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# Physical constraints on the bipartition model of the Universe

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**Summary.** The presence of a void zone surrounding us, as found by Fliche, Souriau and Triay (1982) (FST) suggests a model of a spherical, closed and bipartitioned Universe, one half matter, the other half anti-matter. Taking for granted the matter-antimatter bipartition, we try to check the physical consistency of the model: (1) the high energy radiation in the neighborhood of the contact region between matter and antimatter produces a ionization excess which generates a lag in the decoupling, preventing galaxy formation. The thickness of the zone is compatible with the void zone of FST; (2) the distortion of the Cosmic Microwave Background, due to the annihilation, is negligible; (3) the compatibility of the predicted gamma-ray flux with the observations requires the *ad hoc* hypothesis that the intergalactic matter in the void zone is present in clouds with a filling factor of the order of $\frac{1}{100}$, (4) in which case, the Lyman $\alpha$ absorption lines of the clouds show up only as a contribution to the Lyman $\alpha$ forest.

There is no evidence that the model of FST is physically inconsistent, but the general difficulties of any physical model of the void zone are underlined. Further observational data are needed.

**Key words:** cosmology – formation of galaxies – gamma-rays – quasars – cosmic background radiation

## 1. Introduction

The discovery of the Cosmic Microwave Background Radiation (Penzias and Wilson, 1965) led Harrison (1967) to raise the question of the structure of the Universe for $kT > 1$GeV. At such a temperature, the equilibrium:

$$b + \bar{b} \rightleftarrows \gamma + \gamma \tag{1}$$

leads to about an equal number of baryons and antibaryons. Two models have been proposed at that time in order to explain why our immediate surrounding is made of matter only. According to Harrison (1967), fluctuations in the baryonic number are responsible. But Harrison does not give any model of the generation of the baryonic number. Omnès (1969, 1972) has tried to build a model of a symmetric Universe where fluctuations of the baryonic number have a physical origin. A detailed investigation of the properties of the model has been carried by Stecker and Puget (1972), Aldrovandi et al. (1973), Aly (1974), Aly et al. (1974), Combes et al. (1975), and Aly (1978a,b). The basic idea is that a phase separation between matter and antimatter takes place at $T > T_0$, $T_0 = 350$ MeV, the characteristic size of the emulsion being defined by a diffusion mechanism. When the temperature drops below $T_0$, $T < T_0$, annihilation proceeds. At the end of the diffusion epoch, $T = T_1 = 1$ MeV, the fluctuations define the excess number of matter (or antimatter) regions above the antimatter (or matter) regions, from which the annihilation ratio $N_b/N_\gamma$ is derived as well as the characteristic size of the emulsion at the nucleosynthesis epoch.

A number of difficulties are underlined by Steigman (1976), and also Aly (1974), Aly et al. (1974), Aly (1978a,b), Combes et al. (1975) and Ramani and Puget (1976), which are due to the small characteristic size of the emulsion at $T = T_0$. Furthermore, Steigman (1976) expressed strong doubts even about the possibility of a phase separation taking place at $T > T_0$.

In the mean time, symmetry breaking opened a new way of looking at the baryon asymmetry of the Universe (Weinberg, 1979), without the difficulties of the Harrison-Omnès model. In the classical Weinberg's model, symmetry breaking arising from the grand-unified theory has the same sign everywhere in the Universe. But Sato (1981) and Mohanty and Stecker (1984) have shown that there is no compulsory reason in an inflationary scenario for assuming the same sign of the symmetry breaking all over the Universe. This brings us back to a Universe made of large regions either of matter or antimatter, the main difference with Omnès model being that matter and antimatter regions are not causally related at the time of their formation (Brown and Stecker, 1979).

How large can the regions of matter and antimatter be? We believe that the question is coming up again after the study of the space distribution of quasars by Fliche, Souriau and Triay (1982, hereafter FST). FST have presented evidence for the existence of a void zone, quasar void (called $\mu$-region) in a spherically closed Universe, with a non-zero cosmological constant. The $\mu$-zone separates the Universe into two equal domains and the thickness of this zone surrounding us, estimated at $z = 0$, is constant in all directions and is equal to about 80 Mpc (with $H_0 = 100\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$) corresponding to $\Delta z/z \simeq 0.05$. The eccentric position of the Sun with respect to the center of the void zone, as well as numerical experiments (Triay, 1981) suggest that the $\mu$-zone is probably not a statistical accident, and that it cannot be due to an observational selection effect. FST (1982) suggest that the finite, spherically closed Universe is partitioned, and half matter, half antimatter. Whereas the idea of symmetry breaking

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over a bubble larger than the presently visible Universe is widely accepted in the various inflationary scenarios of the Universe (see e.g. Guth, 1981), it is also closely related to the problem of flatness. In the frame of these theories, it does not seem possible to fit a model of a spherically closed Universe with a cosmological constant. Consequently, we have chosen a different approach. Assuming a spherically closed Universe, in which at an early phase, symmetry breaking took place symmetrically, how did such a universe evolve after appearance of the baryonic and antibaryonic halves?

The contact region of matter and antimatter, and even the free path of rays remain very small compared to the Hubble radius (Aly, 1978a,b) until $1 + z = 10^4$, and we can easily conclude that annihilation in the early phases has little effect on the major part of the Universe, neither on the Cosmic Background Radiation (CBR) nor on the nucleosynthesis. We can then follow the line of thought which has been sketched by Schatzman (1982, 1983a,b) and which we shall summarize in the following way:

(1) before decoupling, at around $1 + z = 10^4$, the mean free path of the gamma-rays produced in the annihilation zone, grows in such a way that there appears a region with an excess ionization. When $1 + z = 1000$, recombination takes place everywhere, except in the neighborhood of the annihilation zone (still very thin compared to all other scales). The effect of the ionization excess (as has been suggested by Jones and Steigman, 1978) is to keep the matter trapped into the radiation field until final dilution of the ionizing radiation allows decoupling to take place. We shall call $\mu$-region the domain with ionization excess. Therefore, it is possible that the condensation time lag explains the existence of the $\mu$-zone, void of quasars.

