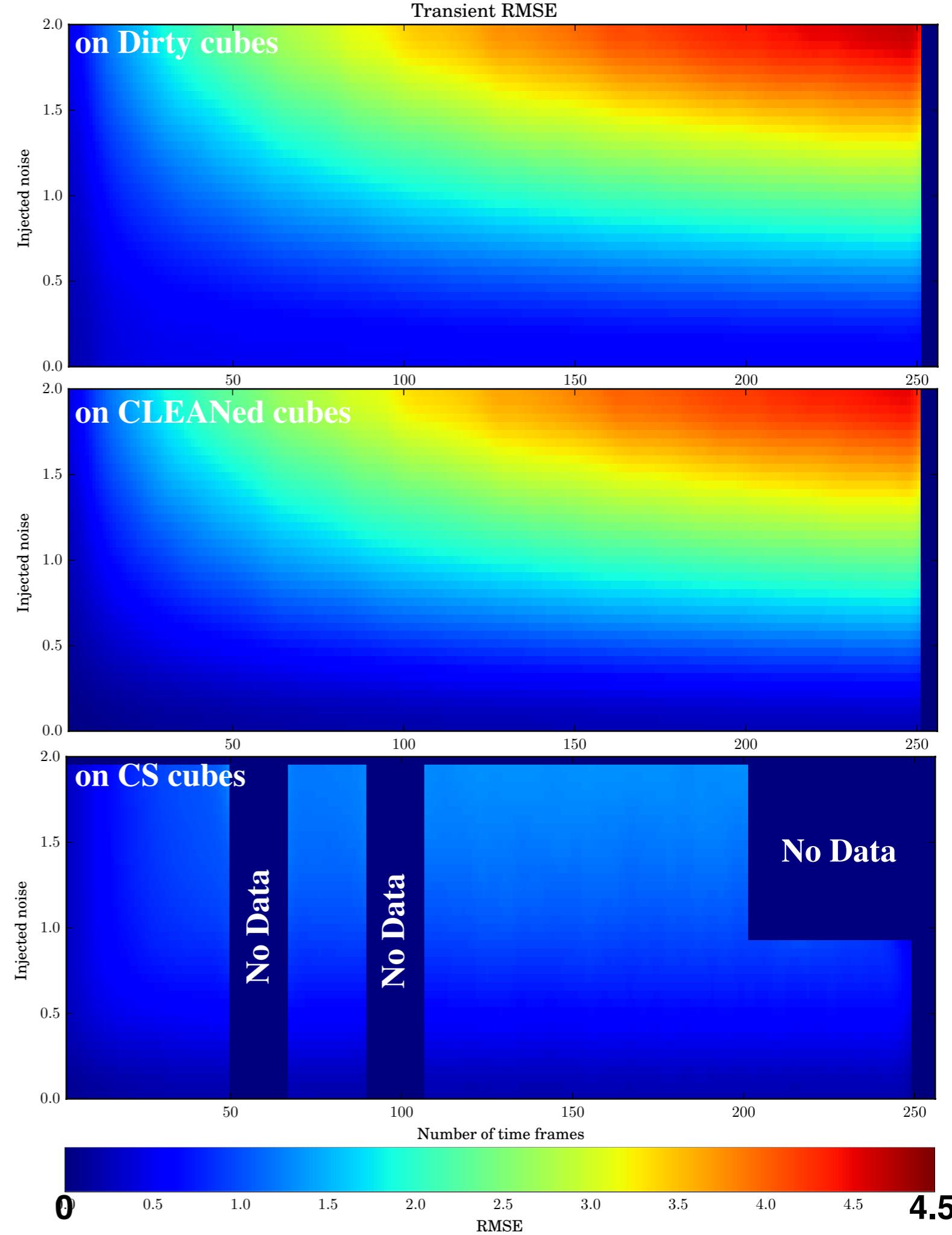
Higher SNR at low & high noise

Slow decrease of SNR due to dilution



**One order of magnitude improvement in SNR**

# Test #2: Time profile

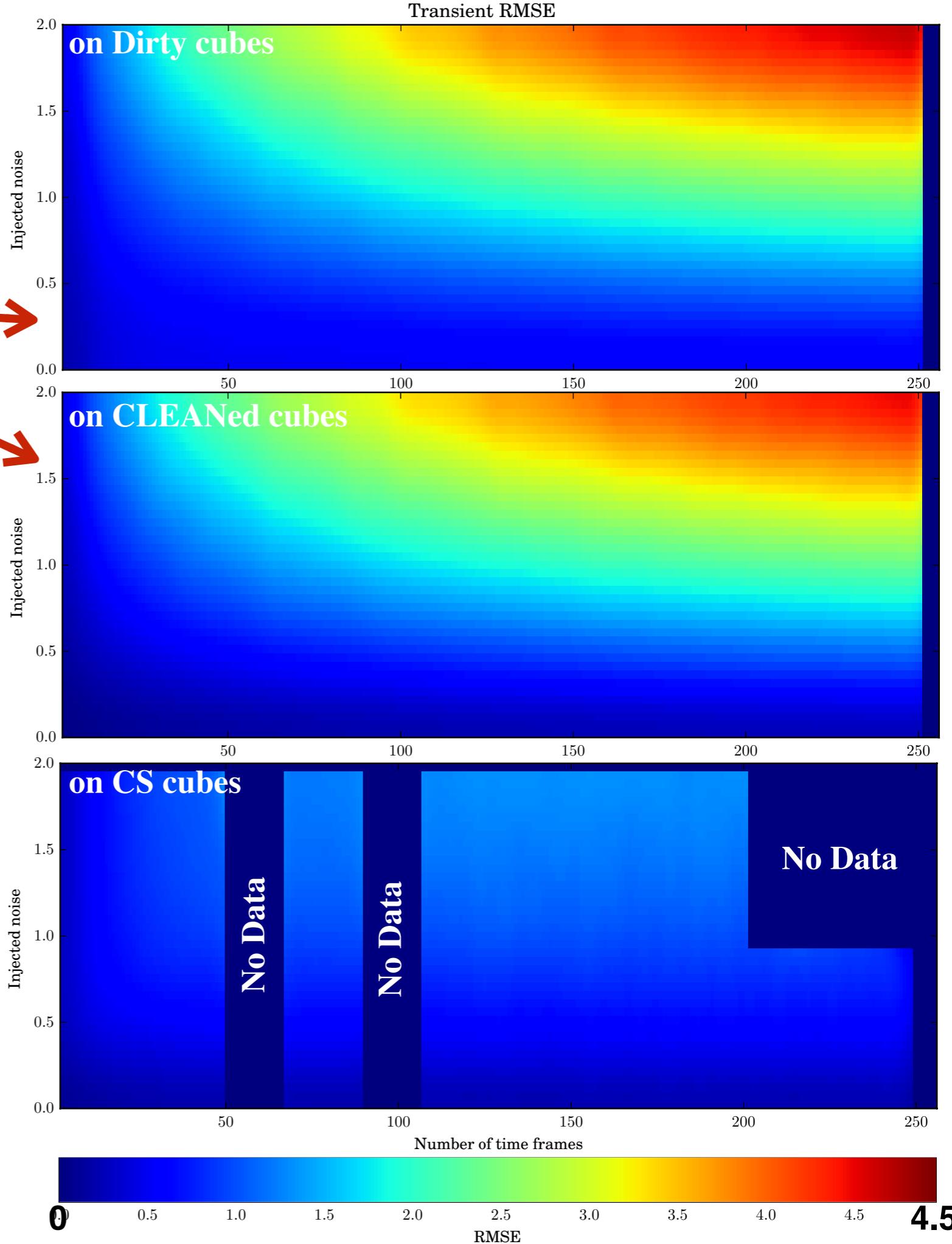


# Test #2: Time profile

## Dirty & CLEANed cubes

Higher RMSE in high noise snapshots

