

# Radiative Transfer

## 1. Introduction

# Outline of the lecture

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1. Introduction
2. Radiative quantities
3. Formal radiative transfer equation
4. Radiation and matter in equilibrium
5. Introduction to scattering
6. Radiative transfer in dusty media
7. Line radiative transfer
8. Numerical methods
9. Other radiative processes

# 1. Aim of the lecture

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- Becoming radiative transfer experts in just under 23 hours
- The main goal is to give an overview, enough background to understand more specialised literature
- Not everything can be covered, so a selection has been made
- Probably biased!
- We will cover some theory, methods of resolution, usual approximations for analytical solutions, numerical solutions... time permitting

# 2. Bibliography

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## Among others

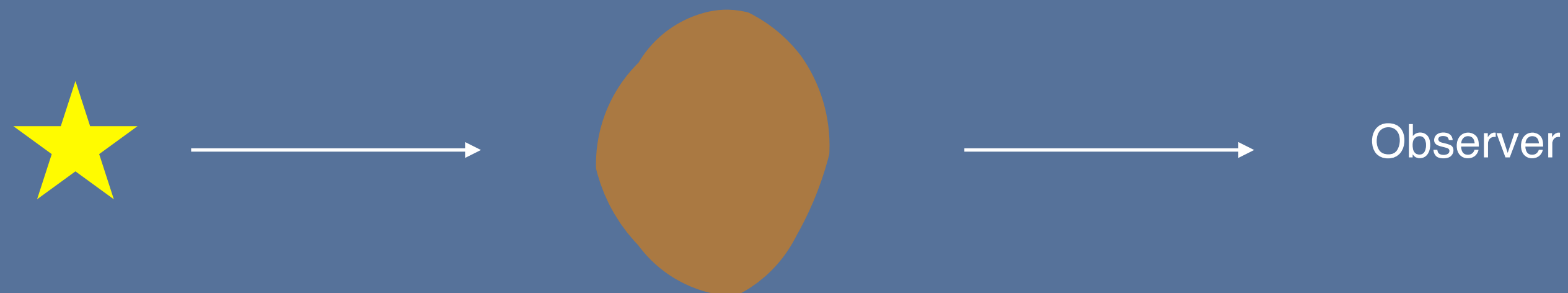
- Rybicki & Lightman, Radiative Processes in Astrophysics, Wiley Interscience (in depth view of radiative processes)
- Rob Rutten's Radiative transfer in stellar atmospheres at Utrecht's Institute for stellar studies
- Kees Dullemond's radiative transfer lecture at the University of Heidelberg (numerically oriented radiative transfer)

## More specialised

- Lambert, Josselin, Nyde, & Faure 2015, A&A, 580, 50
- Steinacker, Baes, Gordon, 2013, ARA&A, 51, 63
- Daniel & Cernicharo 2013, A&A, 553, 70
- Robitaille 2011, A&A, 536, 79
- articles by P. Hauschildt on 3D radiative transfer in stellar and planetary atmospheres

# 3. What is radiative transfer and why it is important

- Radiative transfer is everywhere
- Understanding of astrophysical objects
  - How they work
  - How they are formed
  - How they evolve
- Understanding images and spectra



- Radiative transfer is the study of the emission of photons, their transport in space and their interaction with the matter they encounter

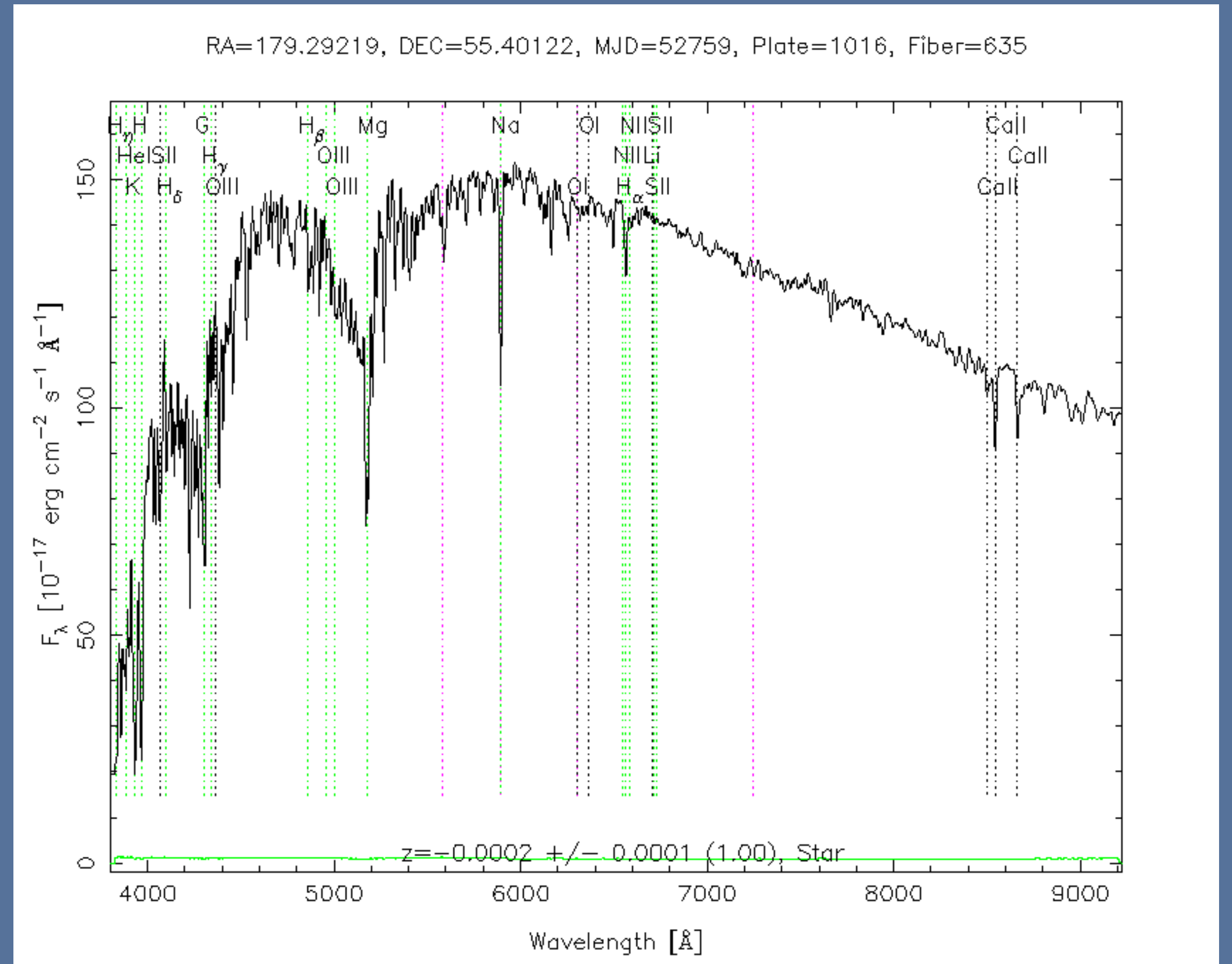
# 3. What is radiative transfer and why it is important

- M dwarf spectrum
- Photospheric (mass) composition:

H 74%

He 25%

Mg 0.05%



# 3. What is radiative transfer and why it is important

- The radiation we receive from astrophysical objects depends on the emission process, on the physical conditions (density, temperature), on the surrounding matter (which can modify it), on the magnetic field, on kinematics...
- We cannot do without radiative transfer
- We have to understand how radiation is emitted, transported, modified by its interactions with matter if we want to interpret correctly the observations
- Radiative transfer is often seen as a necessary evil: we want to make nice images or spectra (if we are observers), or run large simulations (if we are modellers). But without radiative transfer, how can we derive the physical conditions from our image, or get the radiative heating correctly in a protoplanetary disk?
- Radiative transfer is actually the link between observation and theory
- It is not only a carrier of information, but also a very important physical phenomenon that determines the structure of astrophysical objects.

# 3. What is radiative transfer and why it is important

- Radiative transfer is at the same time
  - A diagnostic tool
  - A structuring force for astrophysical objects
- Radiative transfer is used by many astrophysicists
- But is also a research field on its own
- As users of radiative transfer, we need to understand at least the basics, the approximations made, the different codes, why use one and not the others, how to use them.



the Earth's sky



Planetary atmosphere...

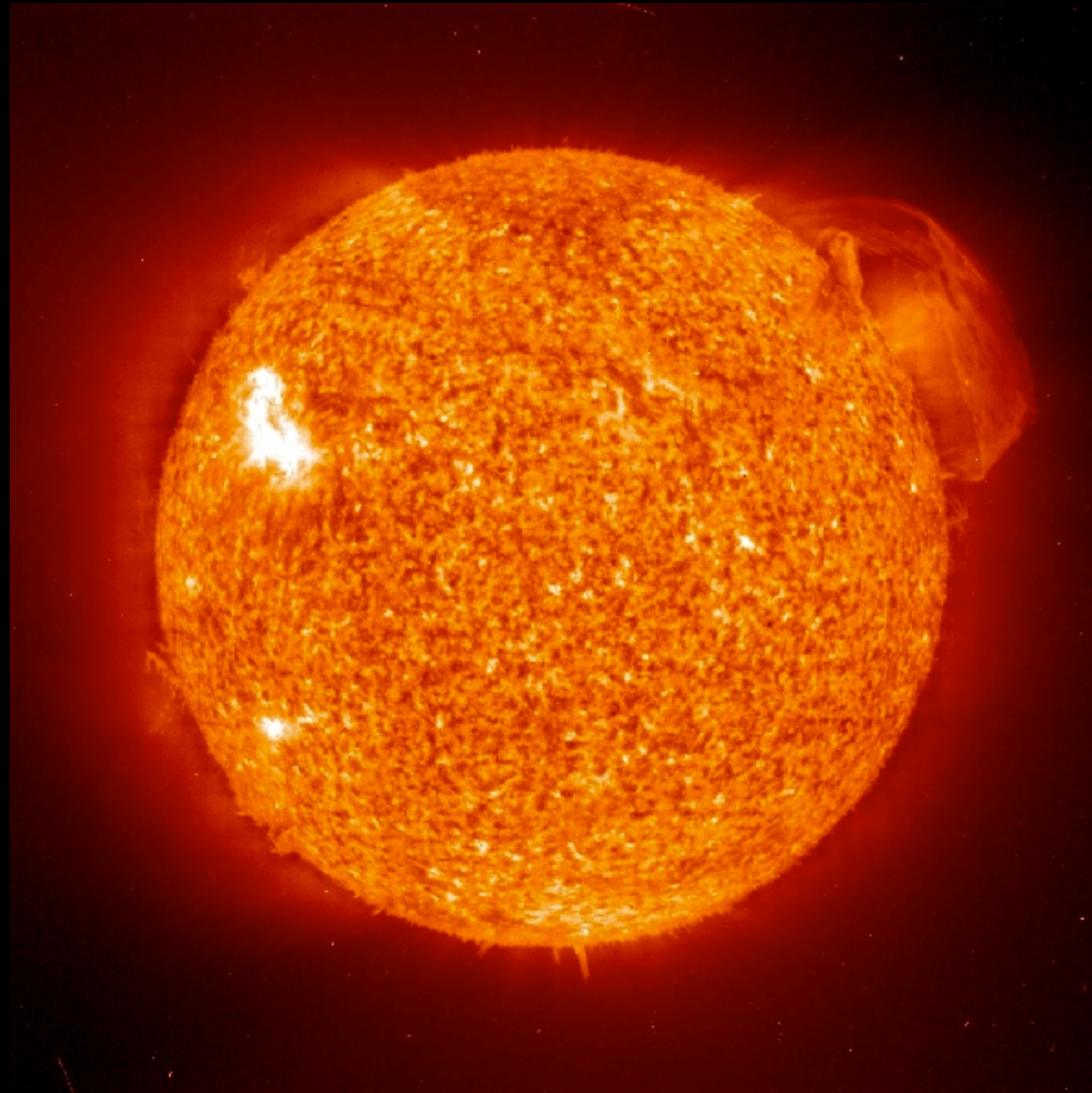


the Earth's sky, again



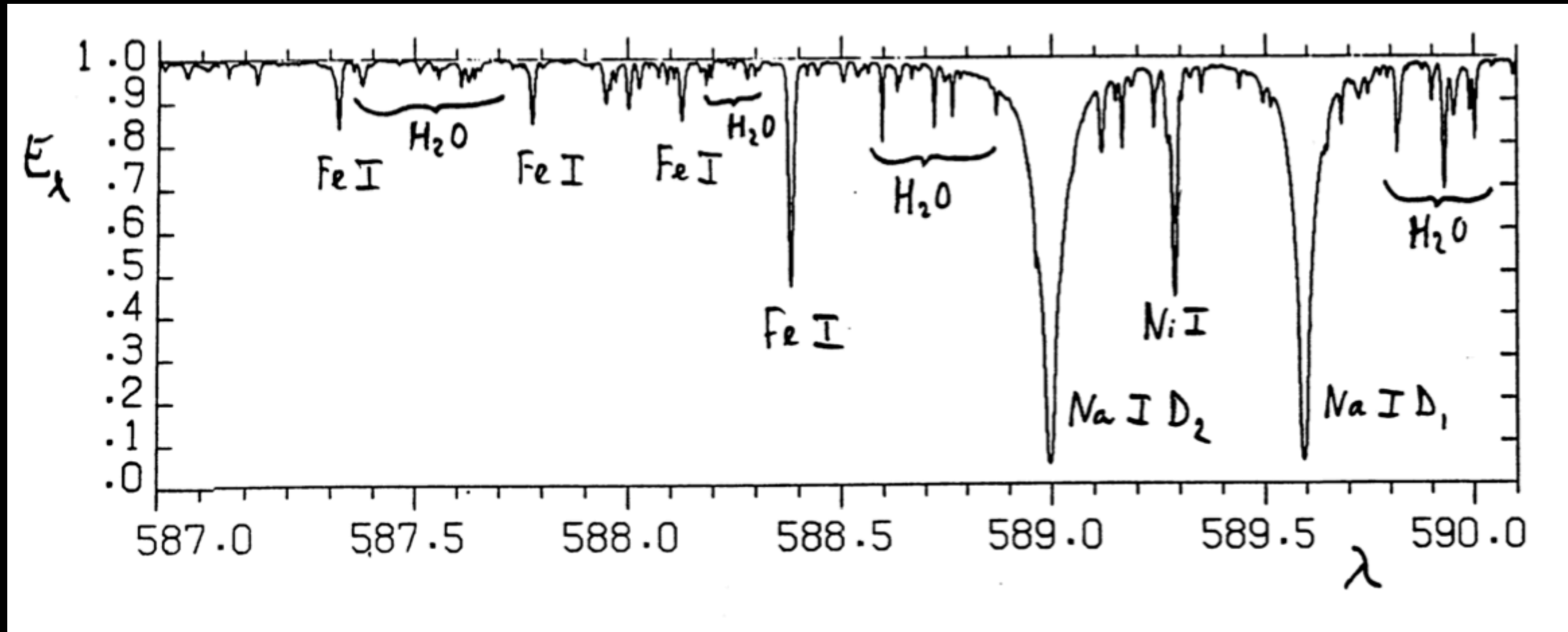
the Sun

extreme UV - 304 Å (He II)