(2) the matter remaining in the $\mu$-region has not condensed into galaxies or quasars. Three effects have to be considered:

(a) the effect of the $\mu$-region on the CBR, with a special interest in a possible anisotropy;

(b) the present production of gamma-rays in the annihilation zone and its relation with the observations of Fichtel et al. (1978);

(c) the presence of absorption lines coming from the $\mu$-region.

It is clear that all observational constraints have to be fulfilled. As we shall see, whereas the generation of the $\mu$-zone, void of quasars, seems to be explained by the recombination time lag, it is more difficult to fit the model to other observational data (gamma-rays and absorption lines especially). But, it seems to us that this is related to the wide class of problem concerning galaxy formation and intergalactic clouds formation.

Starting from a symmetric Universe, already present, for example at the time of nucleosynthesis, we try to follow it up to the time of recombination. The void $\mu$-zone, as shown in Sect. 5 can possibly be a result of the recombination time lag, due to the ionization excess. This result being obtained, we give in Sect. 6 an estimate of the present gamma-ray flux coming from annihilation in the $\mu$-zone. The calculated flux is above the observed one and very anisotropic and should be visible. Then, the annihilation gamma-rays must be a small fraction of the observed background. This puts a severe constraint on the level of the anisotropic contribution of the annihilation region. Another difficulty comes from the complete absence of large Lyman $\alpha$ absorption features, which suggests either a very high degree of ionization of hydrogen in the $\mu$-zone, or the distribution of hydrogen in highly condensed clouds (Sec. 8).

Finally, it can be said that it is difficult to build a consistent physical model of the FST void zone. Further work is needed to determine whether the difficulties can be lifted. And before any new attempt, further observational data should be obtained.

## 2. Matter-antimatter annihilation

For the study of the recombination epoch, we consider a model in which a plane ($\sum$) separates two infinite regions of pure hydrogen and anti-hydrogen respectively, assuming no helium.

Let us briefly summarize the data concerning the annihilation products. Annihilation of a proton with an anti-proton (or p-$\bar{H}$, or H $-$ $\bar{H}$) essentially generates the following particles (Stecker, 1971):

1. $\pi^+, \pi^-$ pions which disintegrate into neutrinos and antineutrinos (50% of the initial energy input: $2m_pc^2$) which will not interact with matter, and relativistic electrons and positrons (20%) which loose their energy by inverse Compton effect on the Cosmic Background Radiation (CBR).

2. $\pi^\circ$ pions which disintegrate into gamma-rays (for the last 30% of the energy).

The annihilation rate on the surface ($\sum$) is defined as the number of annihilations per cm$^2$ and per second. For the period considered ($z \sim 1000$), it is easily calculated by a diffusion model (Bessard et al. 1969) described by Stecker and Puget (1972). It is given by:

$$\Phi = N v_{\text{tha}} (X_a A_i + (1 - X_a) A_n) \tag{2}$$

where $N$ is the number density of matter far away from ($\sum$); $v_{\text{tha}}$ is the thermal average velocity of protons and hydrogen atoms; $X_a$ is the fractional ionization of the matter near ($\sum$); and the coefficients $A_i$ and $A_n$ are given by:

$$A_i = \left[\frac{X_a\sigma^a_{ii} + (1 - X_a)\sigma^a_{in}}{6(X_a\sigma^d_{ii} + (1 - X_a)\sigma^d_{in})}\right]^{1/2}, \qquad A_n = \left[\frac{X_a\sigma^a_{in} + (1 - X_a)\sigma^a_{nn}}{6(X_a\sigma^d_{in} + (1 - X_a)\sigma^d_{nn})}\right]^{1/2}. \tag{3}$$

They are respectively the contributions of proton annihilations and hydrogen atom annihilations. We give in Table 1 the numerical values of annihilation and diffusion cross-sections for p, H, $\bar{p}$, $\bar{H}$ between each other, averaged on a thermal distribution of matter and thus temperature dependent.

**Table 1.** Table of cross-sections for annihilation ($\sigma^a$) and diffusion ($\sigma^d$) in cm$^2$. References are: (a): Aldrovandi and Puget, 1971. (b): Omidvar and Puget, unpublished. (c): Junker and Bardsley, 1972. (d): Spitzer, 1967. (e): Buckingham and Fox, 1962, Dalgarno and Smith, 1962. (f): Landau and Lifshitz, 1958

*Annihilation*

| (cm$^2$) | $\bar{p}$ | $\bar{H}$ |
|---|---|---|
| $p$ | $6\,10^{-15}/T_m$ (a) | $9\,10^{-18}$ (b) |
| $H$ | $9\,10^{-18}$ (b) | $9\,10^{-16}$ (c) |

*Diffusion*

| (cm$^2$) | $p$ | $H$ |
|---|---|---|
| $p$ | $10^{-4}/T_m^2$ (d) | $3\,10^{-13}/T_m^{0.33}$ (f) |
| $H$ | $3\,10^{-13}/T_m^{0.33}$ (f) | $2\,10^{-14}/T_m^{0.17}$ (e) |

We notice that as soon as a small fraction (~1%) of the matter consists of recombined hydrogen atoms, the major contribution to the annihilation rate comes from $H, \bar{H}$ annihilations. It is therefore important to know the level of ionization $X_a$. The depth of the annihilation layer, determined by a diffusive process before annihilation, is given by Omnès (1972): $h = N^{-1}(\sigma_a \sigma_d)^{-1/2}$ and is several orders of magnitude smaller than the mean free path of photons so that the annihilation layer is essentially a plane surface.