Similar profile error

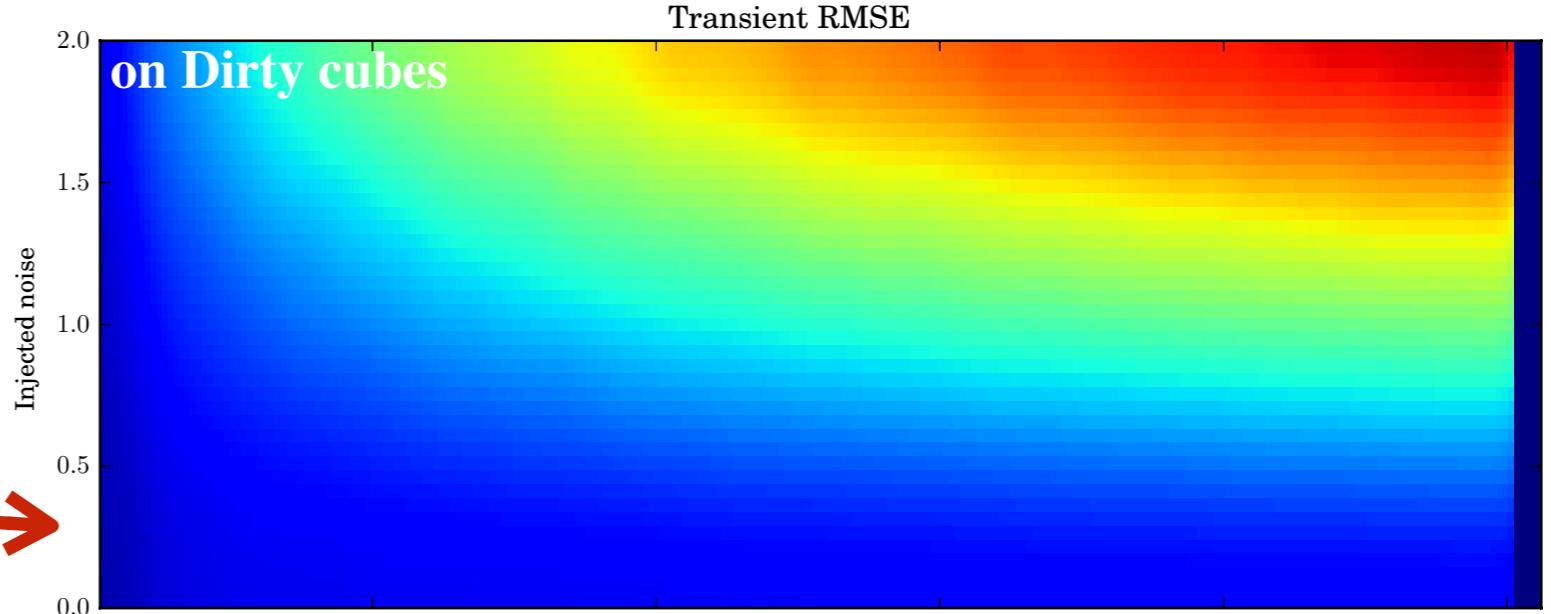


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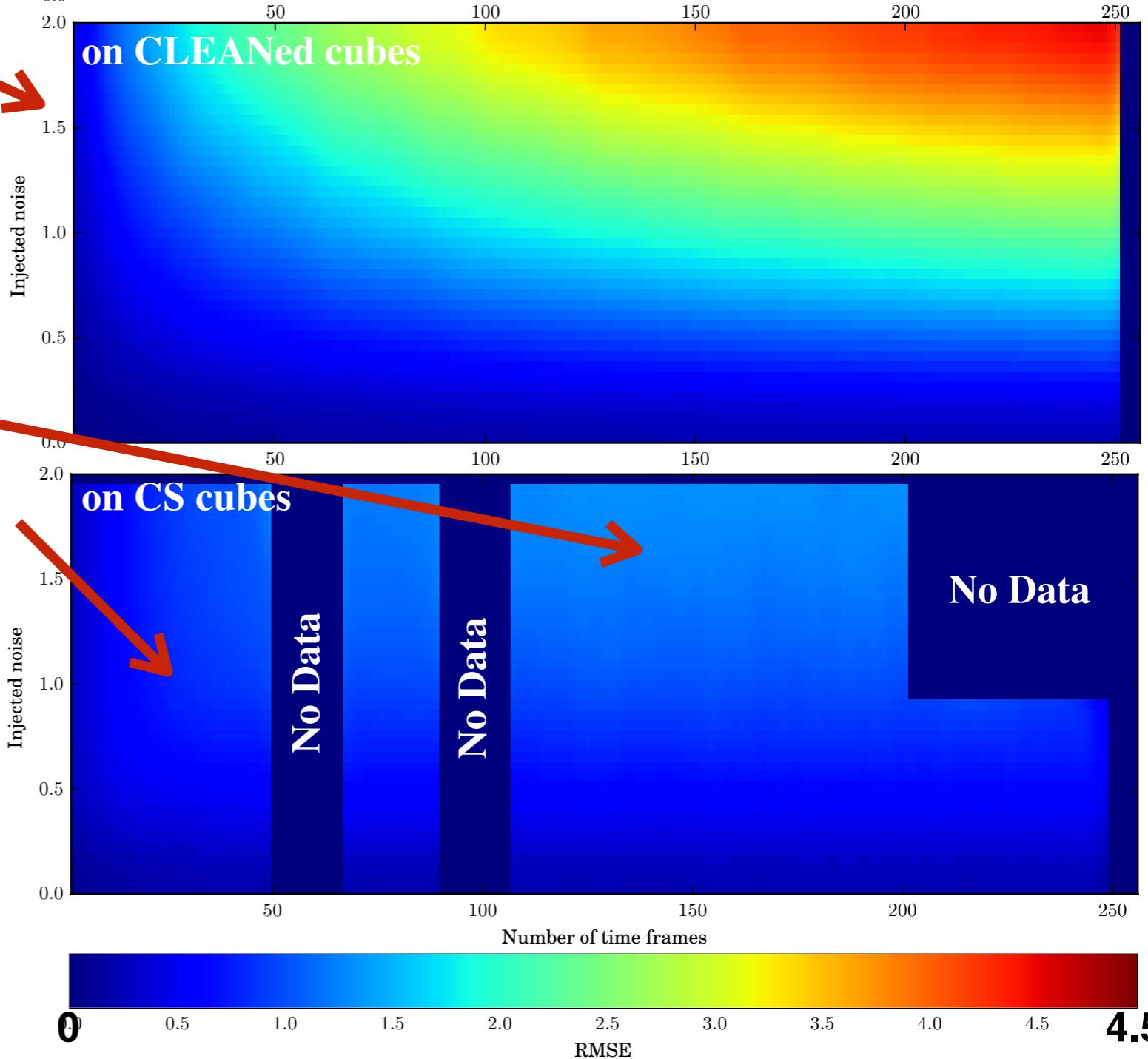
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Higher RMSE in high noise snapshots