SOHO/NASA

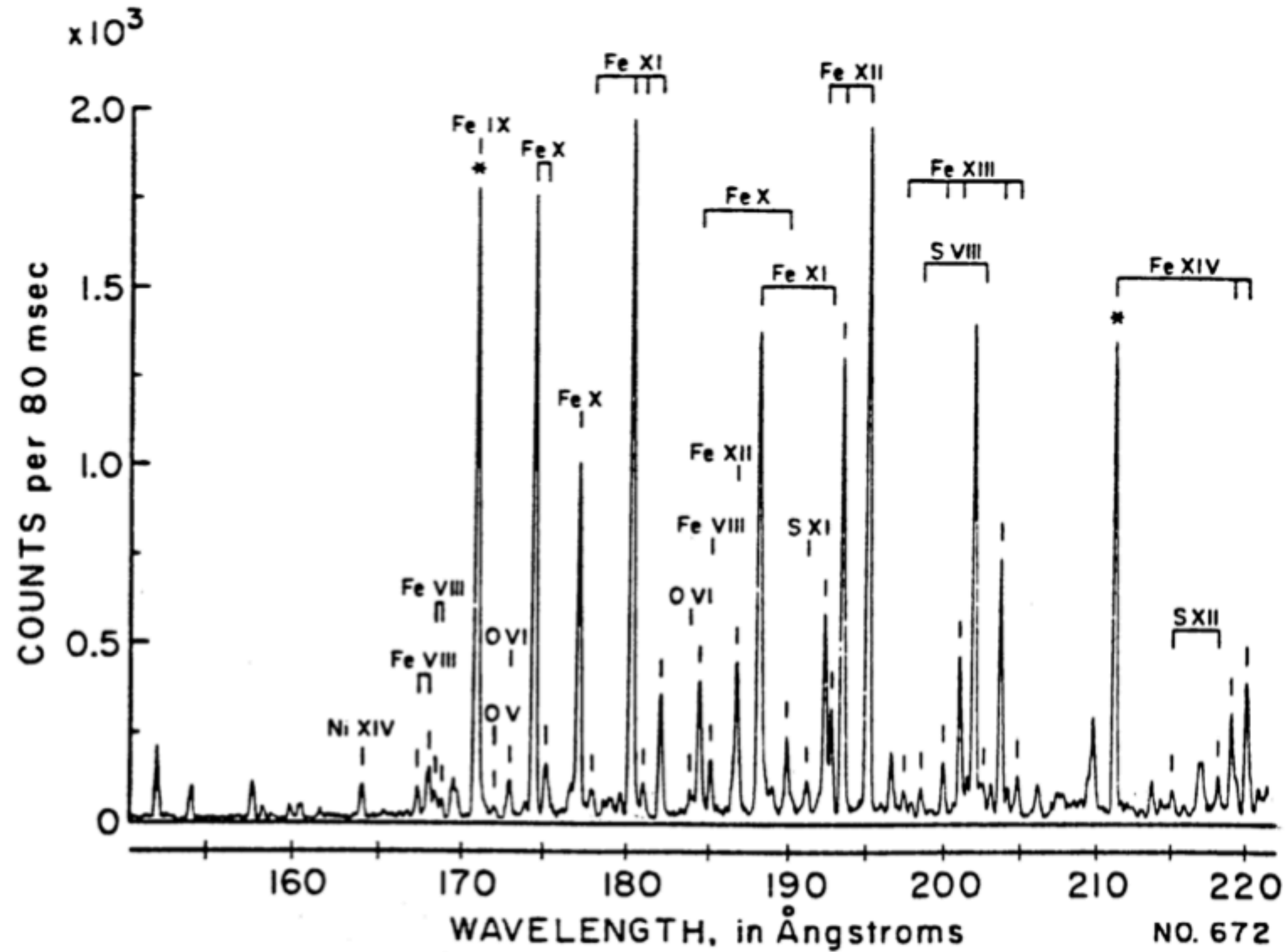
# Solar spectrum - Sodium lines photosphere



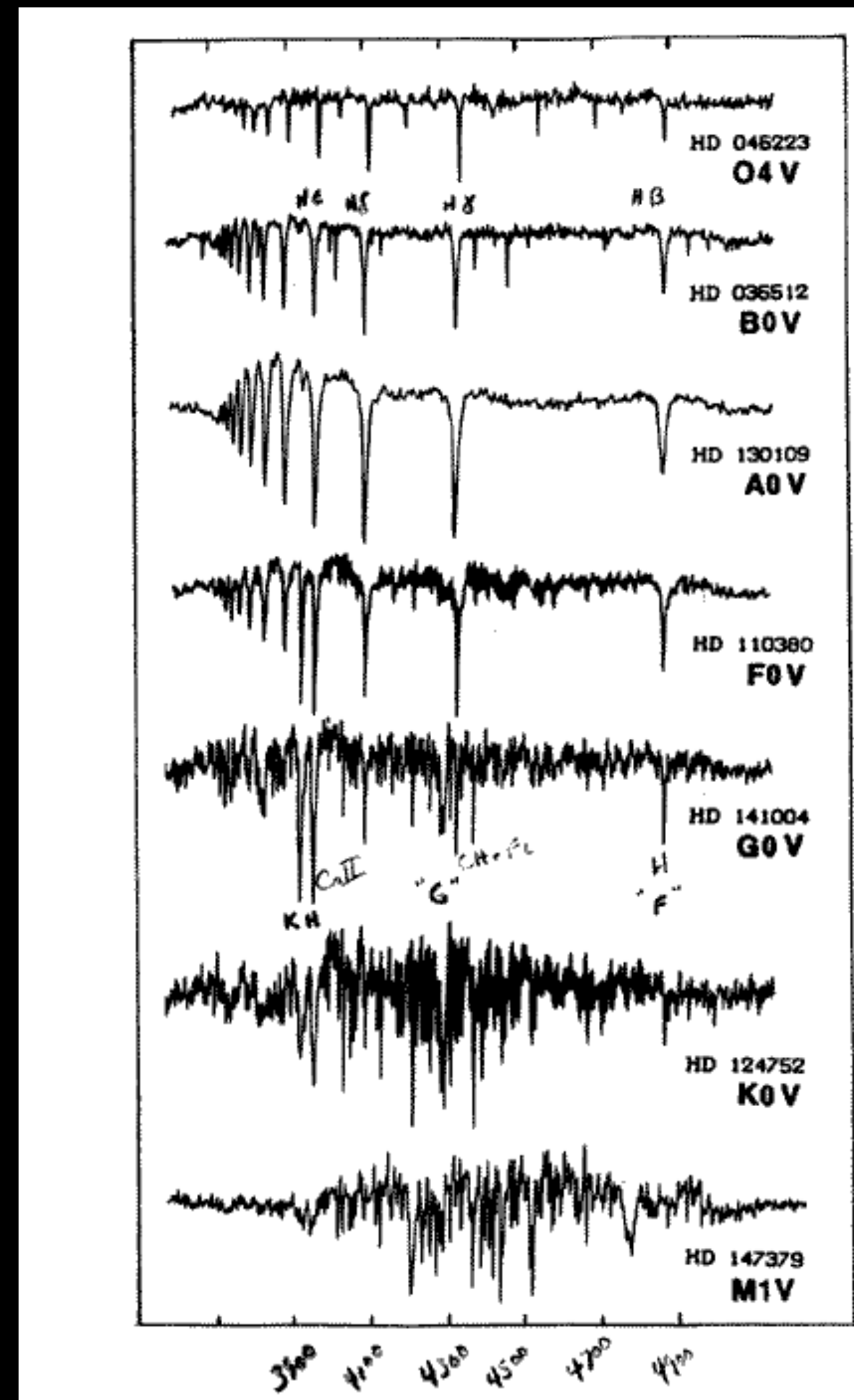
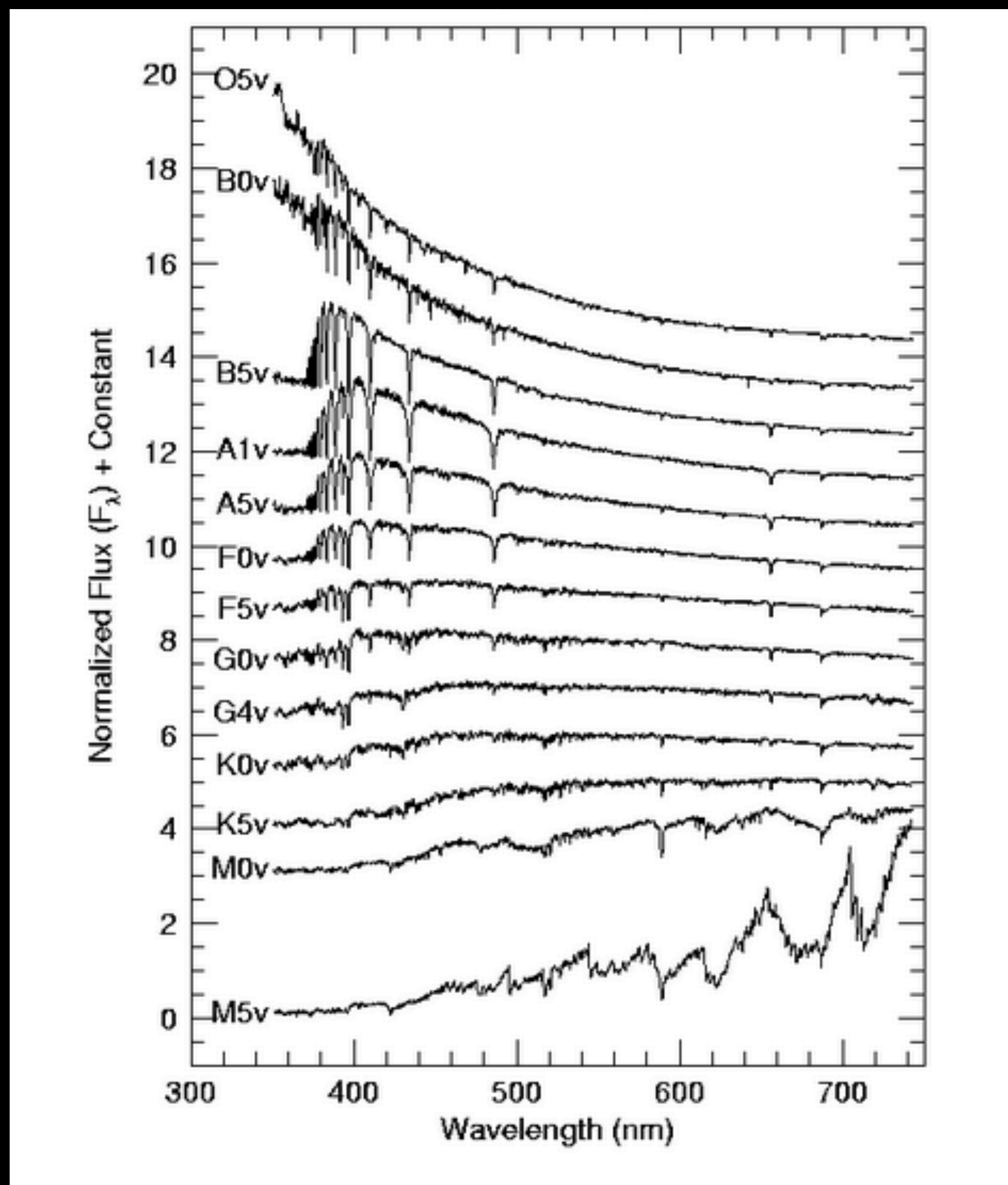
Kurucz et al. (1984)

