Exercice 3

a. The brightness is $I_\nu = F_\nu/\Delta\Omega$, where $\Delta\Omega = \pi(\Delta\theta)^2$. Here $\Delta\theta = \theta/2 = 2.15$ arc min $= 6.25\times10^{-4}$ radian. Thus

$$I_\nu = 1.3\times10^{-13}\ \mathrm{erg\,cm^{-2}s^{-1}Hz^{-1}ster^{-1}}$$

$$T_b = \frac{c^2}{2\nu^2 k} I_\nu = 4.2\times10^7\ K.$$

Since $h\nu \ll kT_b$, the use of the Rayleigh–Jeans approximation is appropriate.

b. $T_b \propto I_\nu \propto (\Delta\theta)^{-2}$. If the true $\Delta\theta$ is smaller, the true T_b will be larger than stated above.

c. From Eq. (1.56b) we find $\nu_{max} = 2.5\times10^{18}$ Hz.

d. The best that can be said is $T > T_b$. This follows from Eq. (1.62) with $T_b(0) = 0$. In general, the maximum emission of any thermal emitter at given temperature T will occur when the source is optically thick (see Problem 1.8 *d*).

Exercice 4