Better overall profile reconstruction

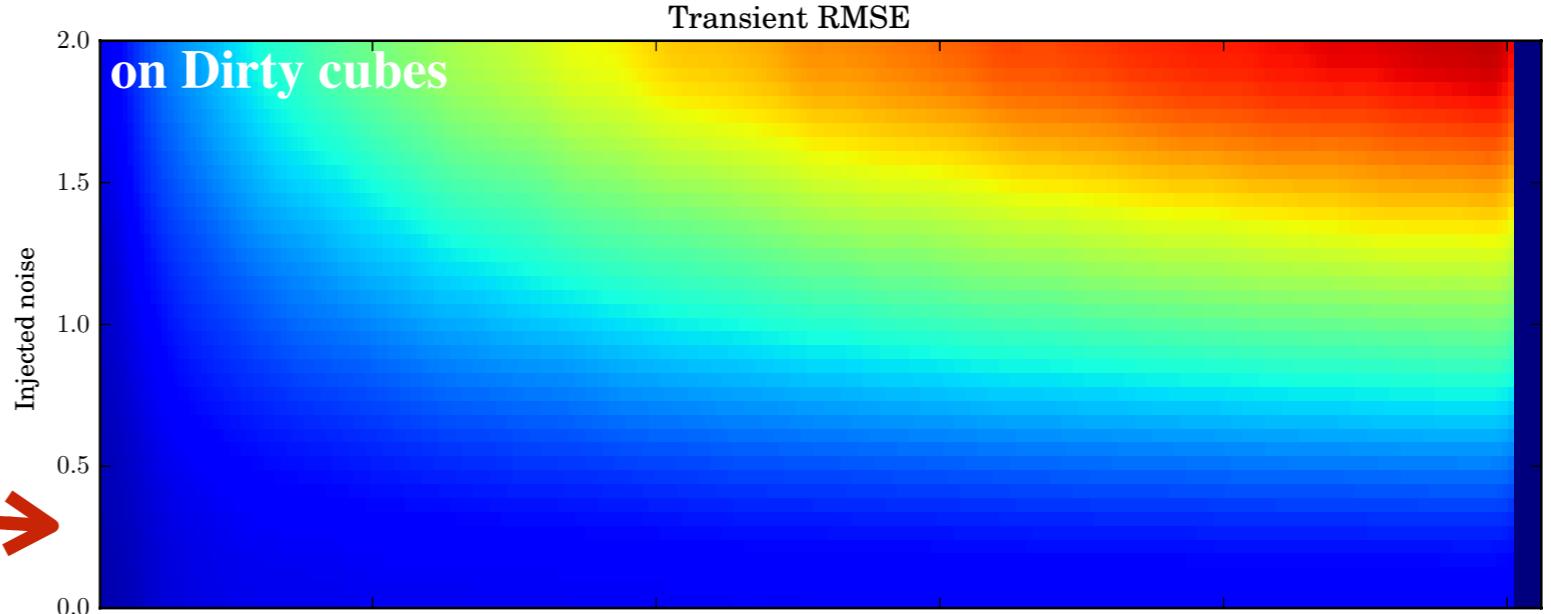


# Test #2: Time profile

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Higher RMSE in high noise snapshots