# X-ray spectrum of the Sun

coronal plasma



# Stellar spectra



# Star colours



Image: G. Parker



# Interstellar reddening



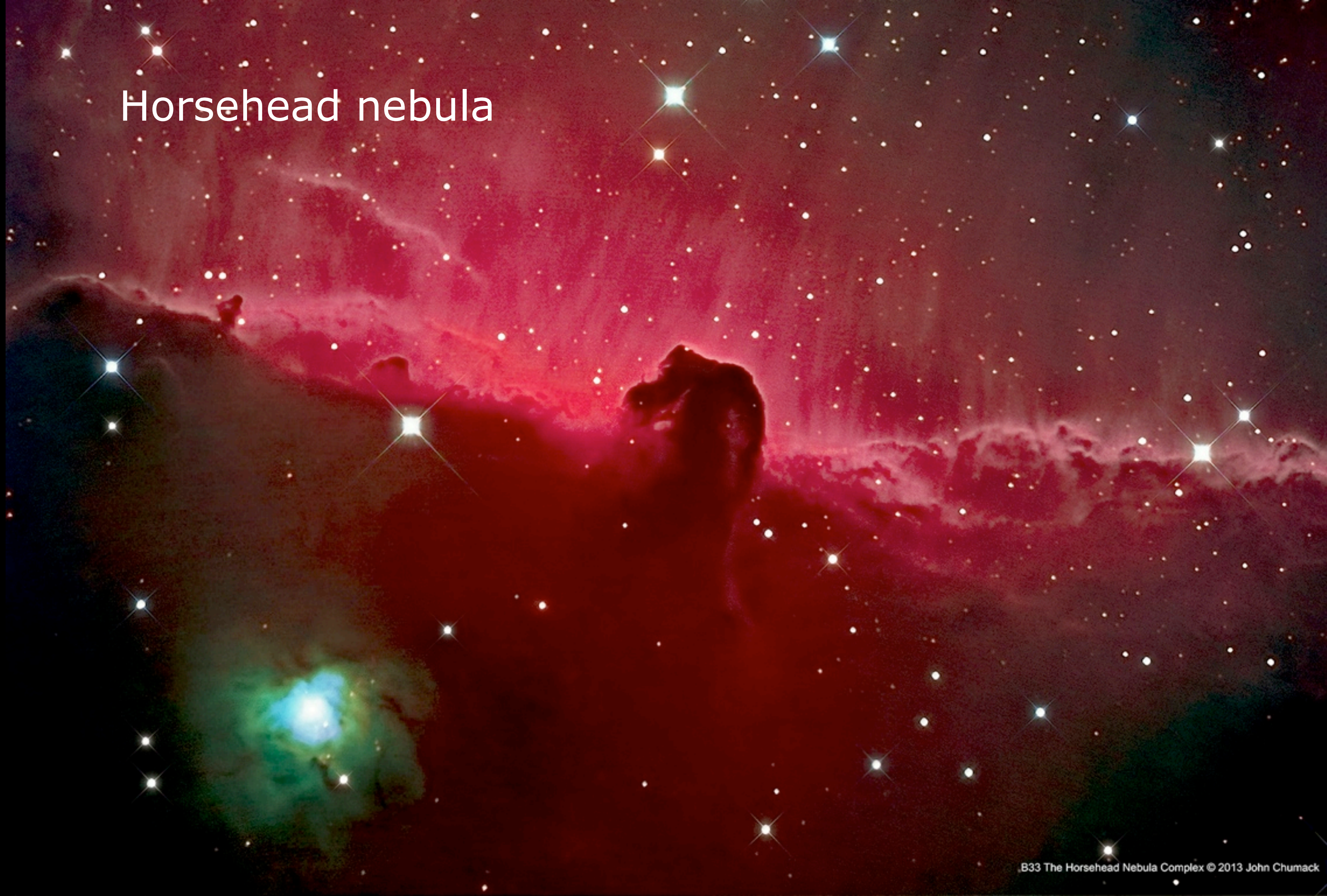
IC4605



Orion nebula



# Horsehead nebula



Pipe nebula and Rho  
Ophiuchus



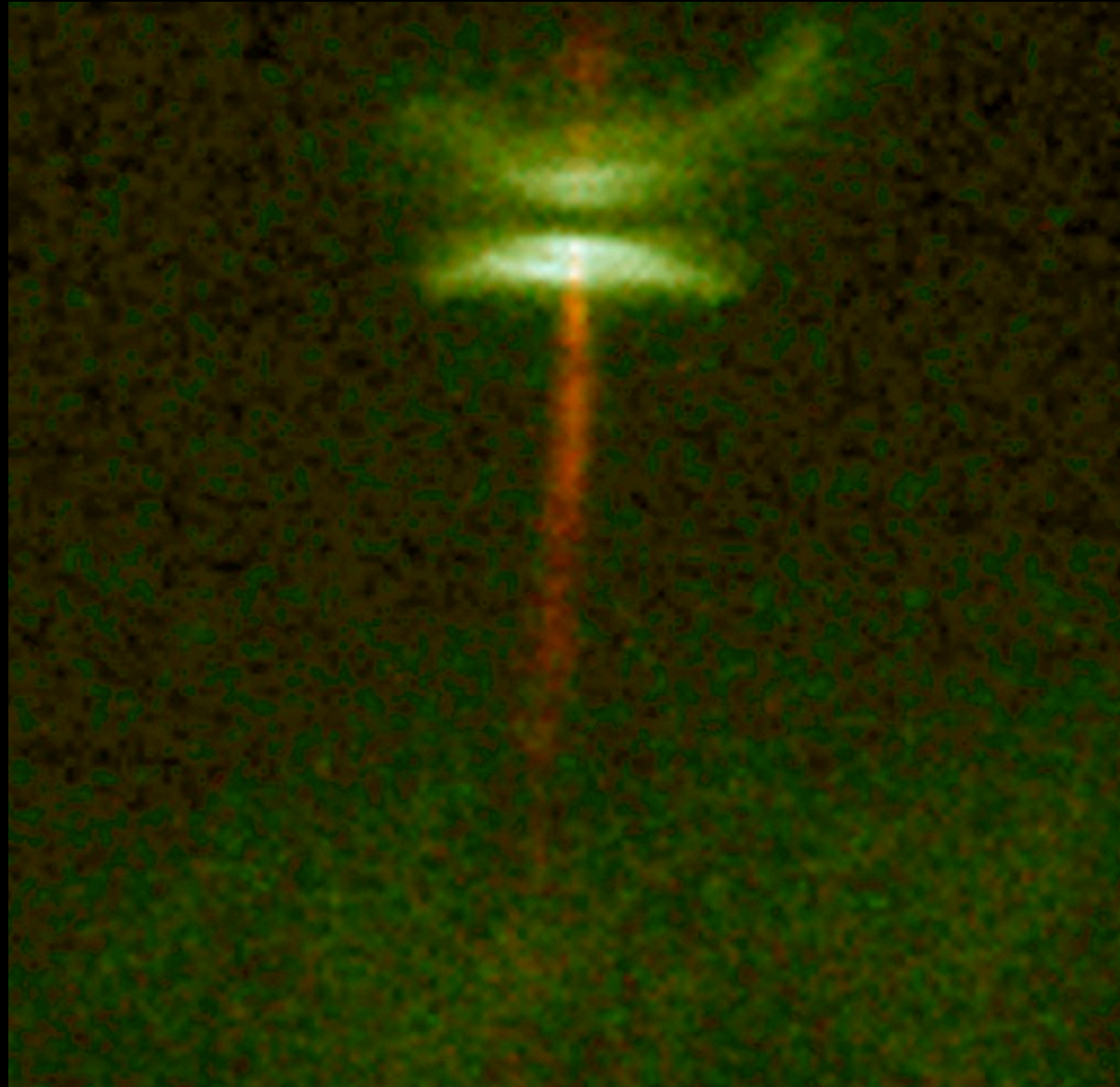
Pipe nebula and Rho  
Ophiuchus



Snake nebula



# HH 30 (T Tauri star)

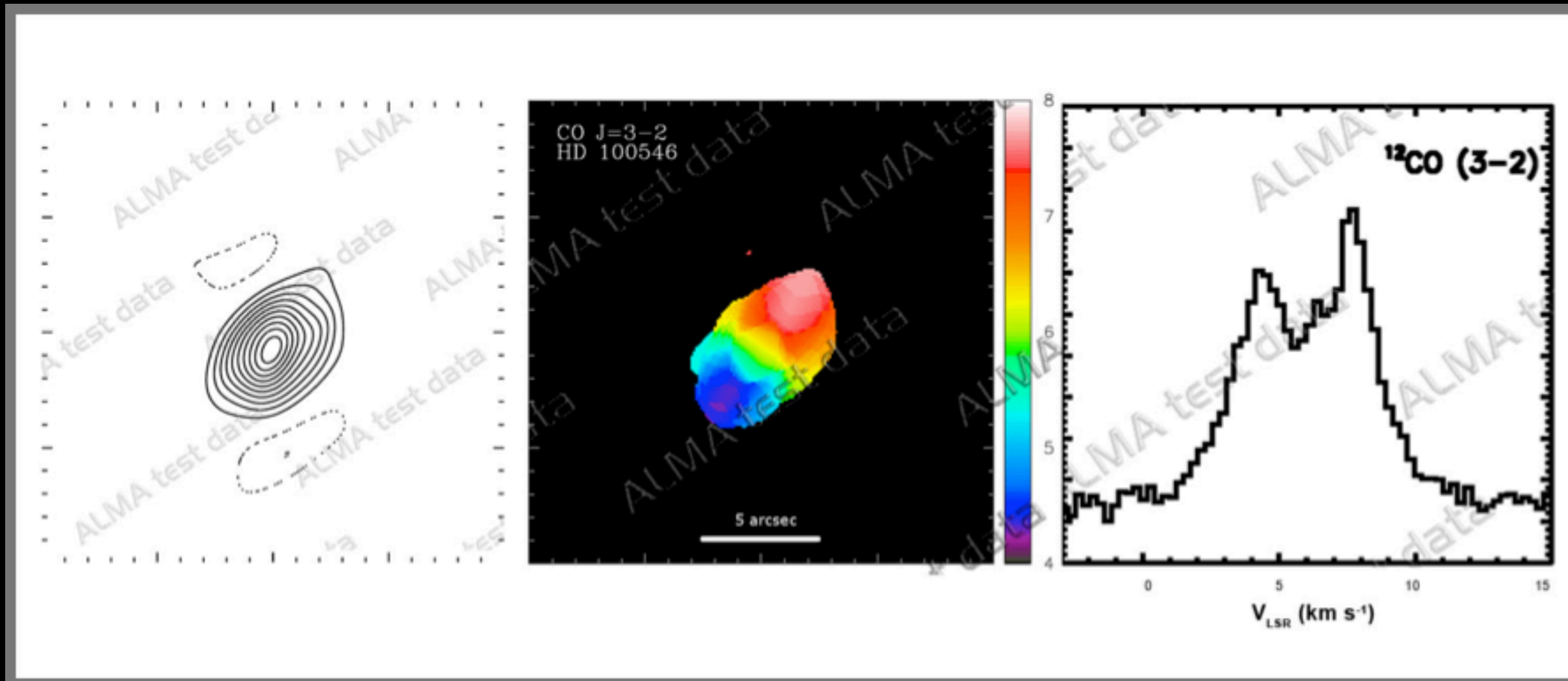


visible

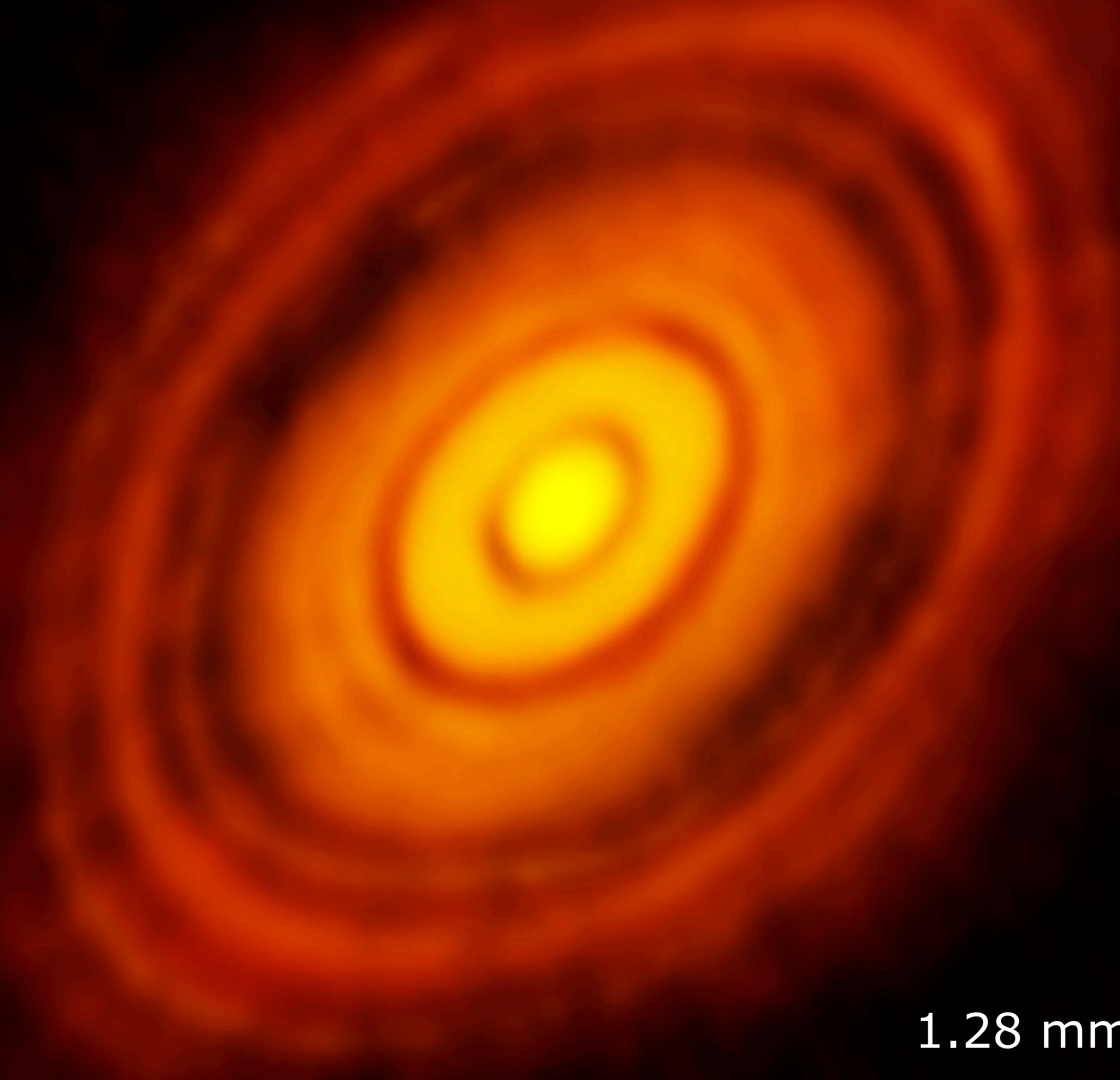
images/HST



# Circumstellar disk HD 100546



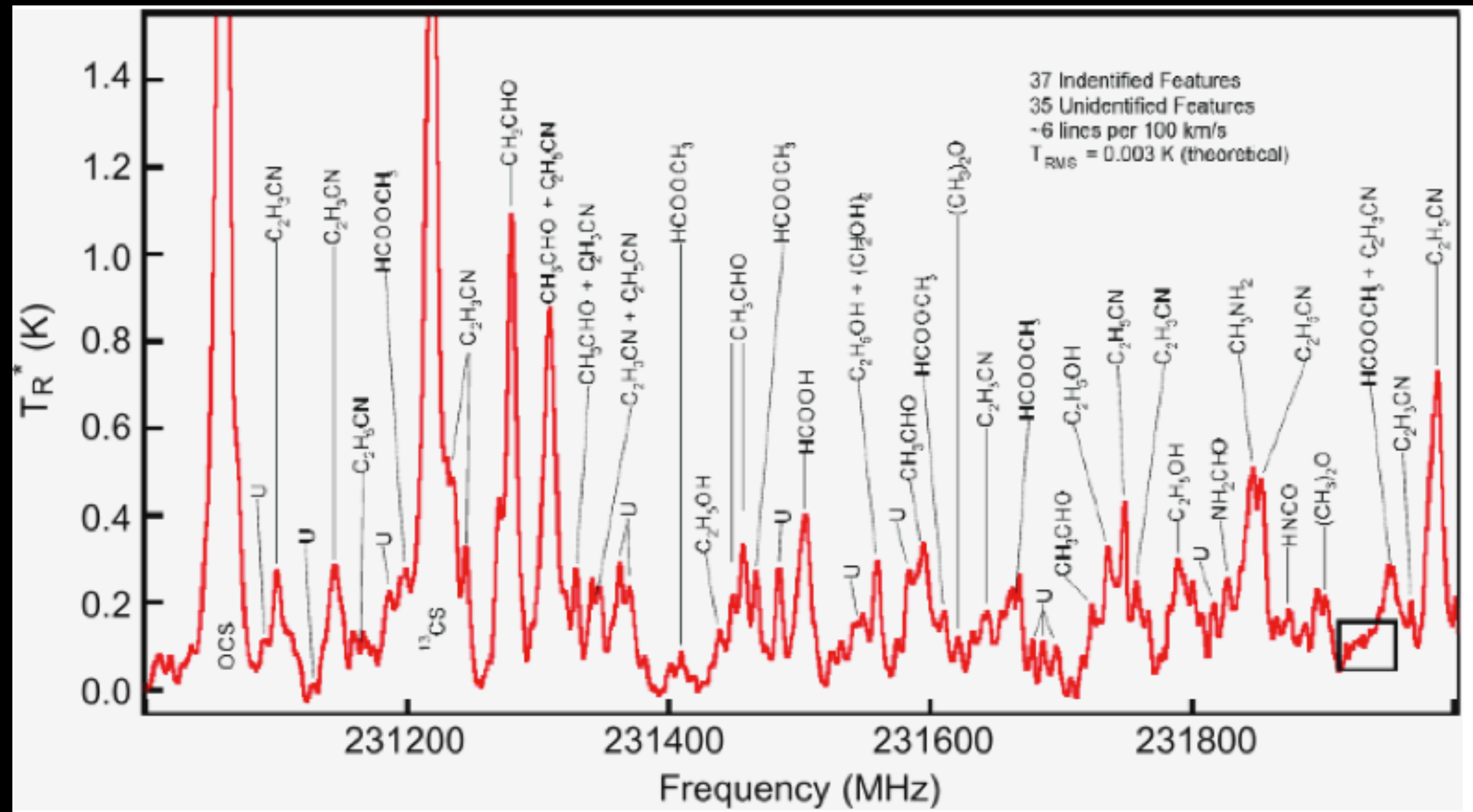
# Protoplanetary disk HL Tau



1.28 mm (233 GHz)

ALMA

# SgrB2 (Centre Galactique)

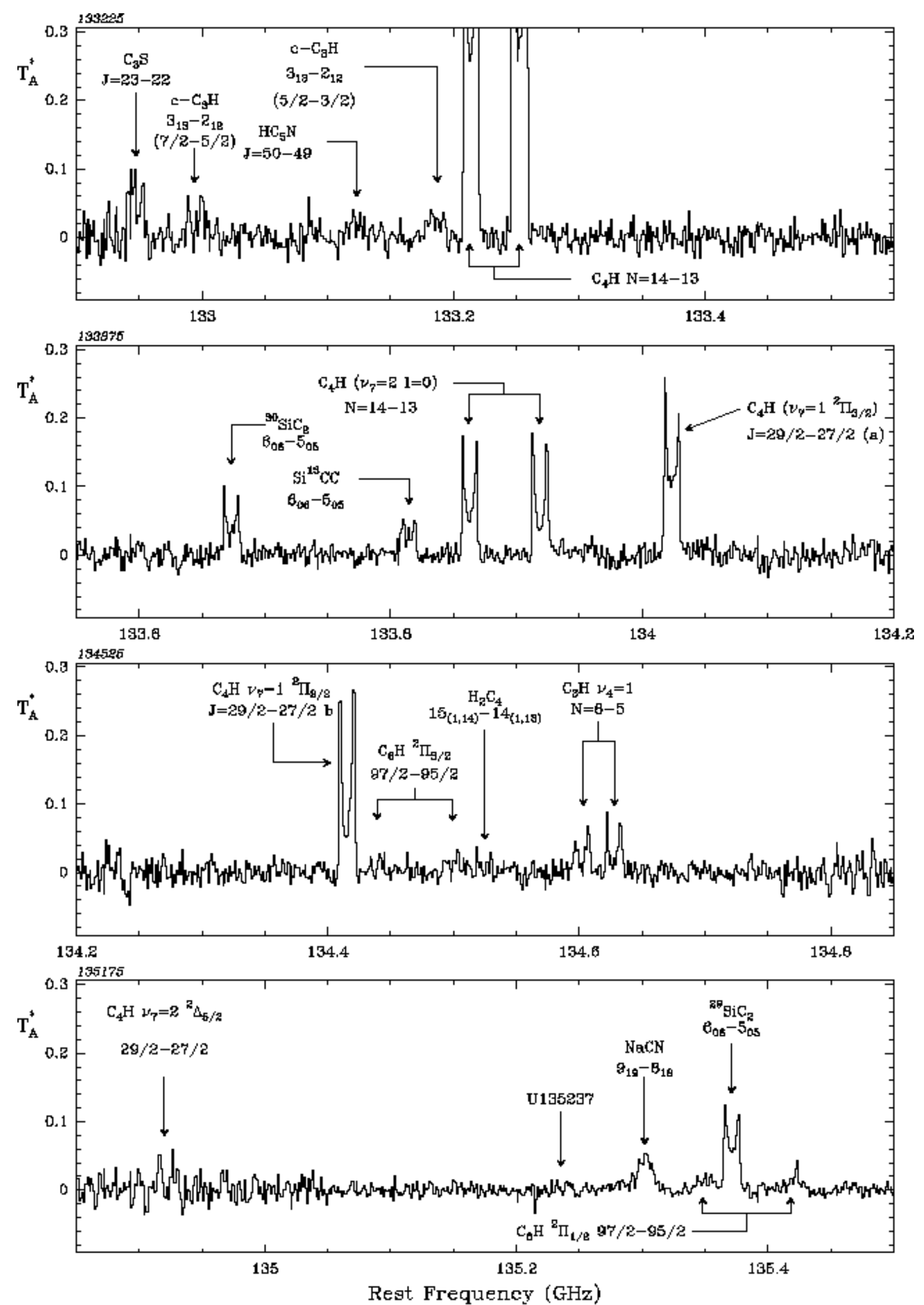


Ziurys et al. (2006)