Now, is this layer ionized? Could it be due to the hard X-rays coming from the interaction of the high energy electrons and positrons produced by $\pi^+, \pi^-$ decay with the CBR? Actually, inverse Compton effect generates too energetic (>100 keV) X-rays for any efficient local ionization. They can escape the annihilation layer, and as a result the major effect is the normal balance of recombination and dissociation in the presence of CBR only.

## 3. Gamma-ray transfer

We are mainly concerned with the propagation of annihilation gamma-rays. They propagate and ionize much farther away from the annihilation surface ($\sum$) than electrons and positrons produced by annihilation. As curvature can be neglected for the study of the recombination epoch, the geometry of the problem is euclidian. Let $n(x, t, \cos\beta, E)$ be the gamma-ray density as a function of time $t$, comoving coordinate $x$ (measured on the normal to ($\sum$)), incident angle $\beta$ with the normal of ($\sum$), and the energy $E$ of gamma-rays at the point $(x, t)$. The transfer equation reads (see e.g. Peebles and Yu, 1970; Arons, 1971):

$$\frac{\partial}{\partial t}\left(\frac{n}{N}\right) + \frac{c\cos\beta}{R}\frac{\partial}{\partial x}\left(\frac{n}{N}\right) = \frac{\dot{R}}{R}\frac{\partial}{\partial E}\left(\frac{En}{N}\right) - D(E)\frac{n}{N}, \tag{4}$$

where $n/N$ is convenient to use because of the universal expansion, the first term on the right-hand side represents the redshift effect and the last term corresponds to absorption by the medium:

$$D(E) = c\sigma(E)N, \tag{5}$$

$D(E)$ being the absorption rate of the photons of energy $E$. Absorption is mainly due to:

1) Compton interaction with thermal electrons which corresponds to Klein-Nishina's cross-section (taken for $E \gg m_e c^2$):

$$\sigma_c(E) = \frac{A}{E}(a + \ln E), \tag{6}$$

with

$$A = \pi r_0^2 m_e c^2 = 1.27\,10^{-25}\,\text{MeV cm}^2.$$

and

$$a = \tfrac{1}{2} + \ln 2 - \ln(m_e c^2) = 1.87.$$

2) Electron-positron pair creation on nuclei. This is important at high energy $E \gg 137 m_e c^2$ for which the cross-section is almost constant:

$$\sigma_p = 9.41\,10^{-27}\,\text{cm}^2. \tag{7}$$

Let us introduce as in retarded potential theory a lag time by the formula:

$$\frac{x}{\cos\beta} = \frac{c}{H_0}\int_{t_a}^{t}\frac{dt}{R} = \frac{2c}{H_0\Omega_0^{1/2}}(R^{1/2} - R_a^{1/2}), \tag{8}$$

where $R_a = R(t_a)$, $R = R(t)$, and $t_a$ is the time of annihilations that produced the photons arriving at the point $x$ at the time $t$. These photons have an energy $E_a$ at the production time, related to the energy $E$ at the time $t$ by:

$$E_a = ER/R_a. \tag{9}$$

Now, we can readily solve Eq. (4):

$$n(x, t, \cos\beta, E) = \frac{1}{4\pi c}\left(\frac{R_a}{R}\right)^2 \Phi(t_a) f(E_a) \exp[-\tau(x, t, \cos\beta, E)], \tag{10}$$

where: $1/(4\pi)$ comes from the emission isotropy, $f(E_a)$ is the annihilation spectrum of gamma-rays at the emission energy (see the appendix and notations for the function $f$) and $\tau$ is the optical depth and it reads:

$$\tau = c\int_{t_a}^{t} dt' N(t')\sigma(E(t')), \tag{11}$$

with

$$E(t') = ER/R(t').$$

Using formulae (6), (7) and (8) allows us to calculate $\tau$:

$$\tau = S(R)\left[\frac{A}{E}[(y-1)(a + \ln E - 2) + 2y\ln y] + \frac{\sigma_p}{3}(y^3 - 1)\right], \tag{12}$$

with

$$y = (R/R_a)^{1/2}, \tag{13}$$

and

$$S(R) = \frac{2cNR^{3/2}}{H_0\Omega_0^{1/2}} = 3ctN.$$

We can notice that, as a consequence of causality, no gamma-ray can reach the point $x$ at the time $t$ if $Rx > 3ct$. Conversely, (8) and (13) imply with $y^{-1} = 1 - Rx/(3ct\cos\beta)$ that: $y > y_0$, with:

$$y_0^{-1} = 1 - Rx/(3ct). \tag{14}$$

## 4. Ionization equilibrium in the $\mu$-region

Actually, gamma photons are not absorbed by the medium after an interaction of the type described by the formulas (6) or (7) but it is true that they loose the greatest part of their energy during the first interaction with matter, leading to cascades of relativistic electrons and X-rays. We adopt as an hypothesis that a gamma-photon of energy $E$ will ultimately product $E/W_I$ ionizations by these cascades with $W_I = 36$ eV (Spitzer, 1958; Stecker and Puget, 1972) in the neighborhood of the first interaction.

Evolution of ionization of the medium due to gamma-rays reads from (4):

$$N\left(\frac{dX}{dt}\right)_\gamma = N\left(\frac{\dot{E}_\gamma}{W_I}\right) = \frac{1}{W_I}\int En(E)D(E)\,dE\,d\Omega, \tag{15}$$

where $\dot{E}_\gamma$ is the rate of energy input by gamma-rays on the matter per nucleus. From (5) we write:

$$\left(\frac{dX}{dt}\right)_\gamma = \frac{\dot{E}_\gamma}{W_I} = \frac{c}{W_I}\int En(E)\sigma(E)\,dE\,d\Omega. \tag{16}$$