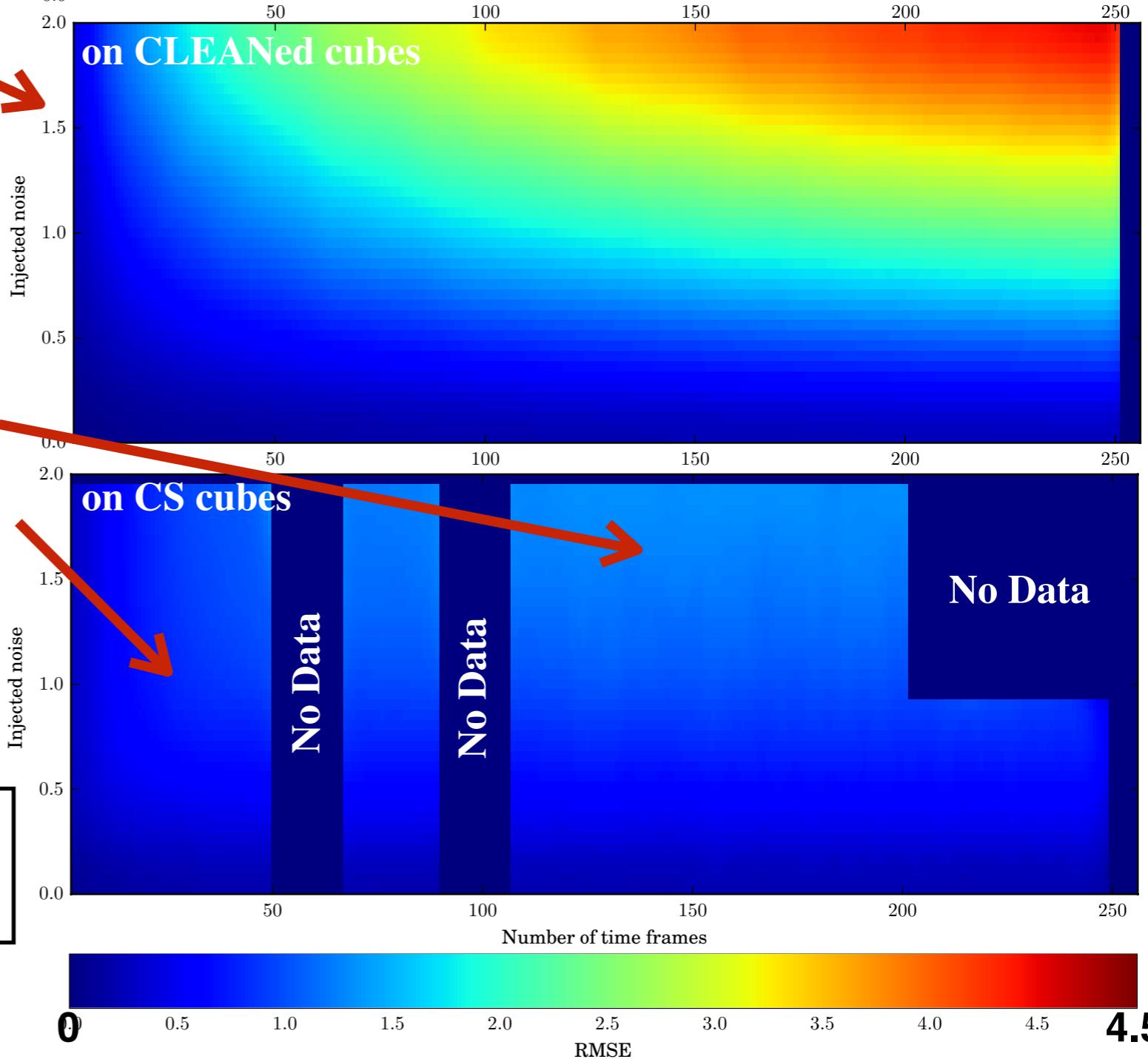
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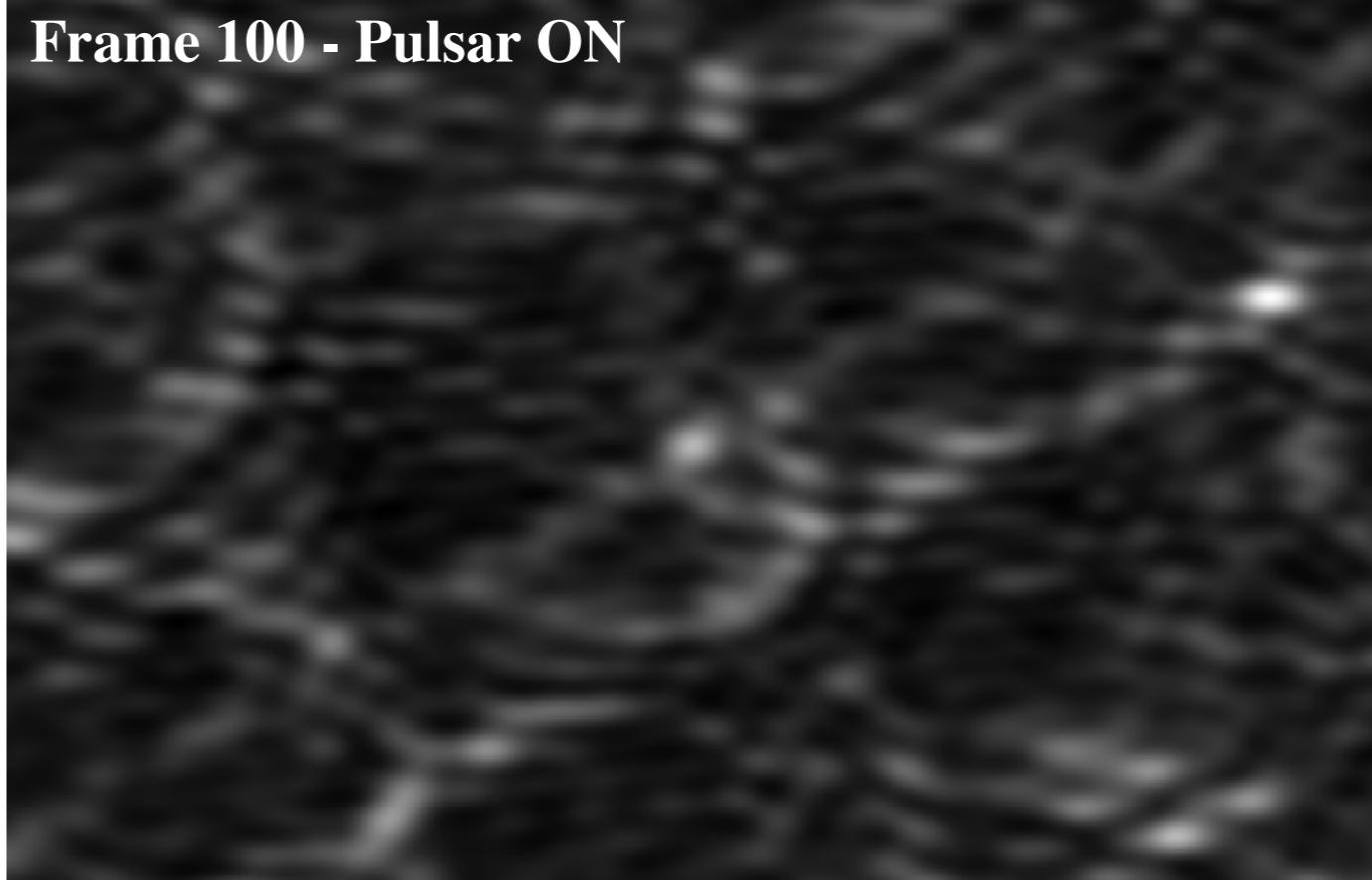
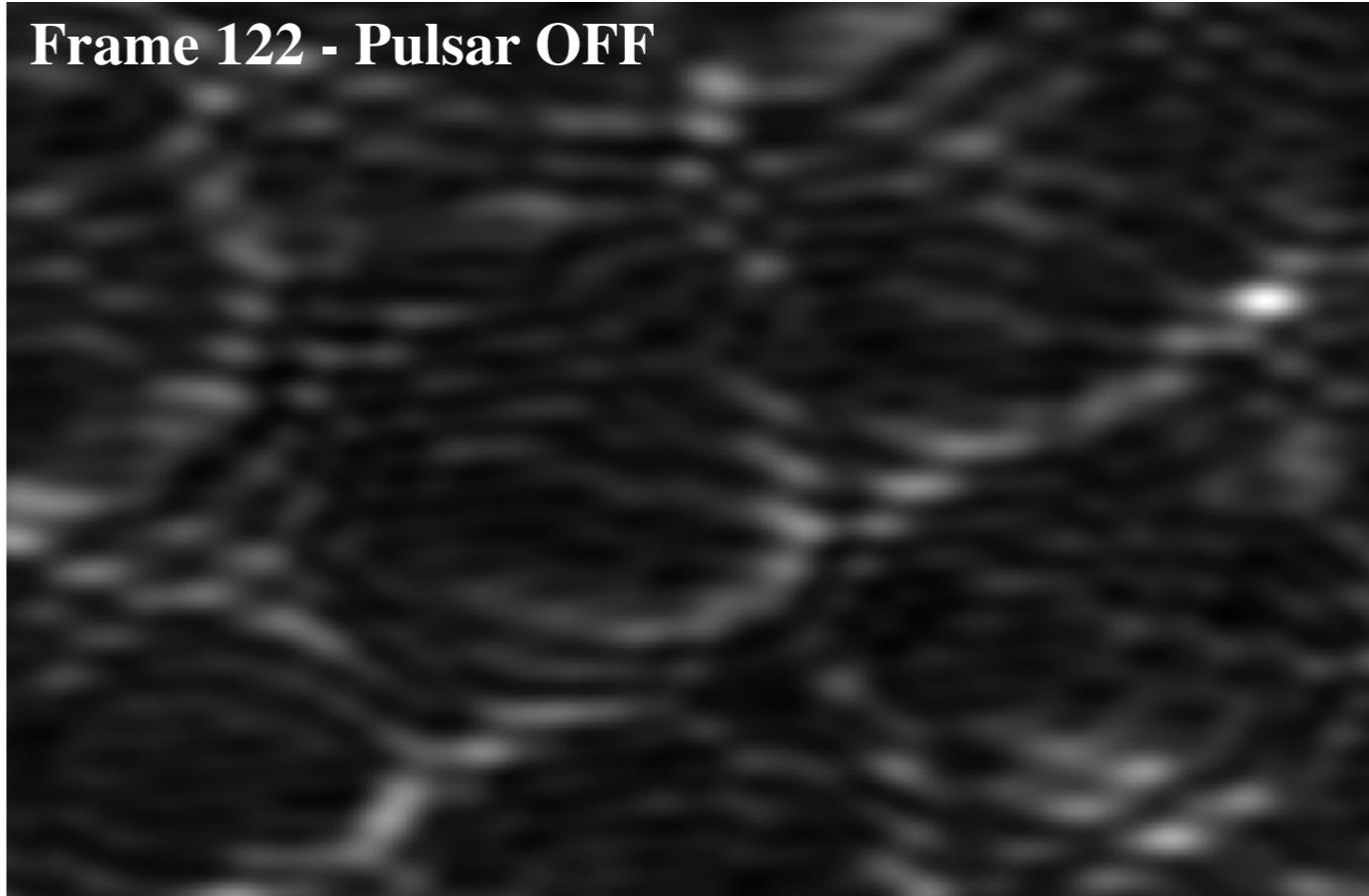
Factor of ~3 reduction of the  
RMSE of transient profile