# IRC 10216 (AGB)

Cernicharo et al. 1999



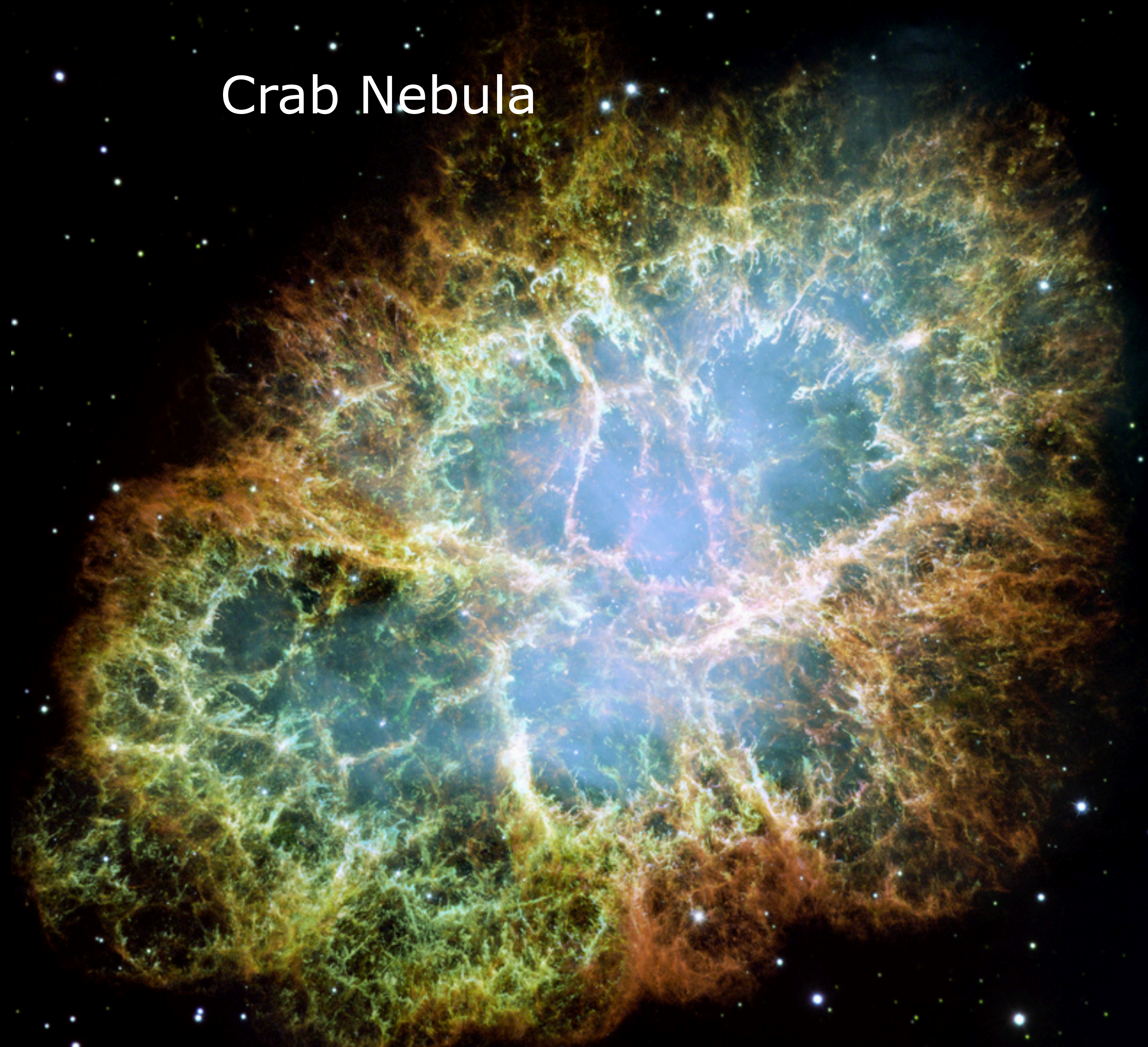
Cat's eye nebula



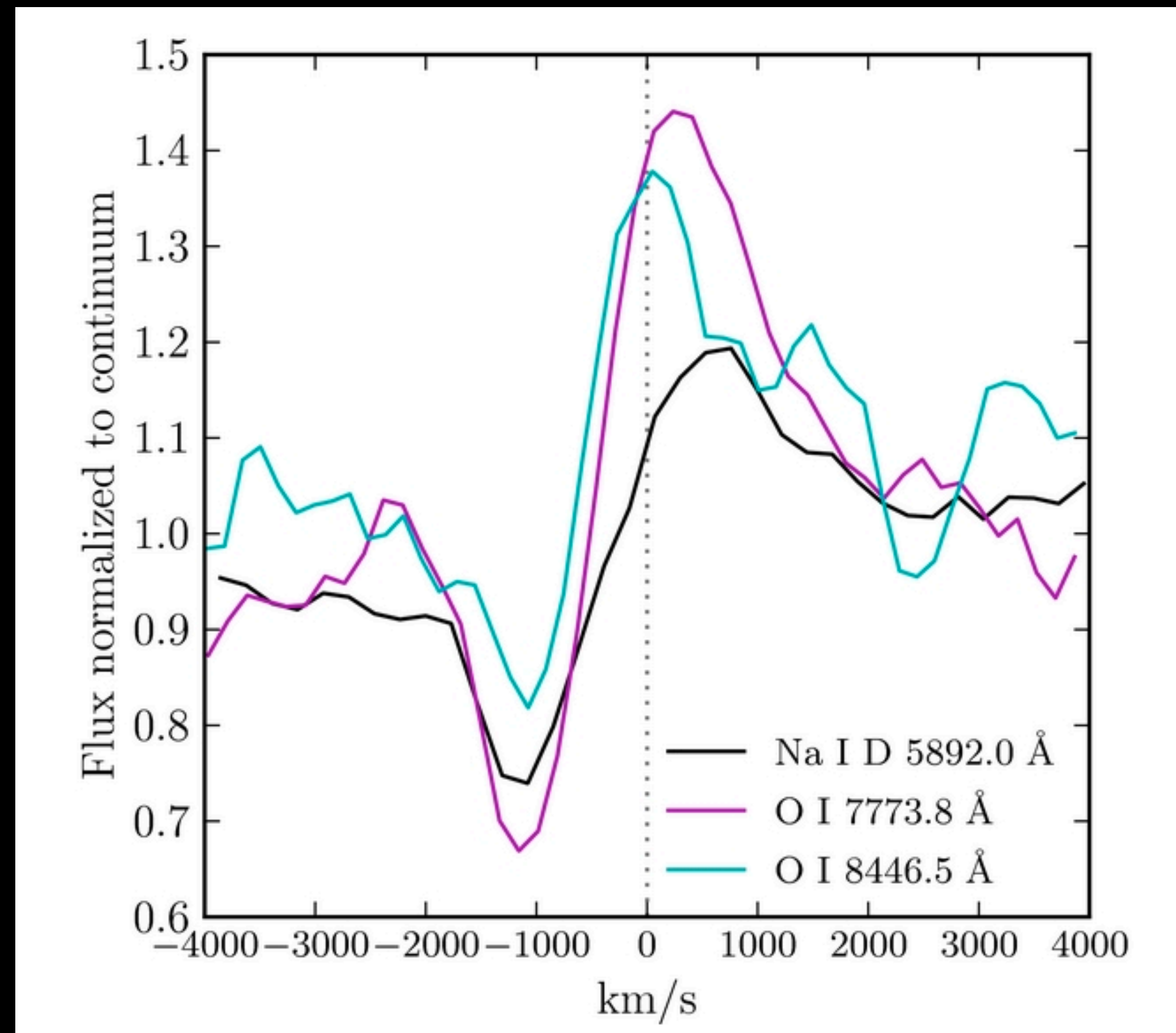
# Crab Nebula

visible

images/HST



# SN 2010 U



# M82 (starburst galaxy)

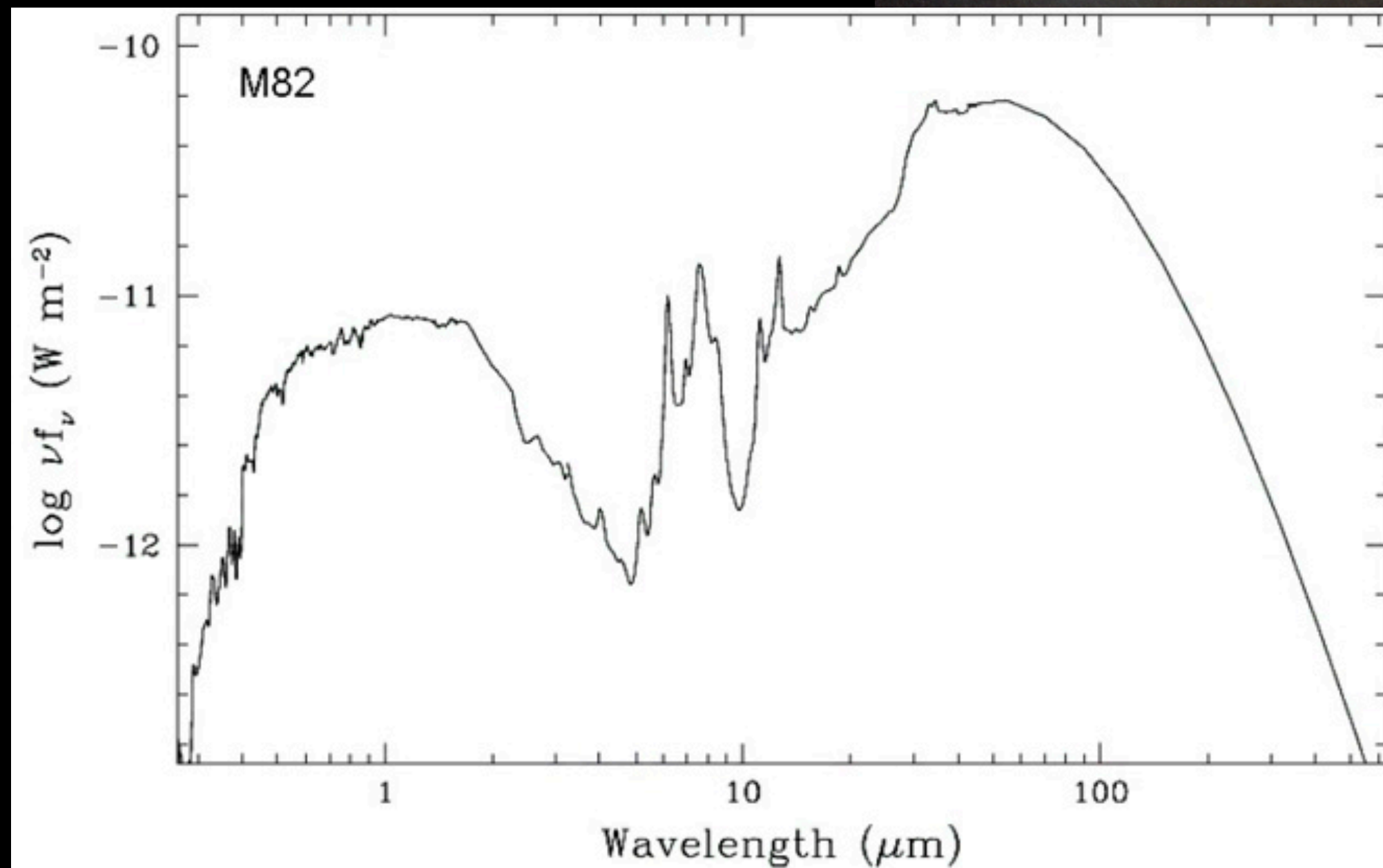
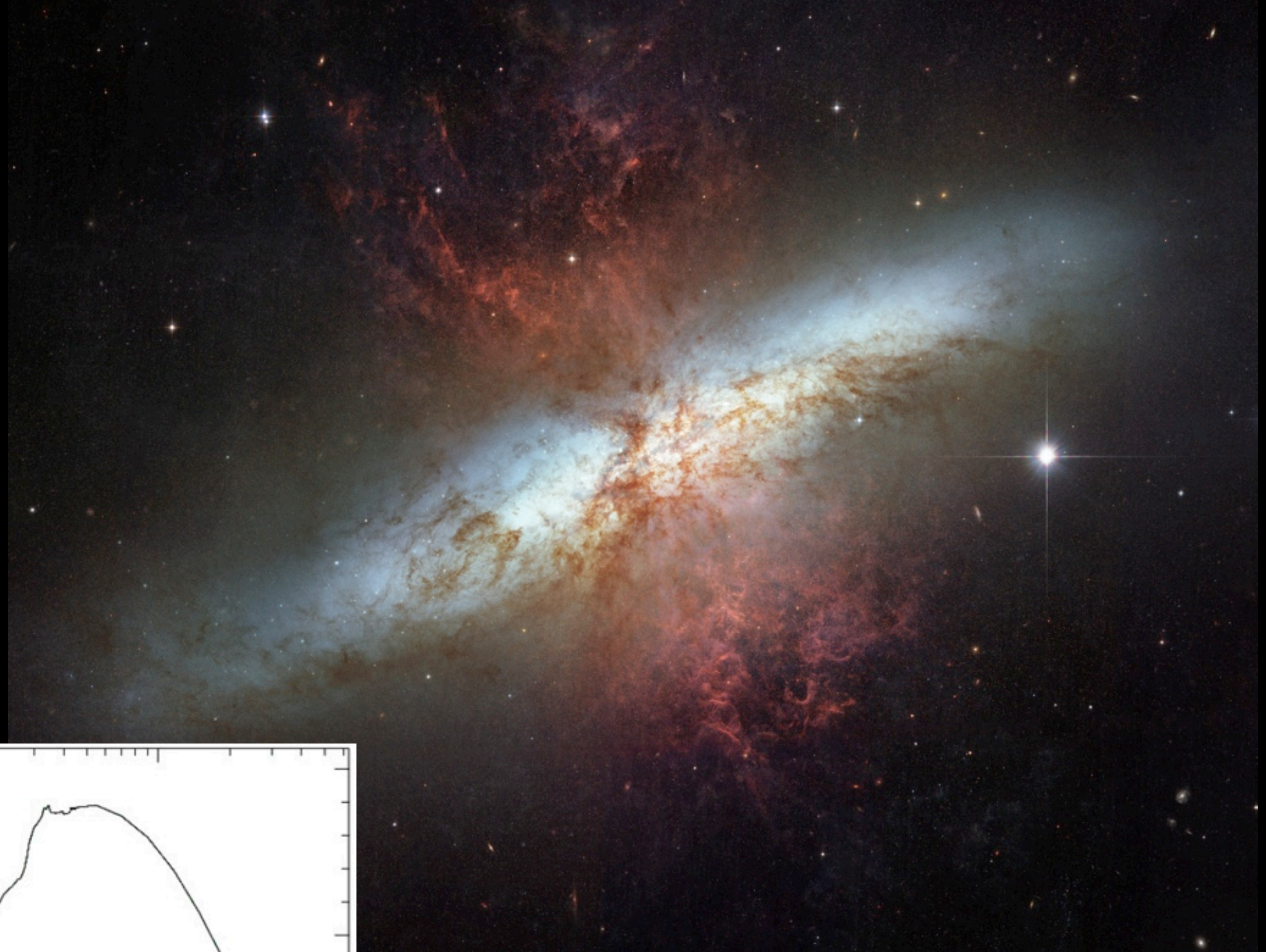
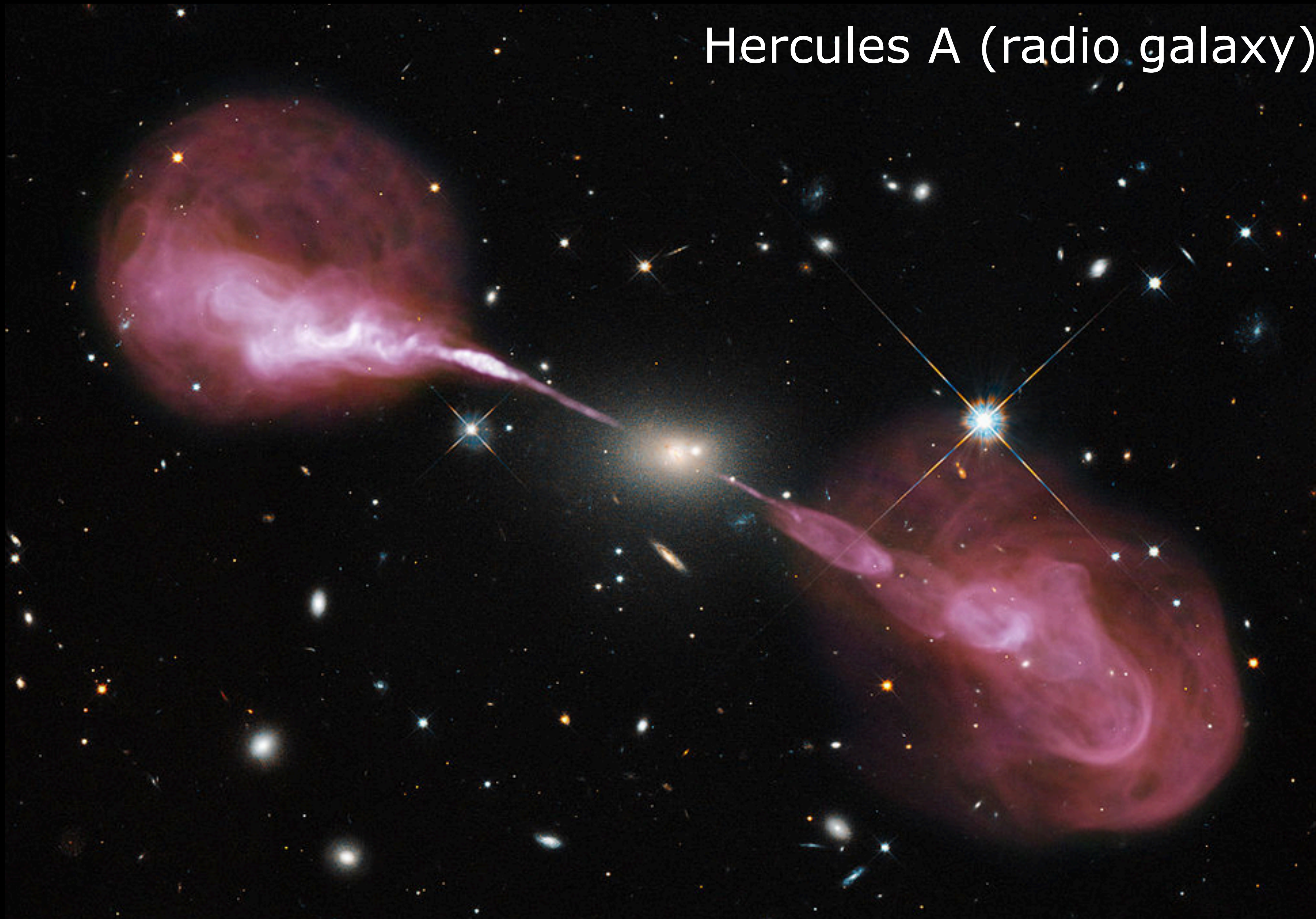