Now, using the solution (10), (12), (13) with changing the variables $(E, \beta)$ into $(E_a, y)$ we obtain the useful relation:

$$\left(\frac{dX}{dt}\right)_\gamma = \frac{1}{2W_I}\frac{Rx}{3ct}\int dE_a E_a f(E_a) \int_{y0}^{\infty} dy\, \sigma(E_a y^{-2}) y^{-8}(y-1)^{-2} \times \Phi(Ry^{-2}) e^{-\tau_a}, \tag{17}$$

with

$$\tau_a = S(R)\left[\frac{Ay^2}{E_a}[(y-1)(a+\ln E_a - 2) + 2\ln y] + \frac{\sigma_p}{3}(y^3-1)\right].$$

Ionization equilibrium obeys the relation (Peebles, 1968):

$$\frac{dX}{dt} = C\alpha(T_m)\left[(1-X)e^{-B_2/k_B T_B}\left(\frac{2\pi m_e k_B T_B}{h_p^2}\right)^{3/2} - X^2 N\right] + \left(\frac{dX}{dt}\right)_\gamma, \tag{18}$$

where we have added the contribution to ionization by the gamma-rays. $T_B$ and $T_m$ are respectively the temperatures of the CBR and the matter. The recombination coefficient is $\alpha(T_m)$. An inhibition factor $C$ is introduced, as the recombination can only be achieved through the decay from the $n=2$ levels. This is possible either through the redshift of resonance line Lyman $\alpha$ $2p-1s$ or through the two-photon transition $2s-1s$ (Spitzer and Greenstein, 1951). $B_2$ is the energy of the $n=2$ level.

From the second principle of thermodynamics, we get the equation giving the temperature of the matter:

$$\frac{dT_m}{dt} = \frac{2T_m}{3(1+X)N}\frac{dN}{dt} + \frac{2}{3(1+X)k_B}\left(\frac{dW_B}{dt} + \frac{dW_a}{dt}\right); \tag{19}$$

$k_B$ is Boltzmann's constant and $dW_B/dt$, given by:

$$\frac{dW_B}{dt} = X\frac{4\sigma_T U_B}{m_e C} k_B(T_B - T_m) \tag{20}$$

is the heating rate by the radiation due to the Thomson drag of electrons through the CBR, the energy density of which is $U_B$ (Weymann, 1965; Peebles, 1971).

$$\frac{dW_a}{dt} = \eta \dot{E}_\gamma \tag{21}$$

is the heating rate from annihilation gamma-ray cascades. Spitzer (1978) gives: $\eta \sim 0.1$. In the energy balance, the contribution of free-free transitions is completely negligible.

## 5. Galaxy formation and the $\mu$-region

As we have mentioned in the introduction, the basic idea is to assume that galaxy formation takes place only when matter is decoupled from the radiation field. As a consequence, galaxy (and anti-galaxy) formation will take place everywhere at $R = R_{\mathrm{cosmic}}$ in the Universe except in the neighborhood of the annihilation surface $(\sum)$.

In this region, the excess of ionization delays the decoupling, which takes place later at $R > R_{\mathrm{cosmic}}$. There are two consequences to the delay in the decoupling.

(1) Primordial density fluctuations corresponding to a mass $M$ are damped by friction on the CBR, if the mass $M$ is lower than a critical mass $M_S$. The mass $M_S$ grows like $R_{\mathrm{rec}}^{15/4}$ (Silk, 1968; Weinberg, 1972). If $R_{\mathrm{rec}} = \frac{1}{500}$ and $R_{\mathrm{cosmic}} = \frac{1}{1000}$, this implies a mass increase by a factor 14.

(2) The growth of density perturbations starts later in the $\mu$-zone and this might have prevented galaxies from being formed at the normal redshift $(1 < z < 10)$.

According to Peebles (1971), collapse can take place when the characteristic damping time of the motion of the matter through the radiation field becomes smaller than the age of the Universe. This is obtained if the degree of ionization $X$ is smaller than the critical value $X_{gf}$:

$$X_{gf} = \frac{3m_p c}{2\sigma_T U_B}\frac{\dot{R}}{R} = 2.95\,10^5 R^{5/2}\left(\frac{h^2\Omega_0}{0.1}\right)^{1/2}. \tag{22}$$

We are then led to compare the degree of ionization $X$ at a distance $x$ with the critical value $X_{gf}$.

For a given $x$, the degree of ionization $X$, derived from Eqs. (18) and (19) can be calculated as well as the critical degree $X_{gf}$. The results are plotted on Fig. 1. It is exactly seen that there is a critical distance $x_c$ such that, for $x > x_c$, there is no delay in the decoupling, whereas for $x < x_c$ the decoupling is retarded. The epoch of decoupling depends on the distance $x$ and the corresponding decoupling is about $R \sim \frac{1}{500}$ for $x < x_c$.

It is important to notice that $2x_c$ is close to 120 Mpc, which is a little higher than the width of the void $\mu$-region found by FST (80 Mpc). Figure 2 shows the temperature of the matter $T_m$ as a function of $R$, depending on the distance $x$. The temperature follows closely the CBR temperature, until the scale factor $R$ equals $\frac{1}{500}$. At this time, the matter is not enough coupled (formula (20)) to the CBR, and as it is heated by the annihilation products (formula (21)), it becomes hotter than the CBR.

Here we have not proved that the $\mu$-region exists. We have just shown that in the frame of standard galaxy formation theory, the annihilation model of the $\mu$-region is compatible with the results of FST.