## Test #3: Real data: pulsar B0355+55 with the VLA

data  $\Delta t = 1.03s$

res.  $\delta t = 5ms$

Period  $P_{\text{B0355+55}} = 156ms$



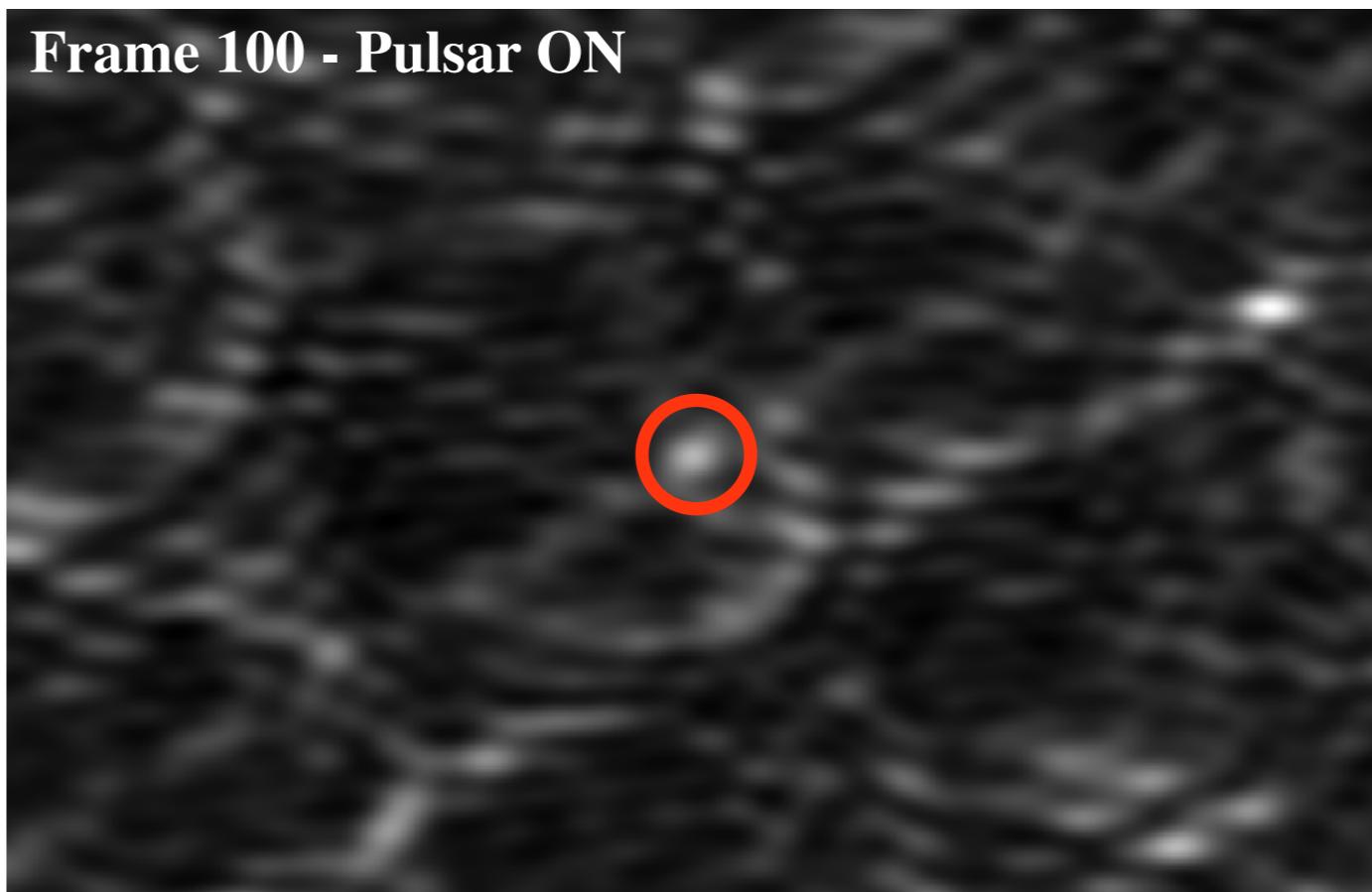
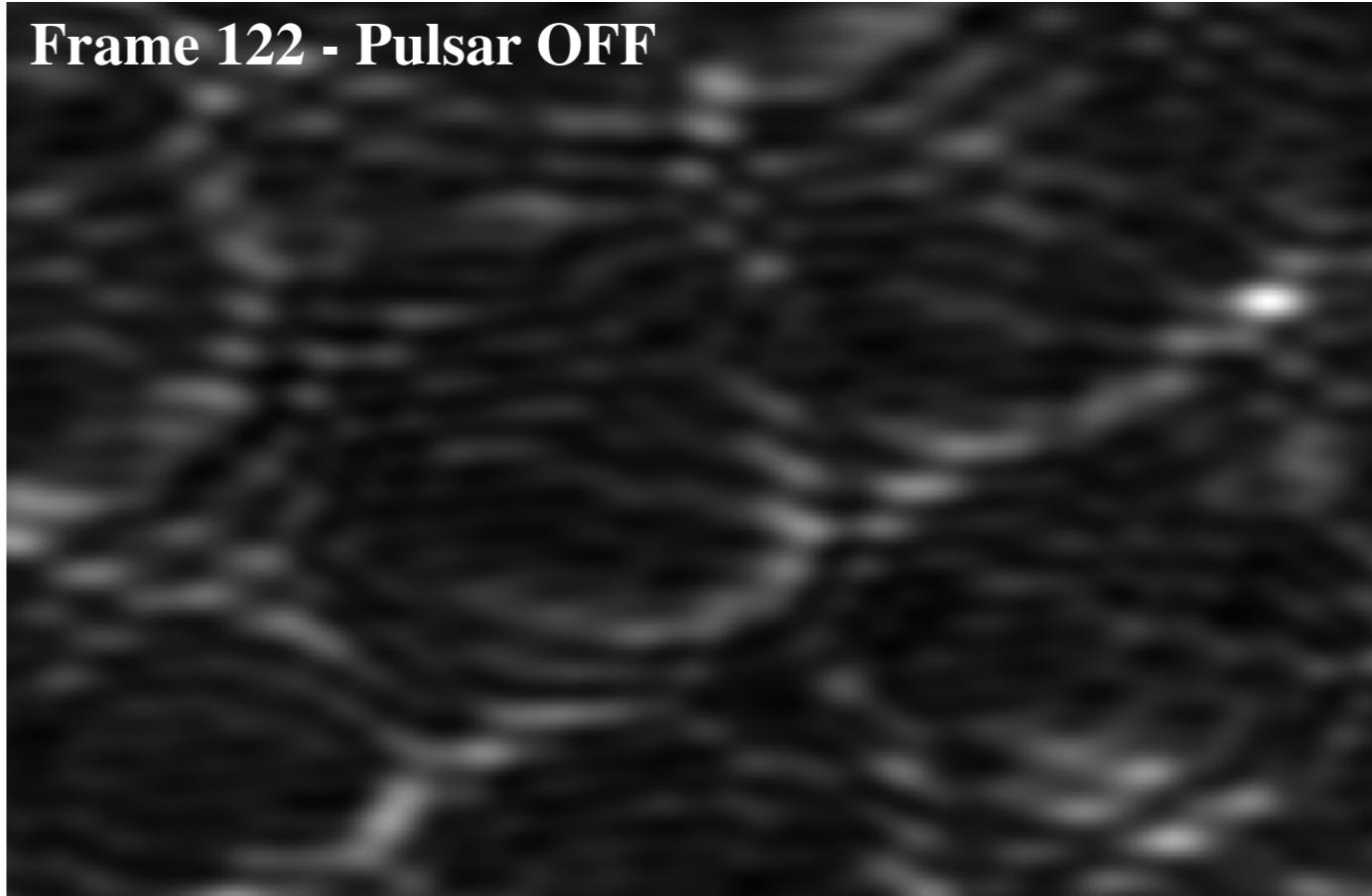
- **Transient** takes the form of the PSF
- **Sidelobes** are also transients !