Image HST  
SED Kennicutt et al. (2003)



Hercules A (radio galaxy)



visible+radio composite image

HST/VLA

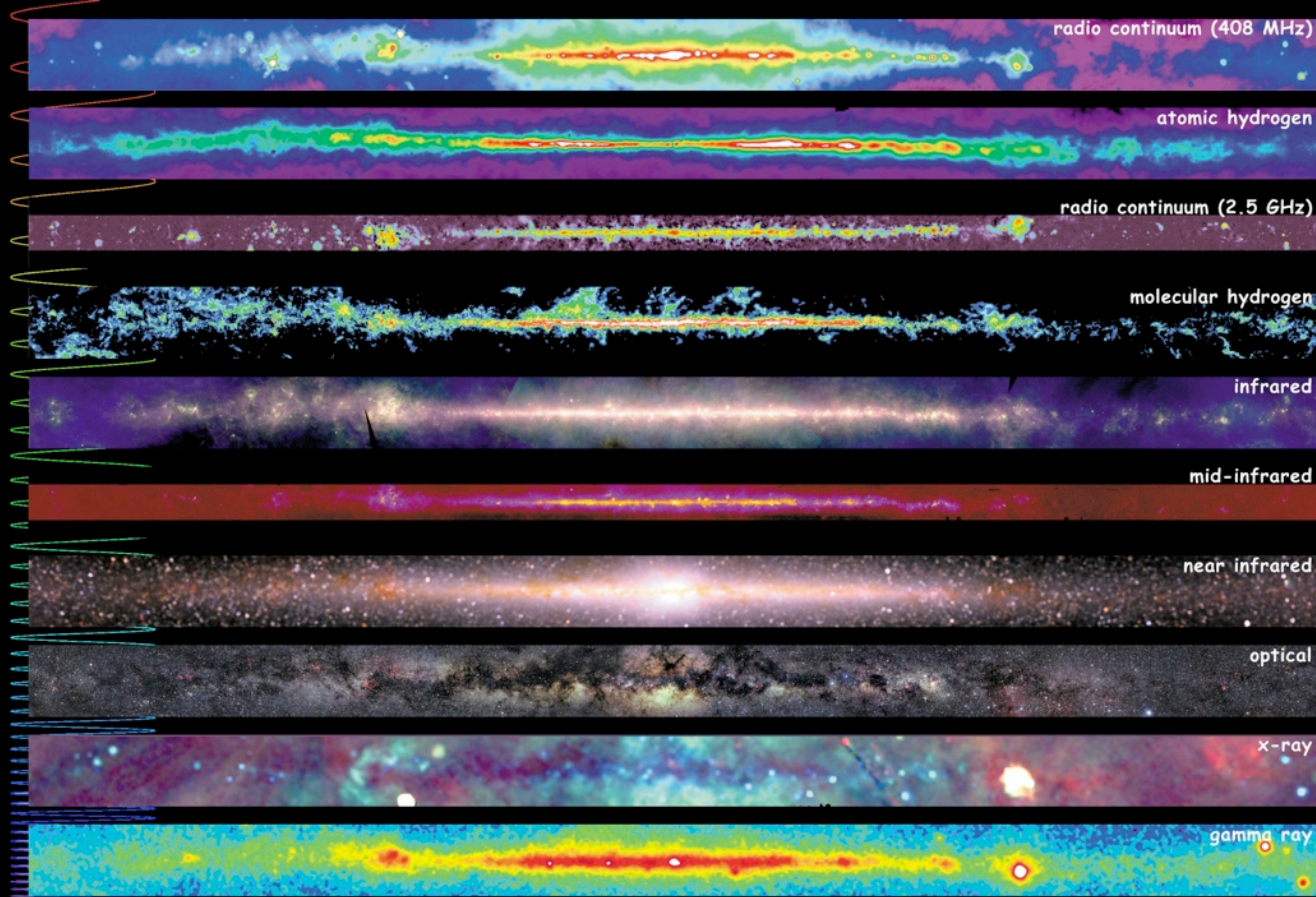
# Centaurus A (starburst galaxy)



# 3.1 Radiative transfer as a diagnostic tool

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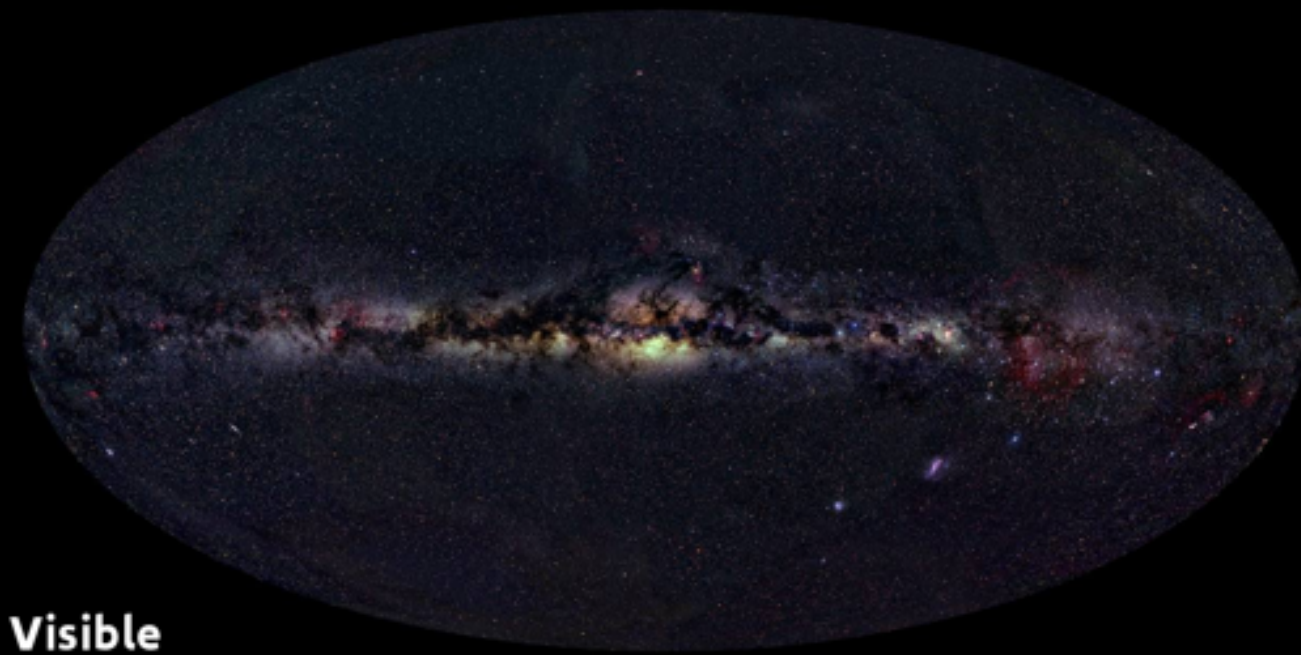
- Most objects are too far to be studied in situ
- Most information from space is in the form of electromagnetic waves (photons)
- Advantages
  - Most objects emit electromagnetic waves
  - photons carry a wealth of information: they cover a very broad energy spectrum.
  - Electromagnetic radiation tells us about processes at the origin of the emission, physical conditions of the astrophysical object, interactions along the path to the observer
  - In vacuum, photons travel at  $c$  (the fastest velocity we know)
  - Photons do not decay (in contrast to some other particles). Photons that do not interact with matter will not be destroyed
- Electromagnetic waves are the oldest diagnostic method