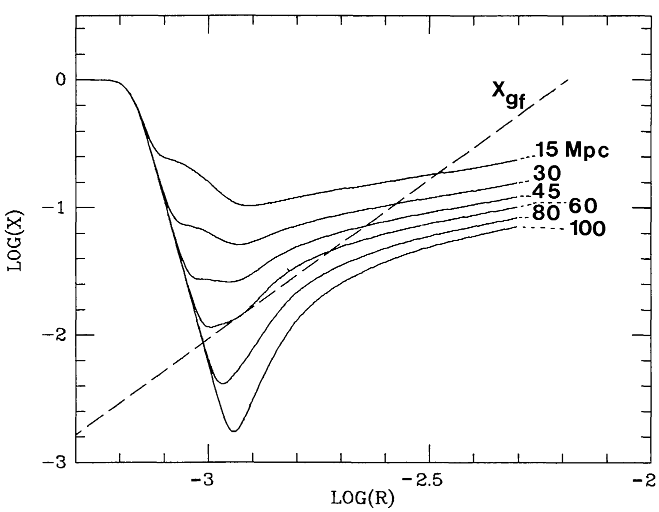


**Fig. 1.** Conditions of galaxy formation. The galaxy formation is supposed to take place when Peebles' conditions on the drag on the Cosmic Background Radiation drops below a critical value. The interrupted line corresponds to the relation (22). For matter lying at various distances $Rx$ from the annihilation surface $(\sum)$, with $x = 15, 30, 45, 60, 80, 100$ Mpc, the curves (continuous line) give the degree of ionization $X$ as a function of the expansion scale factor $R$. For $x = 80$ Mpc it will be noticed that the curve $X(R)$ does not cut again the curve $X_{gf}$ giving the critical degree of ionization. For $x < 60$ Mpc, no decoupling can take place until $R = \frac{1}{500}$.

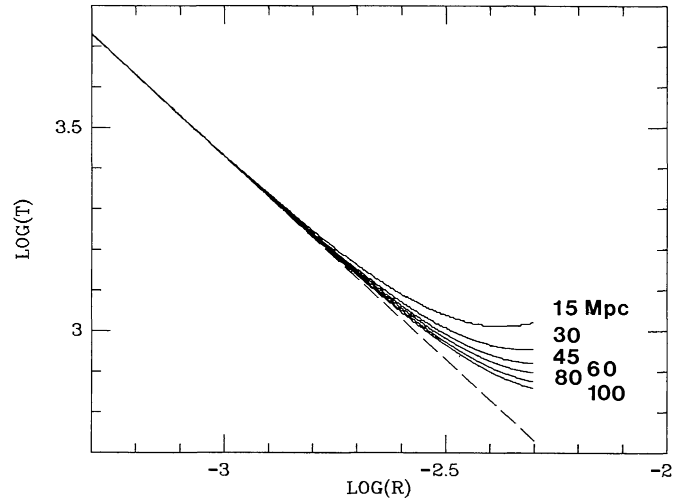


**Fig. 2.** The temperature of the matter. For the same sample of distances as Fig. 1, the evolution of the temperature of the matter as a function of the scale factor $R$. Until $R \sim \frac{1}{500}$, the temperature of the matter follows closely the Cosmic Background Radiation one (interrupted line)

## 6. The gamma-ray emission of the $\mu$-region

A component in the spectrum of the gamma-ray background has been observed between 35 MeV and 200 MeV (Fichtel et al., 1978) which, by its isotropy, is probably of extragalactic origin. We want to show that the level and the isotropy of this gamma-ray background impose several constraints on the symmetric model discussed here. Calculations have already been made for general symmetric models (Stecker et al., 1971). But here, the gamma-radiation comes from a surface ($\sum$) of the Universe rather than from the whole Universe. We can neglect absorption by Compton effect because the optical depth for gamma-rays is less than 0.05 for an emission redshift less than 30.

Let $\theta$ be the angle between the axis PP′ of anisotropy of the model and the line of sight. Because of the symmetry of rotation around PP′, the emission scale factor $R_\mu$ from the annihilation surface ($\sum$) is a function of $\theta$ only. Geometrical arguments give the relation between $\theta$, $\tau(R_\mu)$, which is the angular distance of the emission point, and $L_T$, the cosmic latitude of the Earth (see the Appendix, formula A3):

$$tg(\tau(R_\mu)) = \mathrm{tg}(L_T)/\cos\theta\,. \tag{23}$$

Using relativistic photometry (see e.g. Weinberg, 1972), one can prove that the flux of photons received per steradian and per energy interval is:

$$F(E) = \frac{1}{4\pi} f(E/R_\mu)\Phi(R_\mu)R_\mu^2(\sin^2 L_T + \cos^2 L_T \cos^2\theta)^{-1/2}\,, \tag{24}$$

where $f(E/R_\mu)$ is the redshifted annihilation spectrum and the last term comes from the incident angle $\rho$ of the line of sight with the normal of ($\sum$). We have:

$$\sin\rho = \cos L_T \sin\theta \tag{25}$$

Having computed (24) and using $\lambda_0, \Omega_0$ given by FST (see the Appendix, formulas A4 and A5) we can test any model giving the annihilation rate $\Phi(R_\mu)$ as a function of the scale factor $R_\mu$.

Let us first assume that the $\mu$-zone is filled with an homogeneous gas: anti-hydrogen and hydrogen, which recombined at $R_{\rm rec} = \frac{1}{200}$. Two scenarios seem a priori possible.

1) Suppose that the ionization of the medium in ($\mu$) has not changed since the recombination epoch. Therefore, matter is mainly on the form of hydrogen atoms. Let us assume also the homogeneity of the medium, then formula (2) gives for the annihilation rate:

$$\Phi_n = 5.6\,10^{-3}\, T_m^{1/2}\left(\frac{\Omega_0 h^2}{0.1}\right) R_\mu^{-3}\,\mathrm{cm}^{-2}\,\mathrm{s}^{-1}\,.$$

After decoupling, matter evolves like a Boltzmann gas, so $T_m \propto R^{-2}$ and if we take $T_m(R_{\rm rec}) = T_0 R_{\rm rec}^{-1}$, we get $T_m = T_0 R_{\rm rec} R^{-2}$, so:

$$\Phi_n = 9.2\,10^{-3}\, R_{\rm rec}^{1/2}\left(\frac{\Omega_0 h^2}{0.1}\right) R_\mu^{-4}\,\mathrm{cm}^{-2}\,\mathrm{s}^{-1}\,.$$

As we have seen, $R_{\rm rec} \gtrsim 200^{-1}$, therefore for a neutral medium:

$$\Phi_n \gtrsim 6.5\,10^{-4}\, R_\mu^{-4}\,\mathrm{cm}^{-2}\,\mathrm{s}^{-1}\,. \tag{26}$$

Then, we can obtain: $\langle F\rangle = 3.7\,10^{-4}$ photons $\mathrm{s}^{-1}.\mathrm{cm}^{-2}\,\mathrm{ster}^{-1}$, to be compared with the observed value of $5.7\,10^{-5}$ in the same unit.