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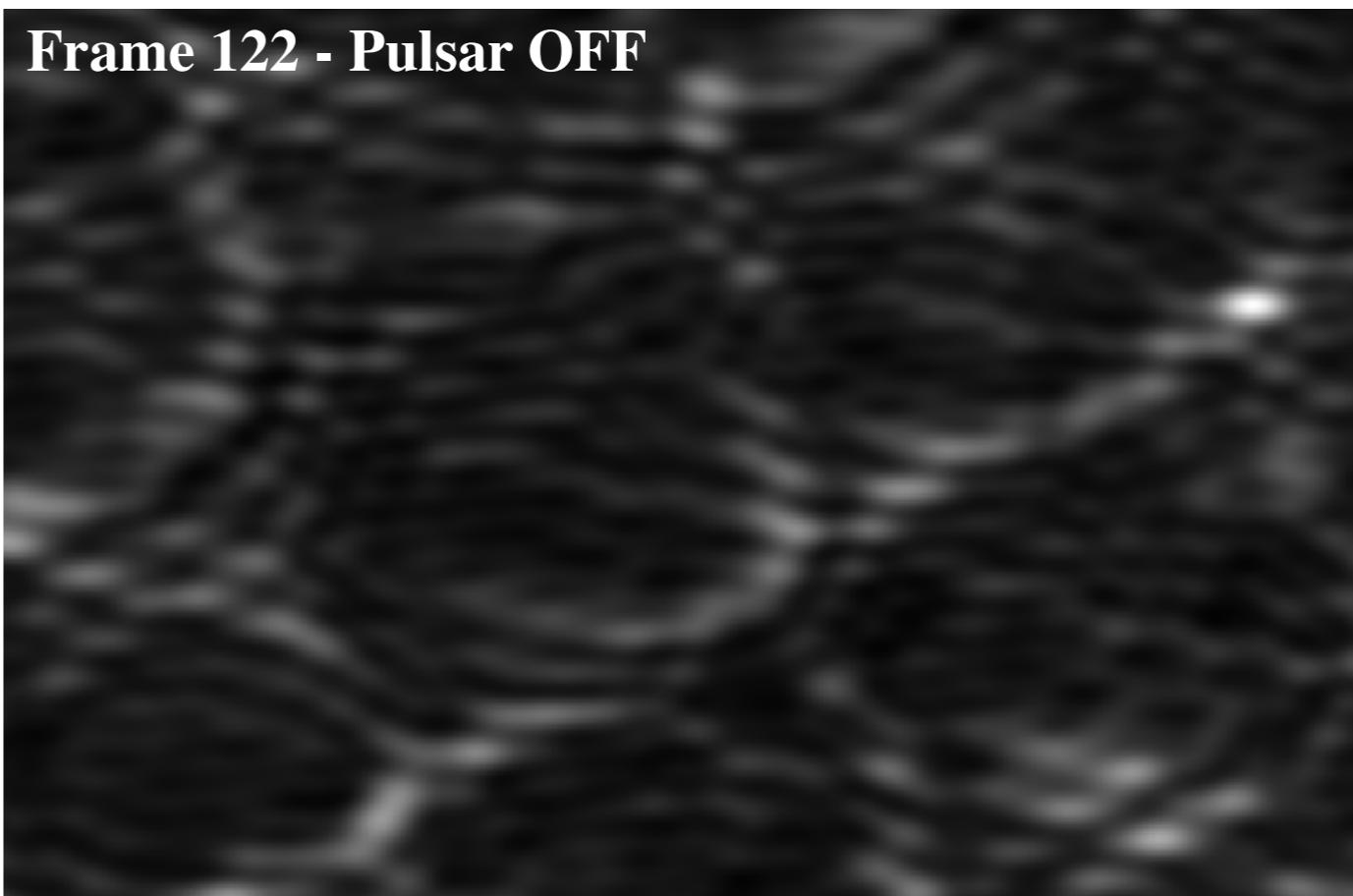
# Test #3: Real data: pulsar B0355+55 with the VLA

data  $\Delta t = 1.03s$

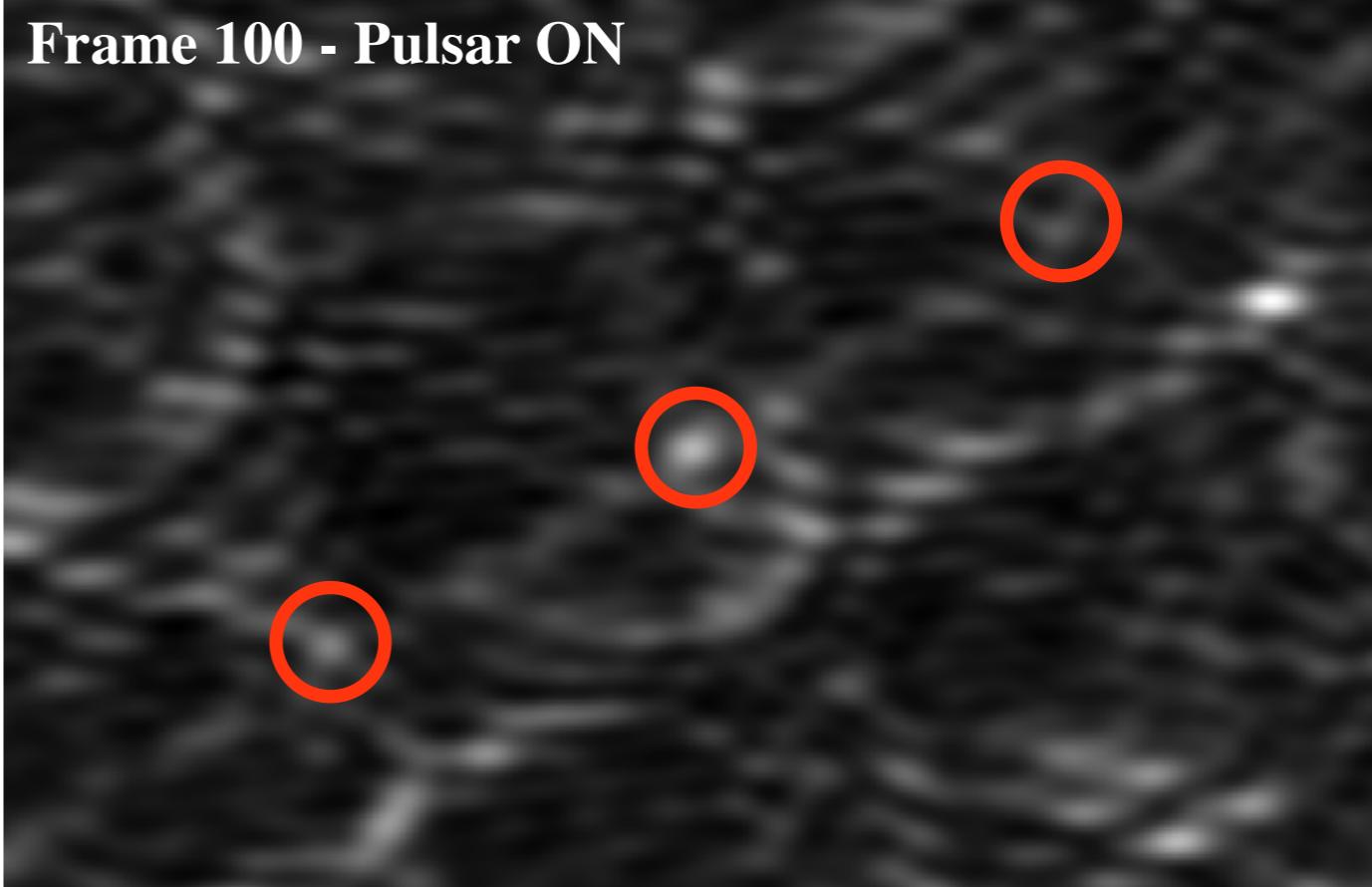
res.  $\delta t = 5ms$

Period  $P_{\text{B0355+55}} = 156ms$

Frame 122 - Pulsar OFF



Frame 100 - Pulsar ON



- **Transient** takes the form of the PSF
- **Sidelobes** are also transients !