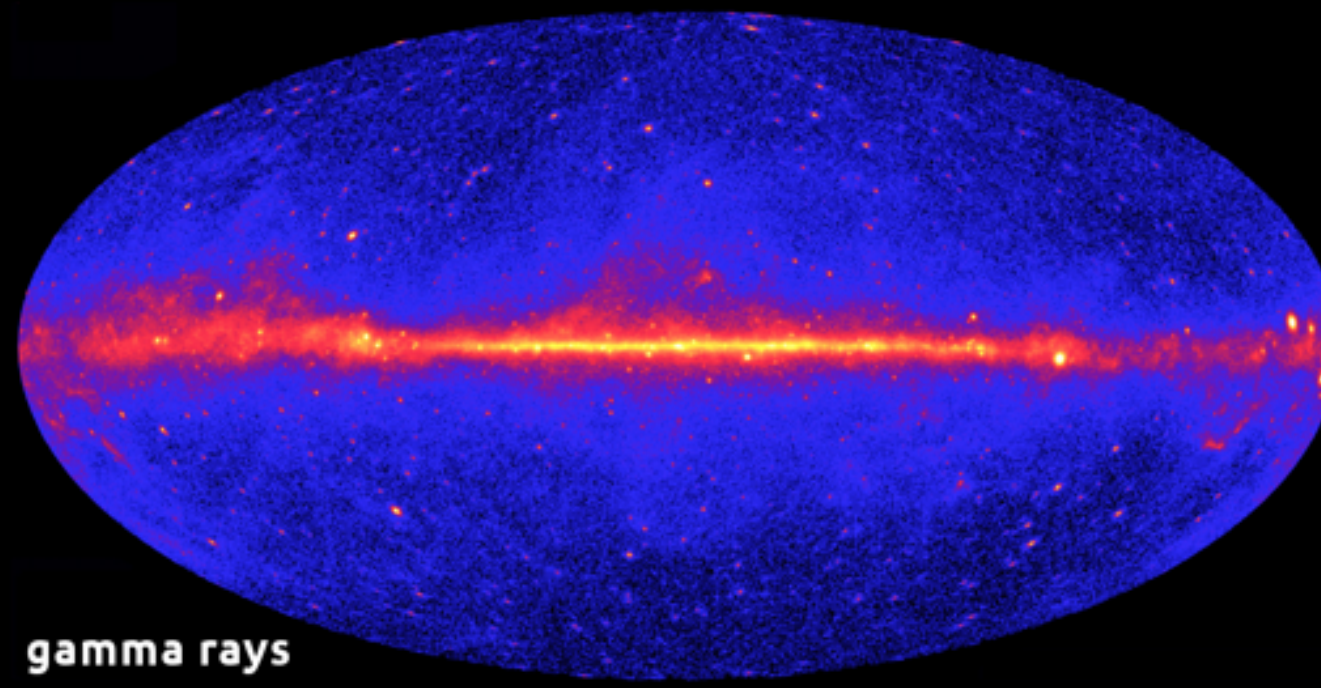
<http://adc.gsfc.nasa.gov/mw>



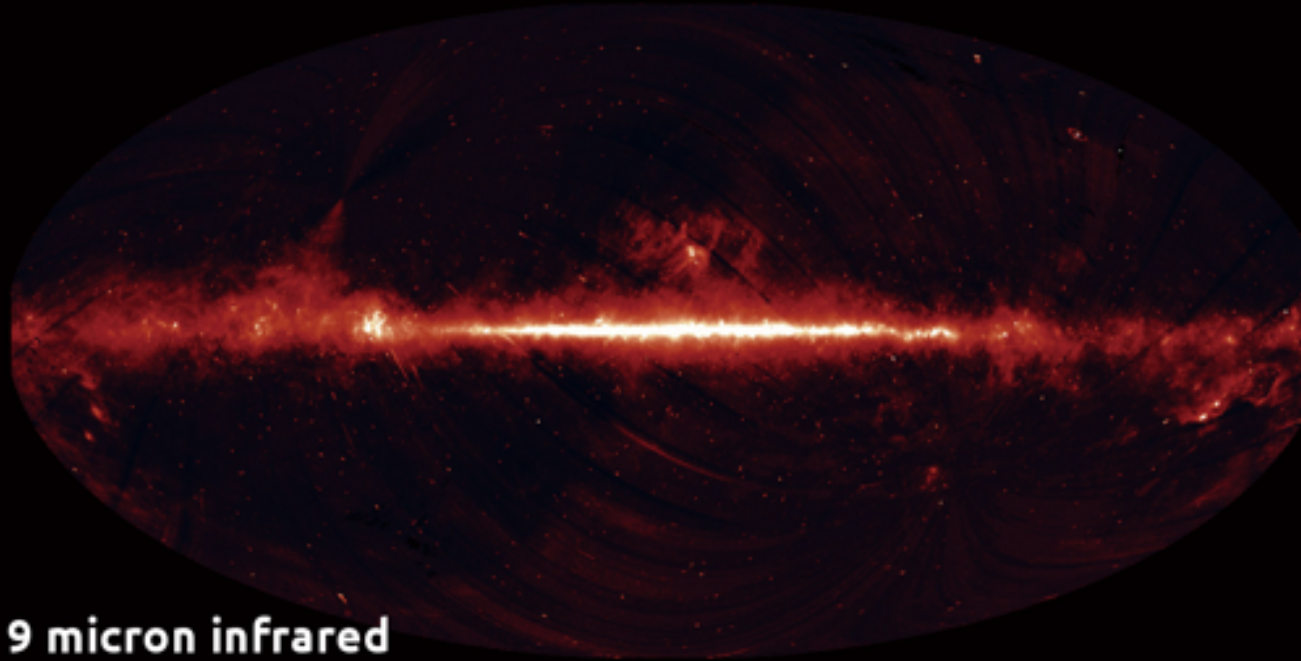
# Multiwavelength Milky Way



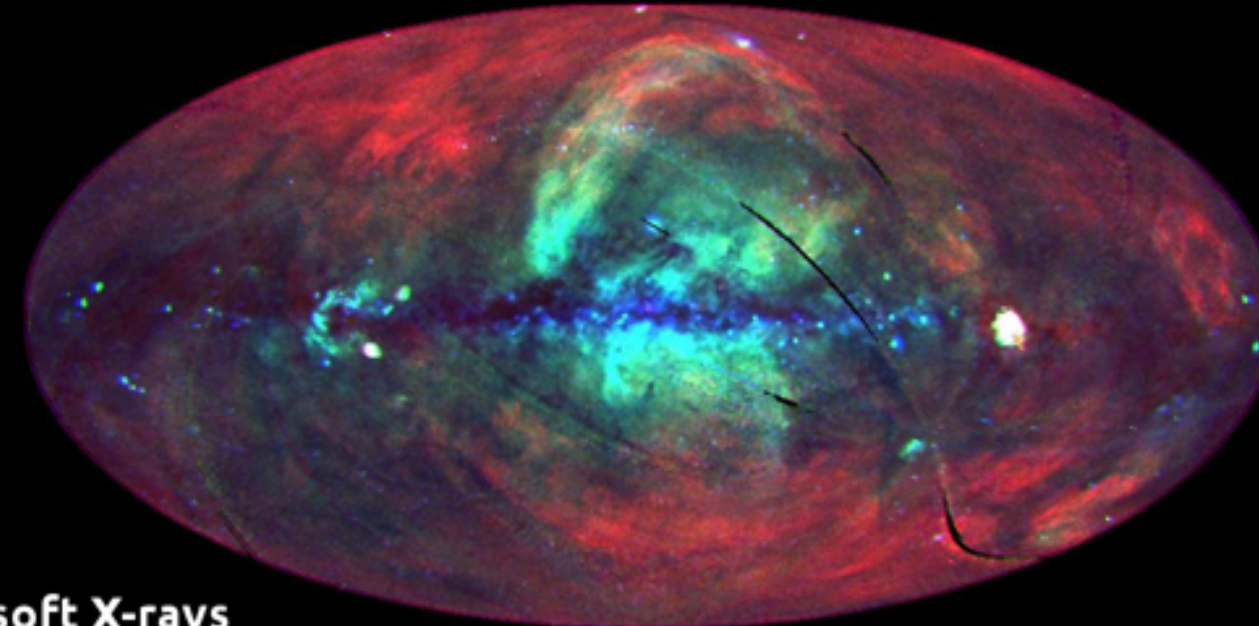
Visible



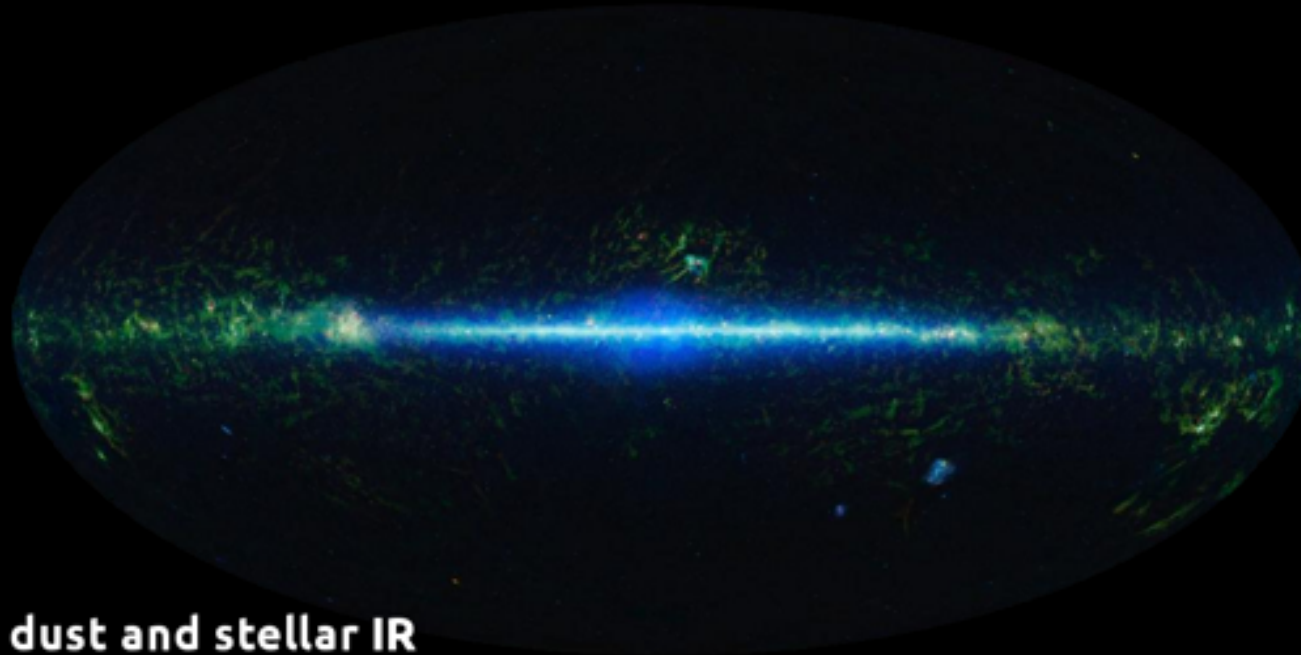
gamma rays



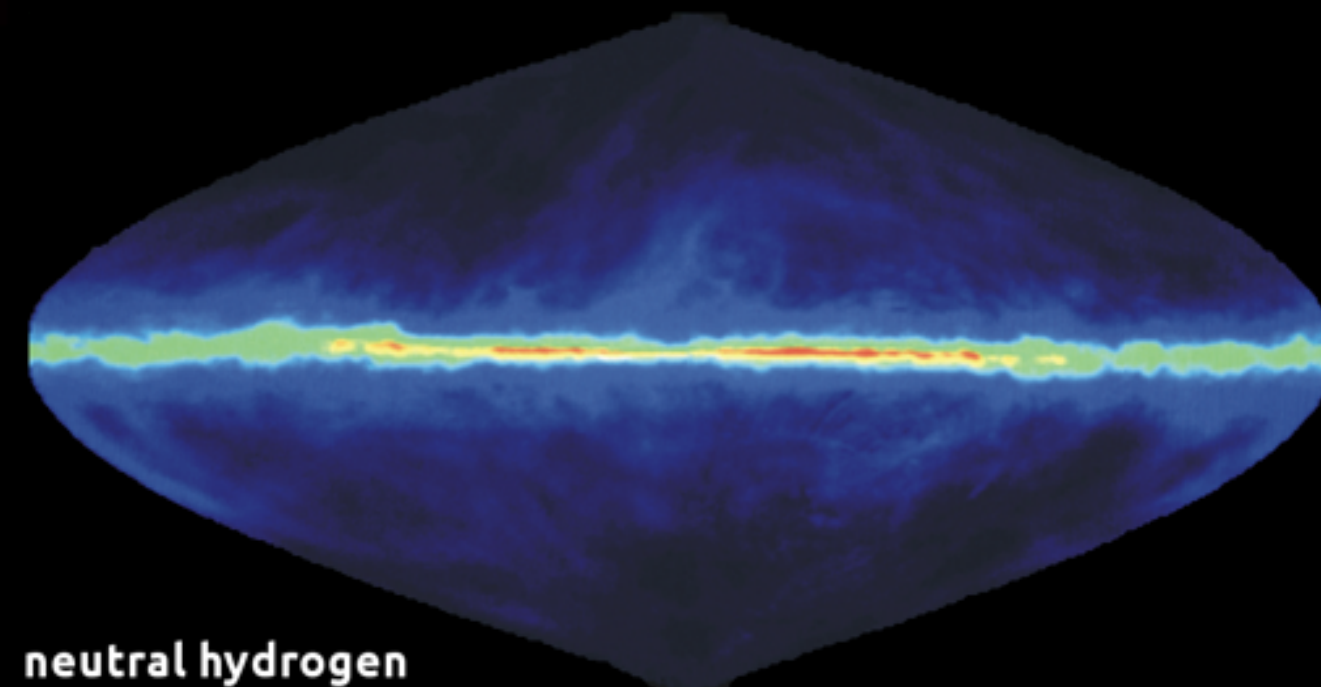
9 micron infrared



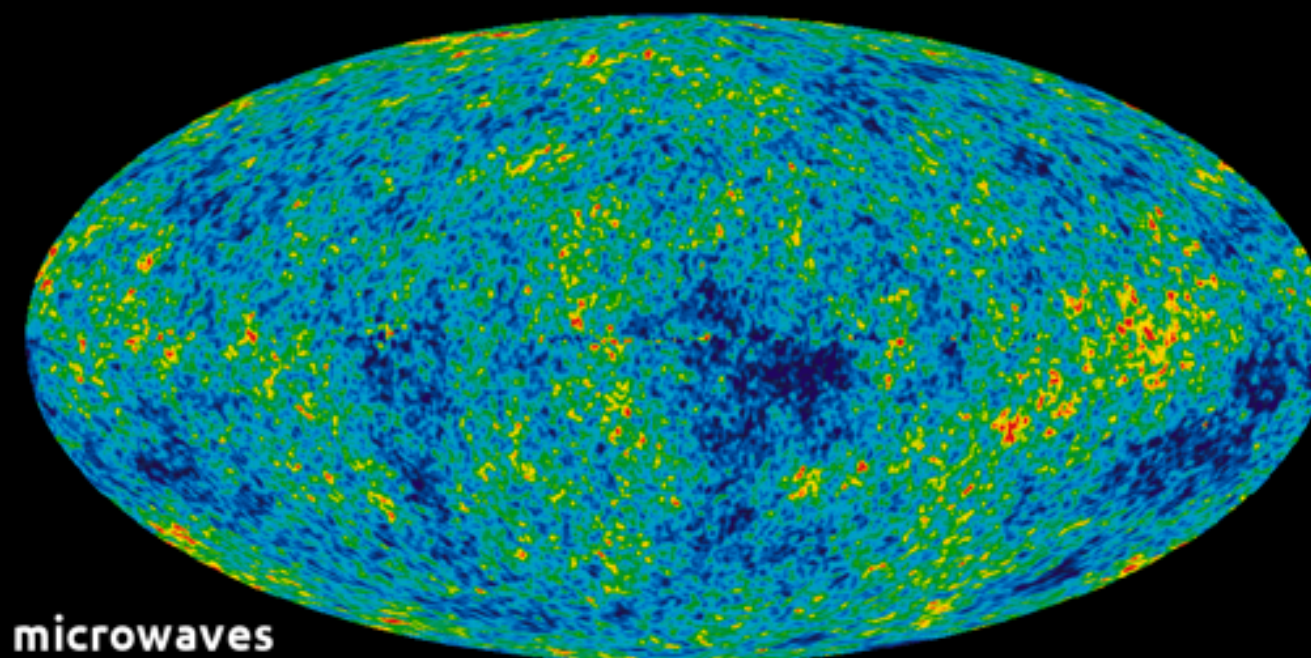
soft X-rays



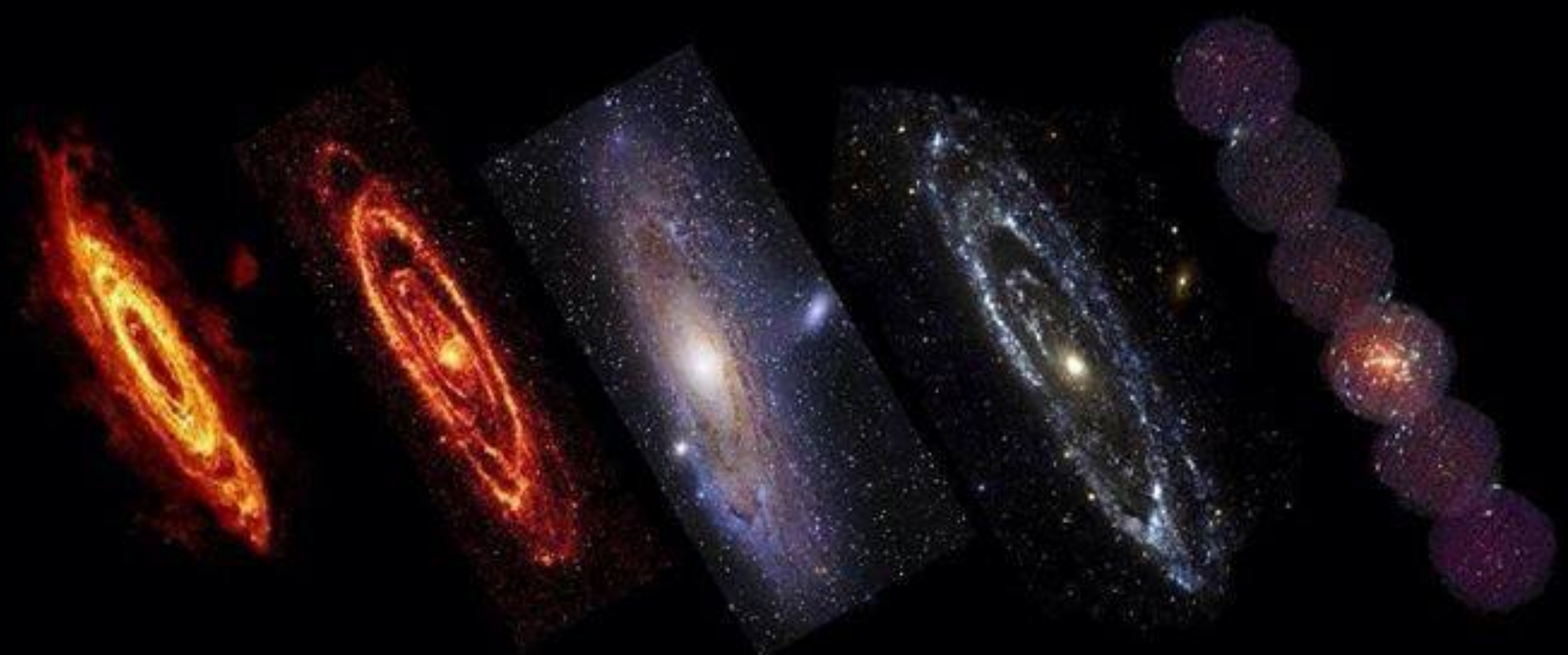
dust and stellar IR



neutral hydrogen



microwaves



**Radio**

**Infrared**

**Visible**

**Ultra-violet**

**X-ray**

# Horsehead Nebula



visible



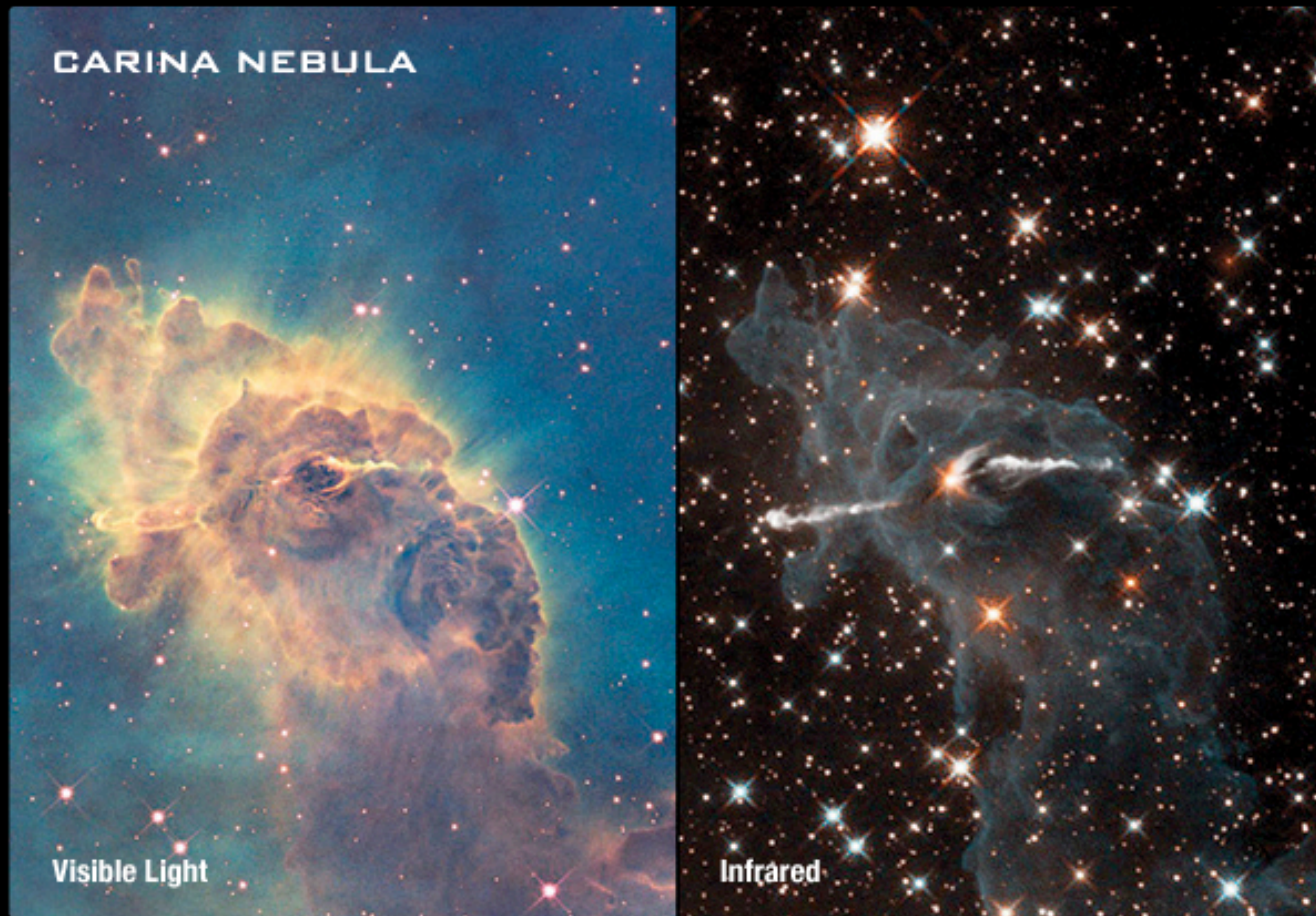
infrared

images/HST

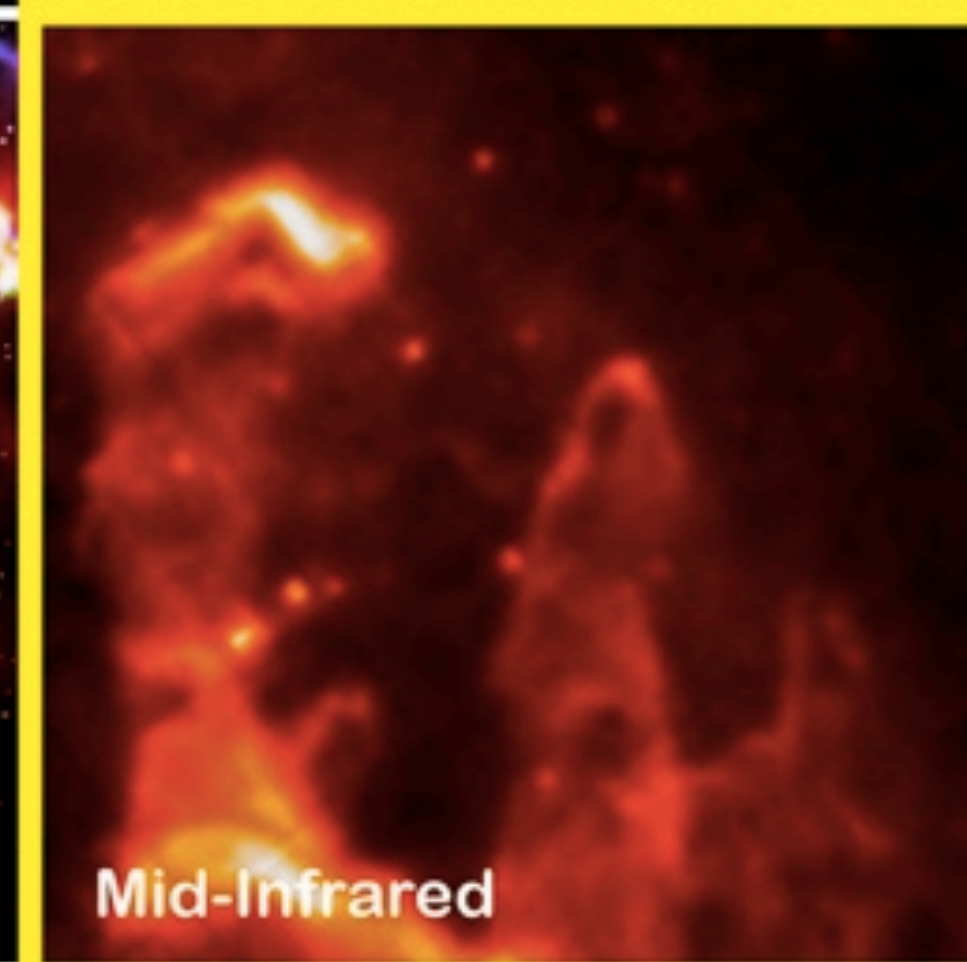
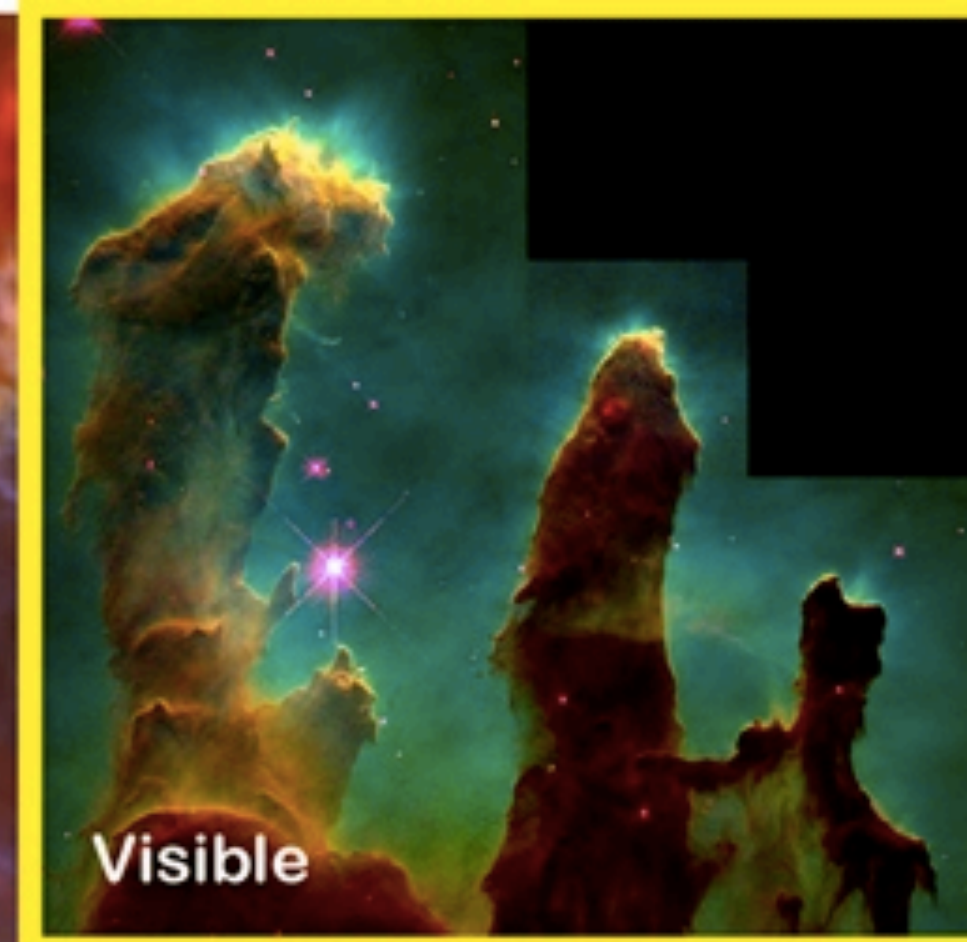
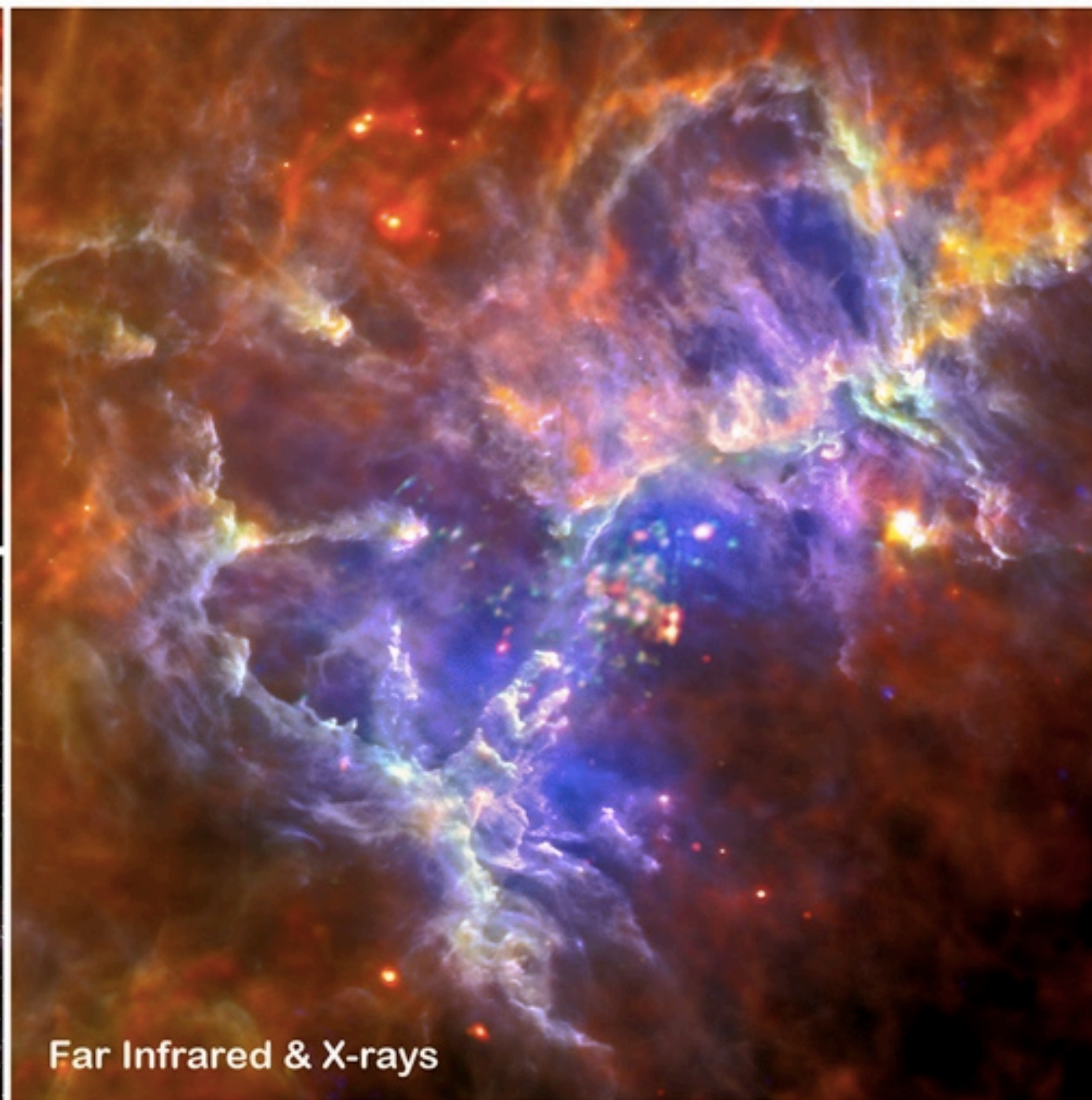
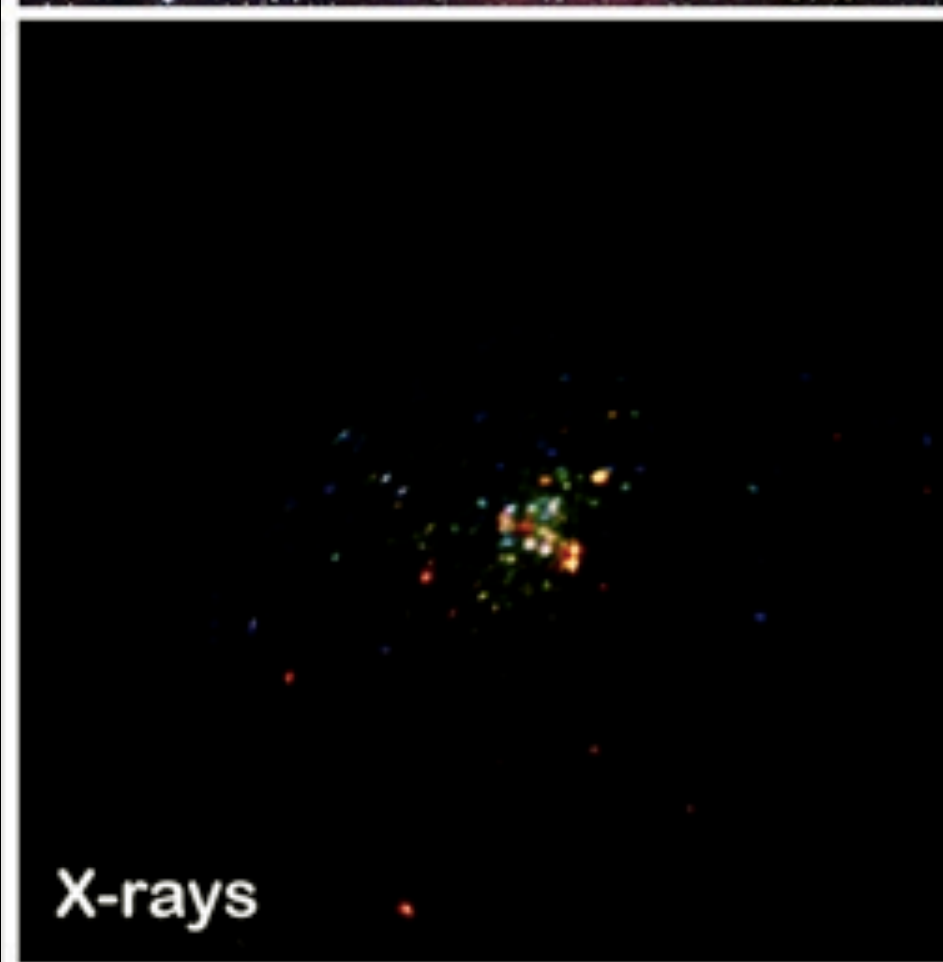
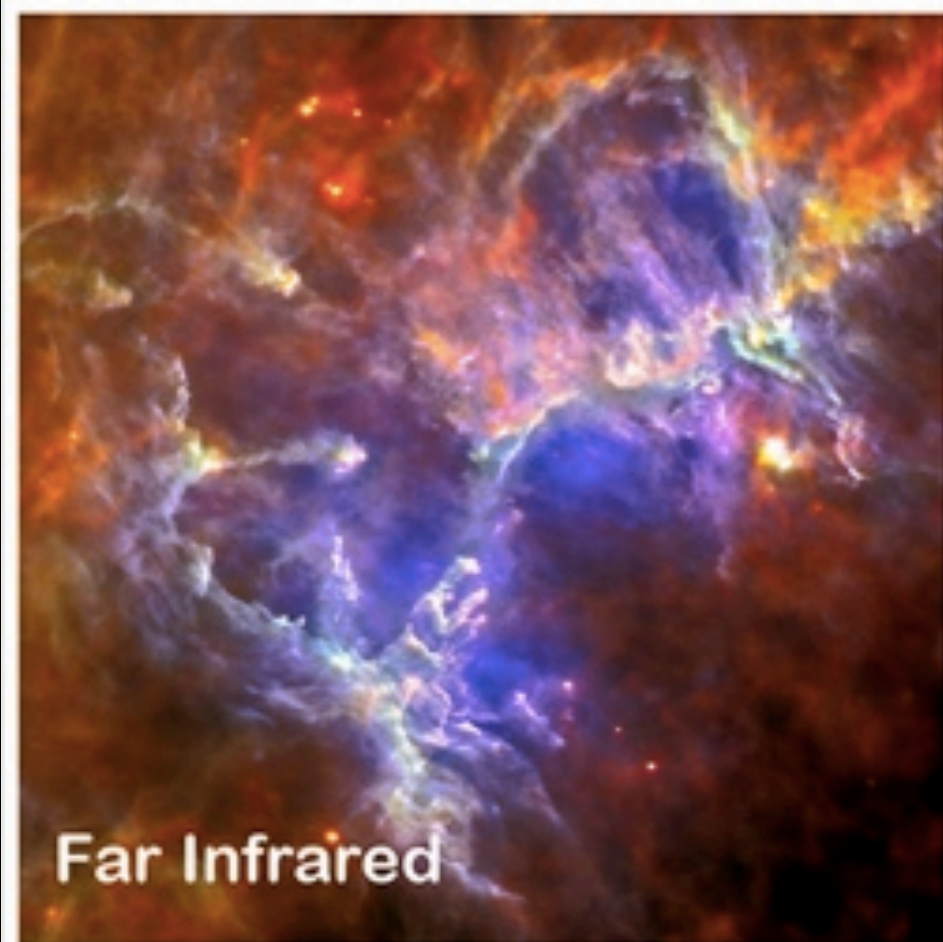
CARINA NEBULA

Visible Light

Infrared