2) Suppose now that the medium in ($\mu$) has been ionized by radiation coming from quasars. The temperature $T_m$ of the medium is estimated to be $T_m = 10^4$ K. Formula (2) gives for the ionized medium:

$$\Phi_i = 9.0\,10^{-4}\, R_\mu^{-3}\left(\frac{\Omega_0 h^2}{0.1}\right)\mathrm{cm}^{-2}\,\mathrm{s}^{-1}\,, \tag{27}$$

and then: $\langle F\rangle = 1.3\,10^{-4}$ photons $\mathrm{s}^{-1}\,\mathrm{cm}^{-2}.\mathrm{ster}^{-1}$.

Both annihilation rates $\Phi_n, \Phi_i$ give a flux of gamma-rays too high by a factor of about 3–6 compared to the observed level of the gamma-radiation ($5.7 \pm 1.3\,10^{-5}$, Fichtel et al., 1978). Let us note that the first scenario seems unlikely to happen because, in the $\mu$-zone, the temperature of matter is always above the CBR one, as it can be seen in Fig. 2.

Furthermore, with $\Phi(R_\mu) \sim R_\mu^{-4}$ (resp. $R_\mu^{-3}$), we get for Fichtel's tests of isotropy of the gamma-ray background:

$$f = 1.02 \quad (\text{resp. } 0.37)\,, \tag{28}$$

with:

$$f = \frac{F(100° < l < 250°;\ 20° < |b| < 40°)}{F(300° < l < 420°;\ 20° < |b| < 40°)}\,,$$

and

$$f' = 1.83 \quad (\text{resp. } 162)\,,$$

with

$$f' = \frac{F(|b| > 60°)}{F(20° < |b| < 40°)}\,,$$

where $F$ is summed over the interval 35–100 MeV, to be compared to the observed value (compatible with isotropy): $f_0 = 1.10 \pm 0.19$ and $f'_0 = 0.87 \pm 0.09$. Another weak point is that the theoretical spectrum in either case is too flat (the slope being equal to 1.9 in the range 35 to 200 MeV) compared to the steep observed spectrum (3.5 in the same range).

The model based on an homogeneous distribution of the gas in the $\mu$-zone fails to represent the observed flux of gamma-rays. In order to fit the observations, it would be necessary to change the model in such a way that the gamma-ray flux would be cut down by a factor 100, in which case there would be no discrepancy, either with the energy spectrum or with the anisotropy, the isotropic part being supposed to come, as usual, from the active galaxies. This question will be discussed in Sect. 8.

## 7. The distortion of the CBR

Before reaching us, the CBR crossed the separation zone ($\mu$) when the scale factor was $R_\mu$ depending on the line of sight angle $\theta$ (see formula (23) implying that $0.9 < z_\mu < 15$). Its spectrum is thus distorted by interaction with the annihilation products. According to Zeldovich and Sunyaev (1969), just one parameter, called $y$ is needed to account for this phenomenon. Let $T_{\text{R-J}}$ be the temperature measured in the Rayleigh-Jeans part of the CBR low (frequencies), and let $U'_B$ be the total energy density of the distorted CBR. The distorted spectrum will be such that:

$$U'_{\mathrm{B}} = aT^4_{\text{R-J}} e^{12y},$$

where $a$ is Stefan's constant, and $y$ accounts for the CBR heating rate $dQ/dt$ per unit volume in the annihilation zone:

$$y = \frac{1}{4} \int \frac{dQ}{dt} \frac{dt}{U_{\mathrm{B}}}. \tag{29}$$

Integration is taken on the line of sight, $U_{\mathrm{B}}$ is the CBR energy density. As gamma-rays and neutrinos do not interact with matter, annihilation electrons only contribute to $y$. So we get:

$$\frac{dQ}{dt} \simeq \frac{0.4 m_p c^2 \Phi(R_\mu)}{\Delta l}, \tag{30}$$

where $\Delta l$ is the depth of the zone on which electrons and positrons loose their energy by inverse-Compton effect on the CBR. Because the redshift does not change very much during the crossing of the $\mu$-zone ($\Delta z/z \sim 0.05$), we can approximate Eqs. (29) and (30) to obtain:

$$y = \frac{1}{4} \frac{\Delta Q}{U_{\mathrm{B}}} = \frac{0.1 m_p c \Phi(R_\mu)}{U_{\mathrm{B}}(R_\mu)} \frac{\Delta l'}{\Delta l}, \tag{31}$$

where $\Delta l'$ is the effective distance crossed by the CBR photons: $\Delta l' = \Delta l / \cos \rho$ with $\rho$ defined by (25).

Noticing that $\cos \rho \geq \sin L_{\mathrm{T}}$ with $L_{\mathrm{T}} = 27°$, we get for both annihilation rates coming from the diffusion model (cf. (26) and (27)):

$$y \lesssim 2\,10^{-5}. \tag{32}$$

This satisfies very well the constraint of the observational limit $y_{\mathrm{lim}}$ (for example Chan and Jones (1975) give $y_{\mathrm{lim}} = 0.055$).

With the values of $\Phi_{\mathrm{i}}$, $\Phi_{\mathrm{n}}$ given in Sect. 6, we obtain a $\Delta T_{\text{R-J}} = 0.1$ m K. This produces some anisotropy in the CBR: a) the dipole vector direction is changed by no more than 2°; b) the generated quadrupole momentum is near the observational limits (Lubin et al., 1983; Fixsen et al., 1983).