# Take-away messages

## Take-away messages

**SKA-class telescopes** → **gain in raw sensitivity**

enable monitoring of your favorite objects (e.g. during quiescence)

bright slow transients      Imaging mode

faint fast transients      Beamforming mode

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Large FoV → Instrumental effects =  $f(t, \nu, \theta, \phi)$

Biased flux

Fake transients

**addressed with Radio Interferometric Measurement Equation**

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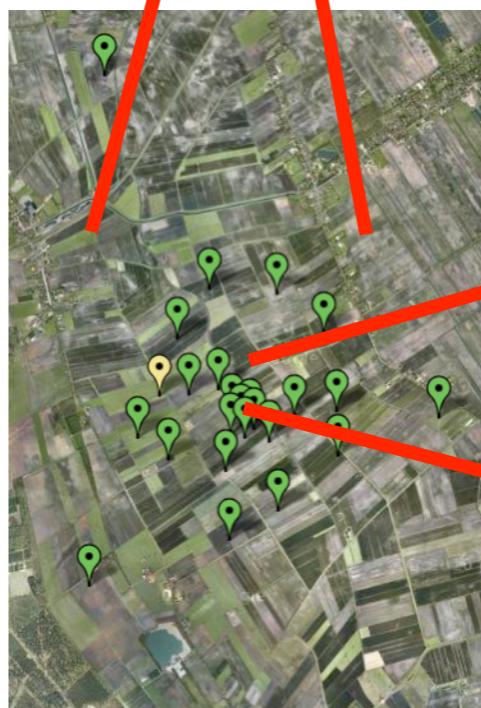
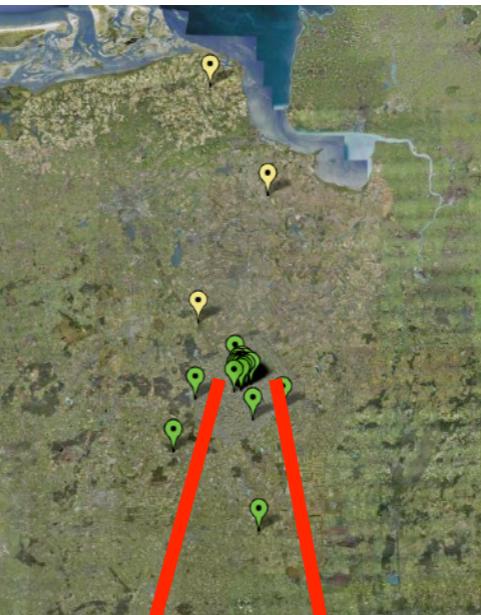
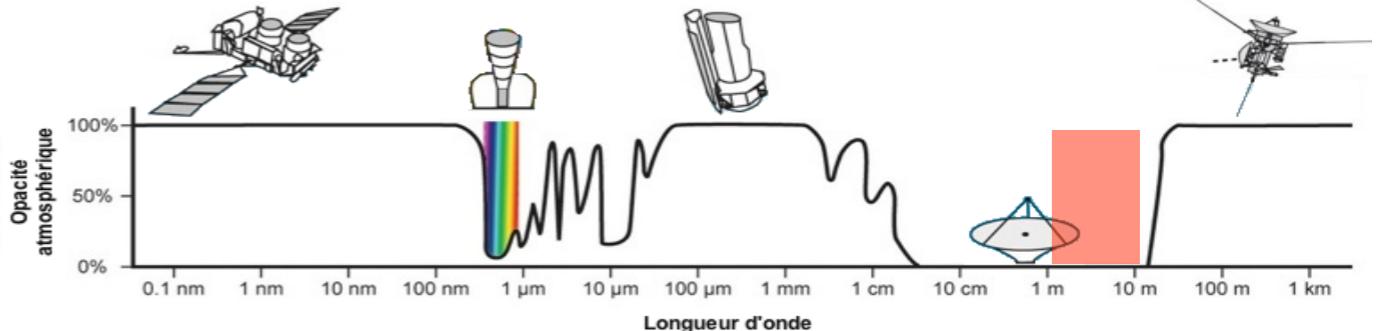
« Compressed Sensing »

Recent signal processing framework to take the best of your data

(also applicable to X-ray and Gamma-ray deconvolution)

# LOFAR

## A multiscale instrument



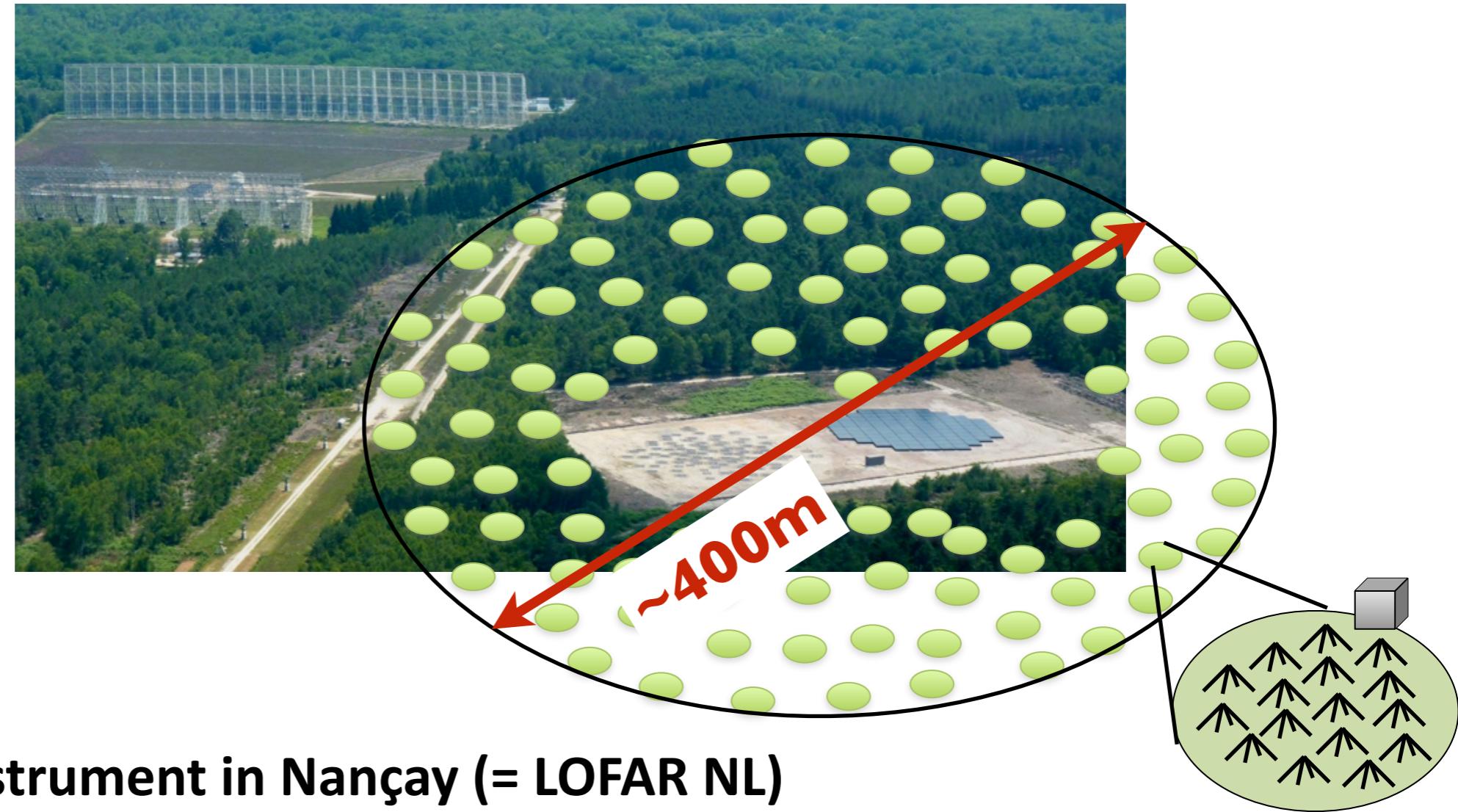
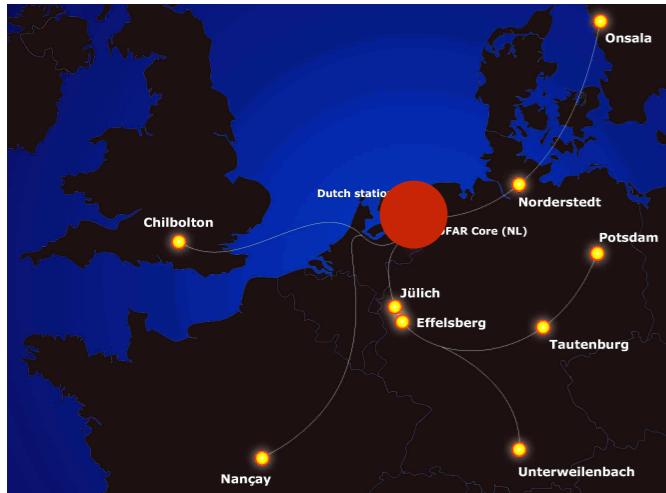
- 1824 antennas / NL tiles + 1248 International
- Imaging, Beamforming, waveforms