## 3.2 Radiative transfer as a source of energy transport in astrophysical objects

- Important role in the energy balance of astrophysical objects: often the main heating and cooling process
- As a consequence, it is a major driver of structure formation
- examples:
  - ▶ Energy transport in stars
  - ▶ Heating of interstellar clouds by stars
  - ▶ Cooling of interstellar clouds by line emission
  - ▶ Stellar winds driven by radiation pressure
  - ▶ Comptonization in accretion disks

# 3.3 Photons and the electromagnetic spectrum

- Electromagnetic waves
- Associated particles: photons
- Solutions of the 4 Maxwell equations

$\vec{k}$  : propagation direction

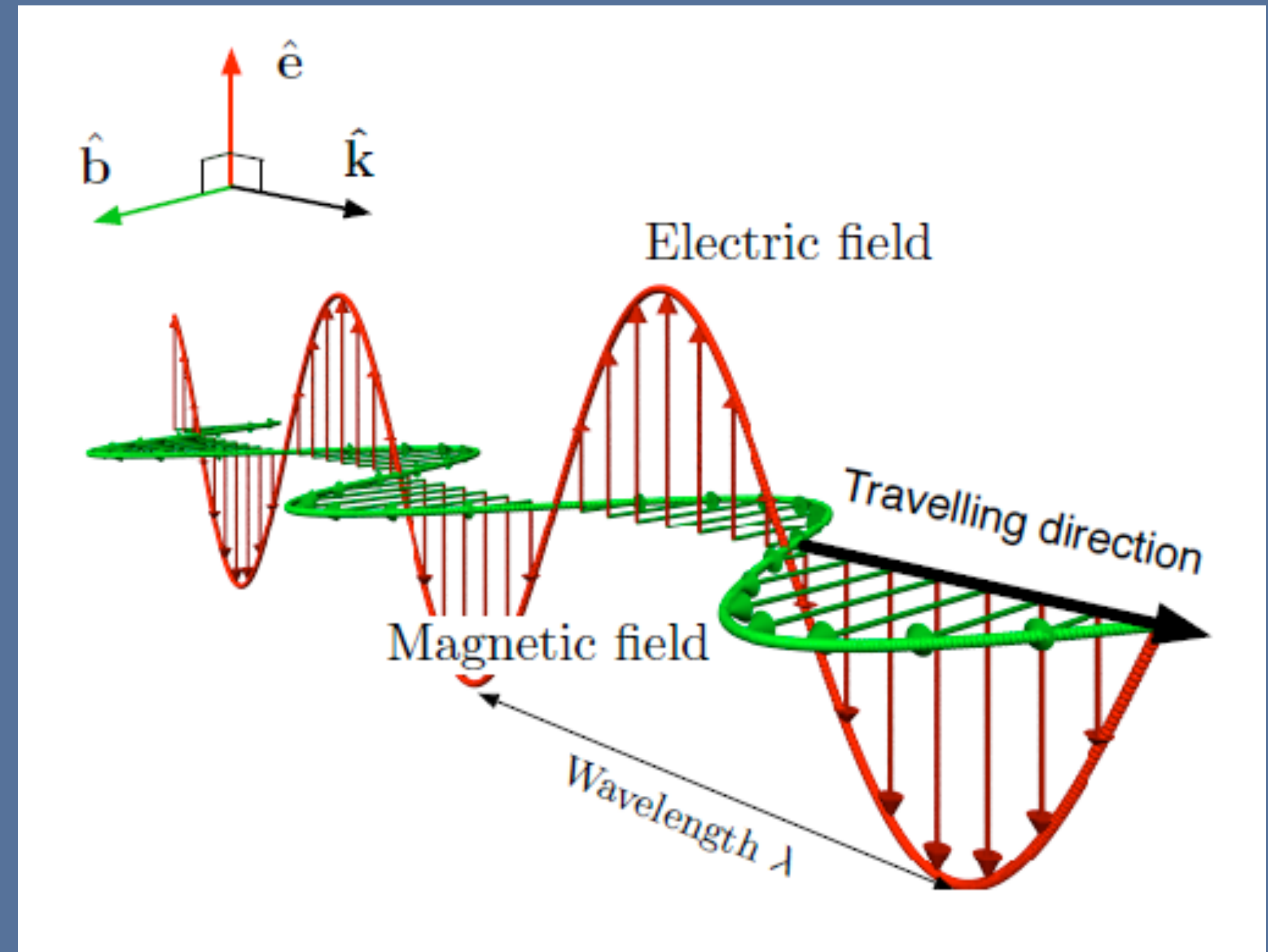
$\vec{E}$  : electric field

$\vec{B}$  : magnetic field

- A transverse wave:

$$\vec{E} \perp \vec{B}$$

$$\vec{B} \perp \vec{k}$$



# 3.3 Photons and the electromagnetic spectrum

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- Photons carry information about