## 8. The state of the matter in the $\mu$-zone

We have seen that a uniform distribution of neutral hydrogen in the $\mu$-zone produces too high a gamma-ray flux and an observable quadrupole anisotropy. Another constraint on the existence of neutral or weakly ionized uniform medium in the $\mu$-zone comes from the absence of Lyman $\alpha$ absorption in the spectra of quasars behind the zone. Here, we do not take into account some broad absorption lines seen in the $\mu$-zone like that of PHL 5200 because it seems probable that they arise as an ejection from the emission quasar with a relativistic velocity. Their redshift does not yield their true position, being much closer to the quasar (see e.g. Weymann and Foltz, 1983). The spectrum of quasar 2256 + 017 ($z_{\mathrm{em}} = 2.66$) (Baldwin and Netzer, 1978) does not show absorption at the redshift expected for the $\mu$-zone in its direction ($z_\mu = 2.26$ to 2.37). We can then deduce that either the matter is clumpy or highly ionized. The degree of ionization in case of a uniform medium must be such that:

$$1 - X \leq 2.5\,10^{-5} \sqrt{P(R_\mu)}\, R_\mu \tau_{\mathrm{lim}} \left(\frac{\Omega_0 h^2}{0.1}\right)^{-1},$$

where $R_\mu = 1/(1 + z_\mu)$ is the scale factor at the redshift $z_\mu$ we look for absorption, $P(R_\mu)$ is the polynomial expression defined in the Appendix (A1) and $\tau_{\mathrm{lim}}$ is the optical depth of the expected trough.

Such a high degree of ionization would require strong radiation sources. Quasars have been shown to be the most likely candidates for ionizing the hypothetical intergalactic medium (Field, 1972) but the degree of ionization needed here seems incompatible with even the most optimistic scenario of quasar luminosity and density evolution, as there is definitely not enough ionizing photons to produce such a high ionization of the remaining hydrogen of the $\mu$-zone.

Two assumptions could lift the difficulties mentioned above.

### 8.1. *Magnetic fields*

If the medium is ionized (but what sources could ionize it?), a magnetic field parallel to the surface of annihilation can be generated (Schatzman, 1970; Puget 1971; Aly, 1973, 1978b). Close to the surface ($\sum$), the medium can be completely ionized, in which case the magnetic field can drastically reduce the annihilation rate, as the diffusion of protons will be reduced by a factor of $3(B/10^{-8}\mathrm{G})^2 R^3$ quite compatible with the possible magnetic fields.

### 8.2. *Clumpiness*

The analysis of the distribution of Ly $\alpha$ absorption lines led Sargent et al. (1980) to suggest the existence of intergalactic Ly $\alpha$ clouds, cosmologically distributed. These clouds are not gravitationally bound, but confined by external pressure. They are highly ionized with $(1 - X) \sim 10^{-5}$. The density in these clouds is 100 times the average density of Universe. The large value of the critical gravitational mass (Silk, 1968; Weinberg, 1972) mentioned in Sect. 5 has no effect at the level of the thermo-chemical instabilities. Extending the results of Field (1965), Ibanez and Paravano (1983) have shown that thermo-chemical instabilities can generate large condensations though below the critical gravitational mass. Ostriker and Ikeuchi (1983) provide the heating mechanism for the intergalactic medium and obtain also the thermo-chemical condensations below the gravitational unstable (Jeans) mass. Their mechanism provides the heating over regions of 100 Mpc in size and even if it started from the border of the $\mu$-zone, it would have easily swept it.

Let us assume that the medium in the $\mu$-zone, with an hydrogen density $N$, condenses into clouds of hydrogen density $N_{\mathrm{c}}$. We assume the same contraction on the antimatter side of ($\sum$). The lower density in the layer of annihilation (see Sect. 2) and the large size of the $\mu$-zone makes it probable that clouds would be made of either matter or antimatter and not a mixture of both. Let $n$ be the the number of clouds per unit volume, $f = N/N_c$ be the filling factor, and $d$ be the average size of the clouds. From matter conservation we get $nN_c d^3 = N$. On $(\sum)$, the surface density of clouds is $nd$, therefore the fraction of surface $(\sum)$ occupied by the clouds is $nd^3 = f$. The surface density of clouds and anticlouds in contact is $fnd$, and for each collision the rate of annihilation is $f^{-1}\Phi d^2$, where $\Phi$ is given by Eq. (2). The new average rate of annihilation is therefore:

$$\Phi_c = (f^{-1}\Phi d^2)(fnd) = f\Phi .$$

It is reduced by a factor $f$. We can see that if $f \approx 10^{-2}$, whatever are the physical conditions in the condensations, there is no more contradiction between the diffusion model and the difficulties underlined above.

## 9. Conclusion

It was quite appealing for one of us (E.S.), a couple of years ago to try to test the FST model on the basis of the physical cosmology.

We do not know yet whether the existence of the $\mu$-zone is a statistical accident or not. The present investigations of FST on 50000 projection planes (Souriau, 1984) suggest that it is not a statistical accident. It is therefore worth trying to check whether the model is physically consistent. Simplifying the geometry, we could say that we are in a "sphere" of matter, surrounded by anti-matter, the center of which is in the direction defined by $(\alpha, \delta)$ (see formula A4).

Assuming as usual that galaxy formation takes place after decoupling between matter and radiation (Peebles, 1971), the hard annihilation gamma-rays produce a lag in the recombination, and therefore, can eventually generate a region where galaxies and then quasars could not be formed. The thickness of the void zone is defined as the range of efficiency of the gamma-rays in increasing sufficiently the ionization level (Eqs. 18 and 22) and turns out to be of the right order of magnitude.