# Instrumentation

Improving LOFAR ? Instantaneous sensitivity Access to large structure  
on angular resolution Field of view

**NenuFAR = New Extension in Nançay Upgrading LOFAR**



**Large stand-alone instrument in Nançay (= LOFAR NL)**

**Multi-scale instrument**

**Under construction (~50% by the end of 2017) - SKA Pathfinder**

Thèse, 2013 [Girard et al., CRAS, 2012](#)

[Girard et al., SF2A, 2012](#)

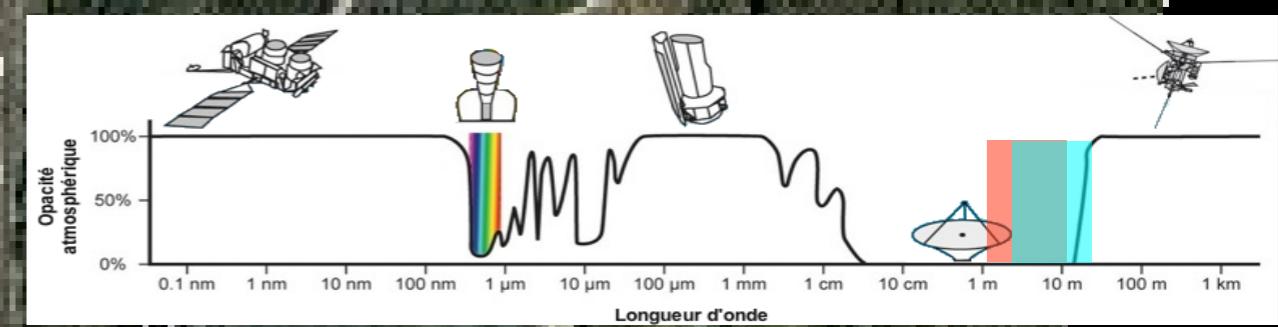
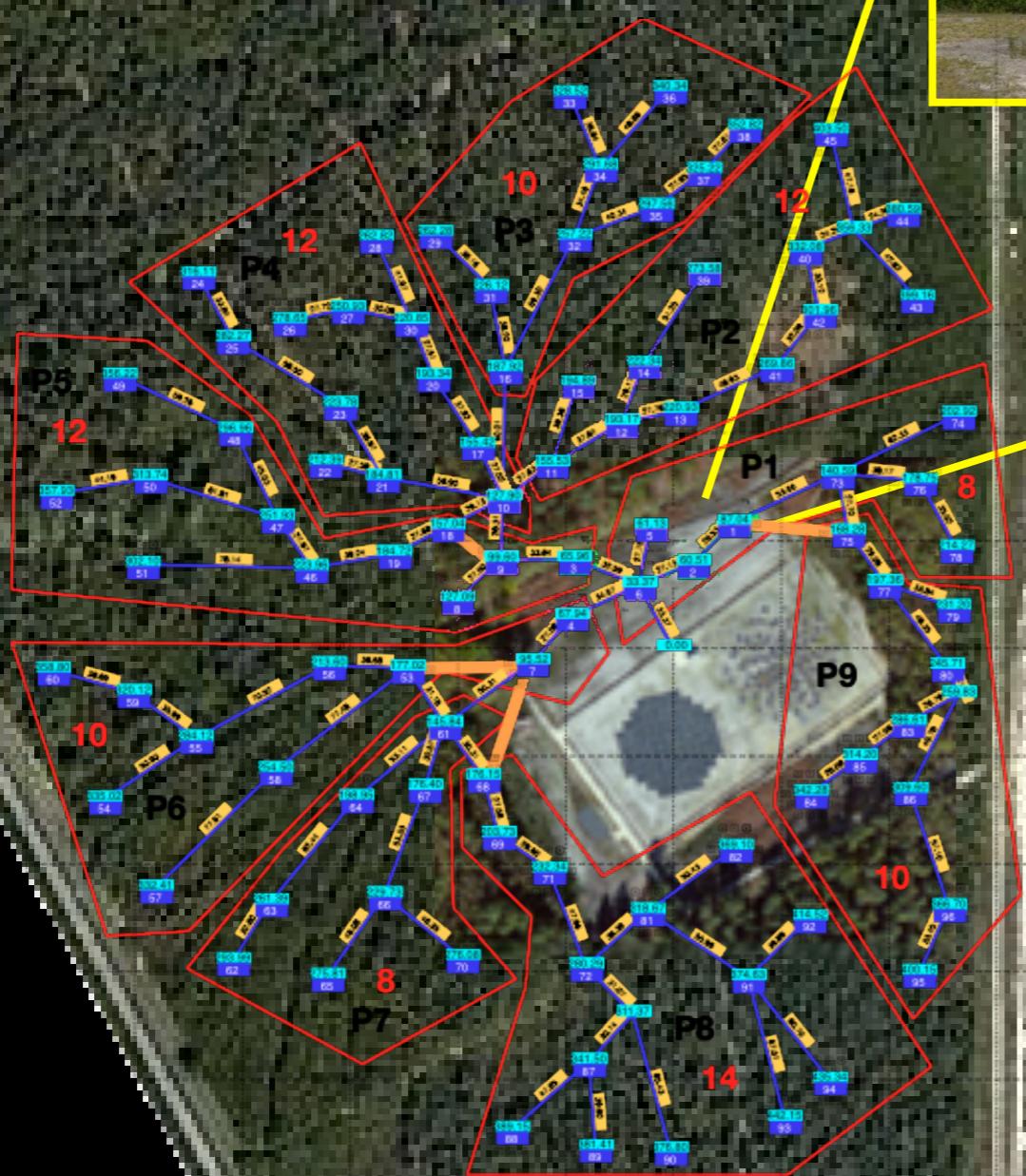
Zyma, [Girard](#), Vasko et al., 2016

Zarka, [Girard](#) et al., SF2A 2012

Zarka, Tagger, Denis, [Girard](#) et al., in prep

# ... NenuFAR

New extension in Nançay upgrading LOFAR



# $A_{\text{eff}}$ compared to LOFAR, LWA