This suggests naturally further tests of the model. The gamma-ray background can have a contribution due to annihilation on the surface $(\sum)$ between matter and antimatter. If the two void regions (hydrogen and anti-hydrogen) were filled with a uniform gas, the rate of annihilation would be too large by a factor 3 to 6, and the anisotropy and the spectrum would be completely incompatible with the results of Fichtel et al. (1978). The distortion of the CBR by the annihilation products turns out to be very small and even its anisotropy at the limit of detectability. If hydrogen were evenly distributed in the $\mu$-zone, the neutral hydrogen column density would be several times $10^{20}\,\mathrm{cm}^{-2}$ and even with the distribution of the absorption over a frequency range $\nu(\mathrm{Ly}\,\alpha)\Delta z/(1+z)$, where $\Delta z$ is the redshift width of the $\mu$-zone, this would give a deep feature easily detectable by a deep absorption in the Lyman $\alpha$ line on the spectra of the QSO's located beyond the $\mu$-zone. For the few of them for which it is the situation, no such absorption is visible on the spectra. The idea that thermo-chemical instability generates clouds below gravitational instability suggests to replace the uniform medium by a system of clouds of matter and clouds of anti-matter. A filling factor $f \sim 10^{-2}$, lifts the contradiction.

Clumpiness in the $\mu$-zone through the presence of clouds similar to the Sargent et al. (1980) Ly $\alpha$ clouds could explain the absence of Ly $\alpha$ absorption lines in the few observed quasars beyond the $\mu$-zone and in the proper direction, and would be compatible (though not explaining it) with the observed extragalactic gamma-ray background of Fichtel et al. (1978). However, it should well be borne in mind that for the time being, there is neither a theory of the formation of such clumps in the $\mu$-zone, nor an observational proof of their existence. It should be stressed that the compatibility of a bipartioned spherically closed model of the Universe with the theory of matter-antimatter interaction rests on the assumption of a present clumpiness of the void $\mu$-zone. Even if we believe that there might be some good physical reason like thermo-chemical instability to generate these clumps of gas, we have to accept the idea that for the time being, this is just an ad hoc assumption and a pure guess.

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## Appendix and notations

In order to introduce some notations and numerical values, we recall briefly the main results of the Lemaître model of the Universe.

Einstein's equations for General Relativity with a cosmological constant $\Lambda$ can be applied to the Robertson-Walker's metric with a curvature $K$. For a perfect fluid they give the behavior of the scale factor $R(t)$:

$$H = \frac{\dot{R}}{R} = \frac{H_0}{R^2}\sqrt{P(R)}, \tag{A1}$$

with

$$P(\mathrm{R}) = \lambda_0 R^4 - k_0 R^2 + \Omega_0 R ,$$

where $H_0 = 100h\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$ is the present value of Hubble's constant, $\lambda_0 = \dfrac{\Lambda}{3H_0^2}$ is the reduced cosmological constant, $k_0 = K/H_0^2$ is the reduced curvature and $\Omega_0 = 8\pi G\rho_0/3H_0^2$ is the reduced density of matter.

As a convention $R_0 = 1$, $(A1)$ implies $P(R_0) = 1$, therefore $\lambda_0 - k_0 + \Omega_0 = 1$. This leaves us with two free parameters, for example $\lambda_0$ and $k_0$. Lemaître's model is characterized by $\lambda_0 > 0$ and $k_0 > 0$.

The age of the Universe is:

$$t_0 = H_0^{-1} \int_0^1 \frac{R\,dR}{\sqrt{P(R)}} . \tag{A2}$$

The angular distance (comoving) of an object of redshift $z$ is:

$$\tau = \sqrt{k_0} \int_{1/(1+z)}^{1} \frac{dR}{\sqrt{P(R)}} . \tag{A3}$$

The $\mu$-zone discovered by FST is characterized by an axis of symmetry defined by:

$$\alpha = 5^{\mathrm{h}}41^{\mathrm{m}}, \quad \delta = +7°27' , \tag{A4}$$

and by

$$\Omega_0 = 0.08 , \quad \lambda_0 = 1.18 , \tag{A5}$$

and by the fact that ($\mu$) separates the Universe into two equal halves with the Earth at a cosmic latitude $L_T$ (i.e. the minimal angular distance to a point of ($\mu$)) the value of which is $L_T = 29°$.

Finally, let us define some physical quantities:

$T_B = T_0/R$ is the temperature of the Cosmic Microwave Background Radiation (CBR)

$U_B = 0.25(T_0/2.7\,K)^4 R^{-4}$ eV cm$^{-3}$ is the CBR energy density.

$t(R)$ is the age of the Universe when the scale factor is $R$.

$x$ is the comoving coordinate taken on the normal to the annihilation "plane" and normalized to the present cosmic distance scale.

$T_m(x,t)$ is the matter temperature.

$N = 1.17\,10^{-5}\,\Omega_0 h^2 R^{-3}$ cm$^{-3}$ is the baryon density.

$\sigma_T = 6.65\,10^{-25}$ cm$^2$ is Thompson's cross-section.

$m_p$ and $m_e$ are the masses of proton and electron.

$X(x,t)$ is the degree of ionization of the matter: it is the ratio of the density of free electrons to the density of baryons.

$X_a = X(0,t)$ is the degree of ionization on the annihilation surface ($\Sigma$).

$\Phi(R)$ is the annihilation rate on the surface ($\Sigma$).

$v_{tha}$ is the thermal velocity of baryons in the $\mu$-zone.

$f(E)\,dE$ is the number of gamma-photons the energy of which is between $E$ and $E + dE$ emitted by one annihilation (isotropic emission). We have chosen an analytical approximation to the curve given by Stecker (1971):

$$f(E) = 0.05 \exp\left[-9.78/(6.73 - \ln^2(E/68\,\text{MeV}))\right]$$

for $5.3\,\text{MeV} < E < 865\,\text{MeV}$.

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