- ▶ Their propagation direction: image

- ▶ Their frequency  $\nu$ , unit Hz or s<sup>-1</sup> or equivalently

- Wavelength  $\lambda$ , unit cm or Å (1Å = 10<sup>-8</sup> cm) or nm (1 nm = 10Å)

Relation:  $\nu = c_m / \lambda$

In vacuum:  $c = 3 \cdot 10^{10}$  cm/s

In a medium:  $c_m = c/n$ , with  $n$  the refraction index of the medium

- Wavenumber  $\sigma$ , unit cm<sup>-1</sup>

Relation:  $\sigma = \nu / c$

$\sigma$  and  $\nu$  are constant and do not change in a medium with index of refraction  $n$

- Angular frequency:  $\omega$  (e.g. in Maxwell equations)

relation:  $\omega = 2\pi\nu$

- The energy of a photon with a frequency  $\nu$  is:  $E = h\nu$

where  $h$  is the Planck constant  $h = 6.626 \cdot 10^{-27}$  erg s (1 erg = 10<sup>-7</sup> J)

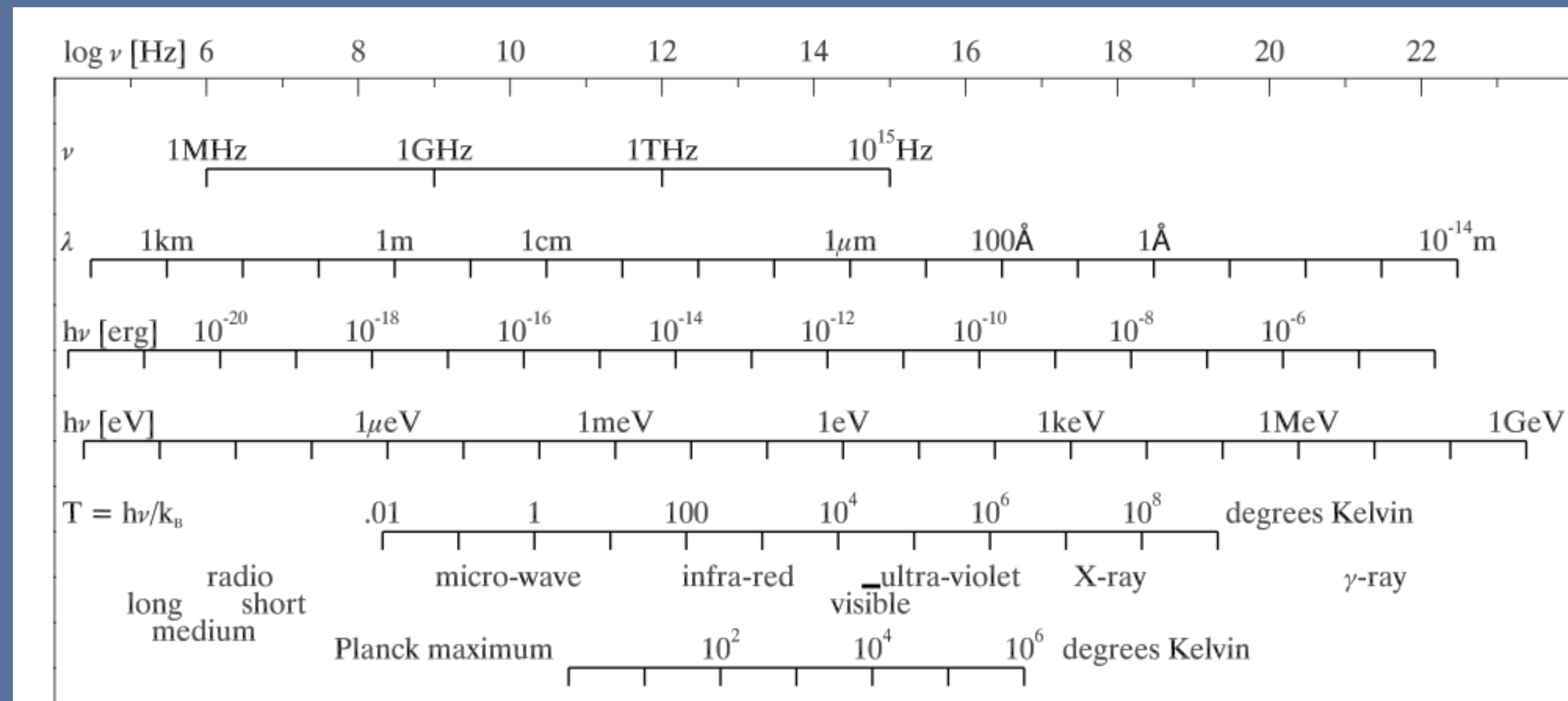
# 3.3 Photons and the electromagnetic spectrum

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- Photons carry information about
  - ▶ Their polarisation
    - Oscillation direction of the electromagnetic wave
    - The polarisation of radiation contains information on emission mechanisms, for example the intensity and the direction of the magnetic field, or the distribution of interstellar grains
  - ▶ The signal we receive can also be time dependent
    - In this lecture, we are only concerned with time-independent radiative transfer

# 3.3 Photons and the electromagnetic spectrum

- Spectrum: amount of energy received as a function of wavelength
- Important information about the astrophysical object is carried by its spectrum
  - Characterisation of emission processes
  - Information on kinematics, temperature, density of emitting or absorbing medium



# 3.4 Approximation of radiative transfer

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- The good news: we do not need to solve the Maxwell equations (... in most cases)
- If the object is much larger than the wavelength, the **laws of geometric optics** apply.
- Most of the time, we use the particle description of electromagnetic radiation and ignore diffraction
  - Exception: scattering properties of small particles
- **Approximation of diluted medium**
  - Index of refraction is set to 1. This is a good approximation in stellar/planetary atmospheres or ISM
  - Light travels strictly in straight lines
  - In case of scattering, light travels in straight lines between two scattering events
  - Photons can be also absorbed (destroyed) or created and added to the direction of propagation
- Radiative transfer consists in looking at how many photons are removed (absorption, scattering) and how many are added (emission, scattering) to the direction of propagation

# 3.5 Challenges of radiative transfer

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- Much progress in the past 15 years (increase of computer power)
- But it is still a very computing-time intensive problem, even for modern computers, and often we will have to make assumptions
- RT is a 6D problem
  - Geometry is an obvious difficulty, and assumptions on the geometry are not always justified
- Coupling
  - Of frequencies
  - Of different regions
  - Of different phenomena (eg photoelectric effect, convection, hydrodynamics)



Carina Nebula



blue: O III, green:  
H $\alpha$ +N II, red: S II

HST/NASA/ESA

## 3.6 Notation

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- The convention used in this lecture is (mostly) that of stellar atmospheres
- Care should be taken for the definitions of various coefficients, as different authors use different ones: are they defined per unit volume? Per unit solid angle?
- Use of cgs system in many cases

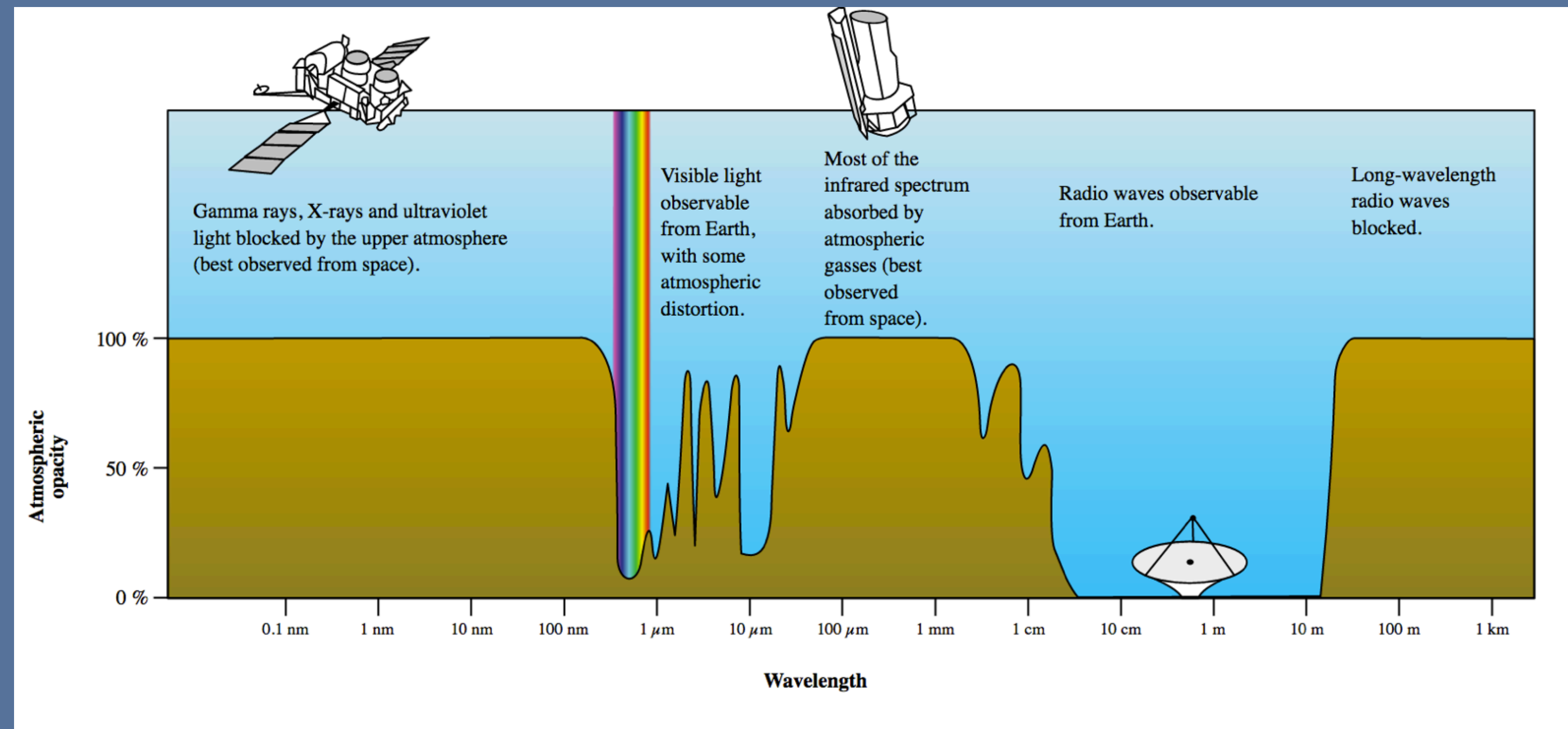
# 4. Alteration of radiation

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- The radiation coming from astrophysical objects is altered in different ways

# 4.1 Atmosphere

- The main modification of the radiation coming from space is the atmosphere around the Earth



Absorption

Only certain wavelengths get through → atmospheric windows (but transmission not always 100%): visible, some windows in NIR and MIR, and from mm into radio domain

Reflection

On the ionosphere ( $\lambda > 30$  m)

# 4.1.1 Atmospheric extinction

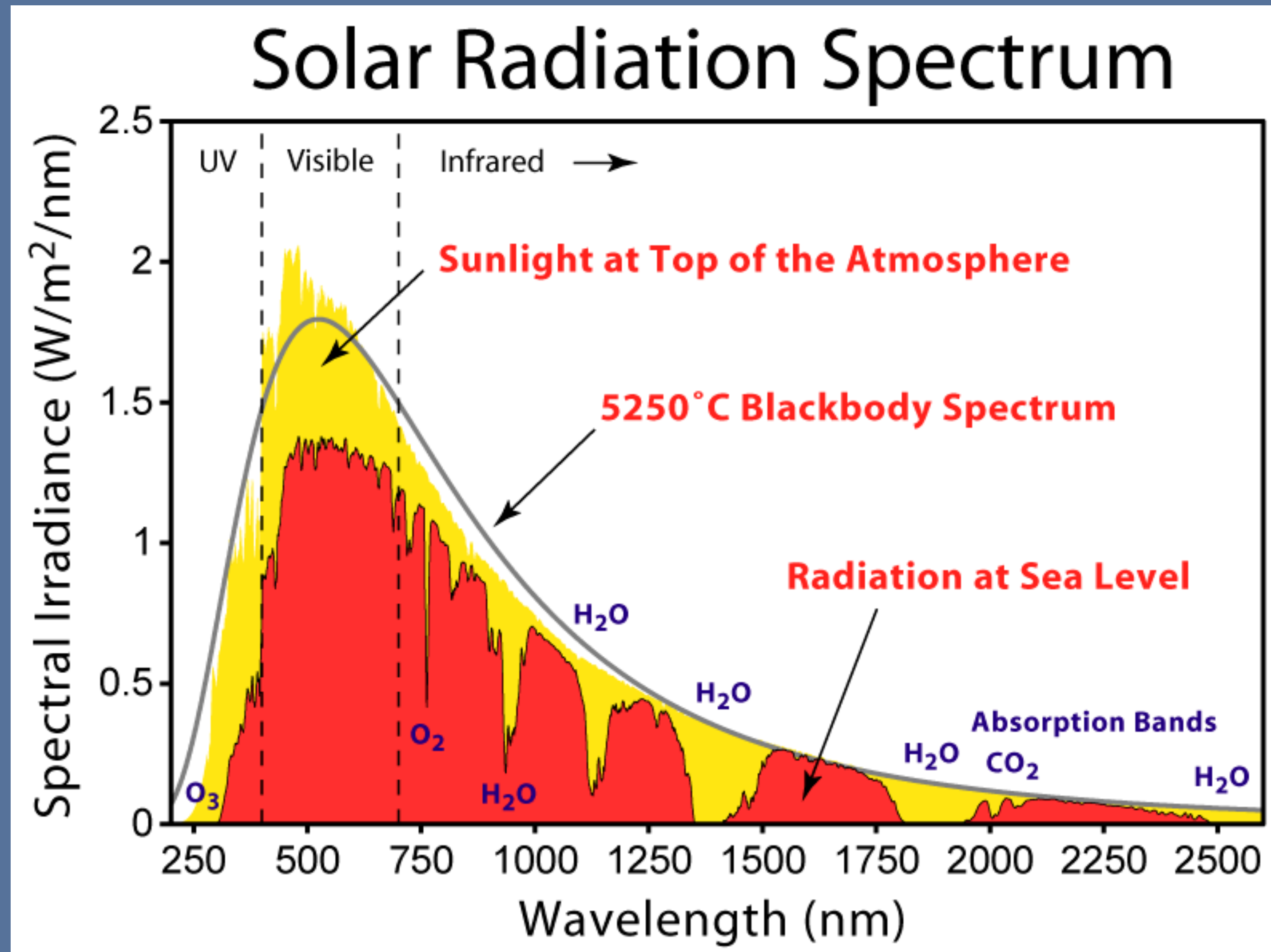
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- Atmospheric extinction can be
  - ▶ Selective

Absorption of the incoming radiation by atmospheric constituents

    - Millimeter and infrared: H<sub>2</sub>O, CO<sub>2</sub>, O<sub>2</sub>, O<sub>3</sub>
    - UV: O<sub>2</sub>, O<sub>3</sub>
    - X ray: photoelectric absorption on N<sub>2</sub> and O<sub>2</sub> (continuous absorption)
  - ▶ Non selective
    - Rayleigh scattering on molecular electrons (extinction coefficient in  $\lambda^{-4}$ )
    - Mie scattering by water droplets and dust particles
- Satellite observations are mandatory for certain wavelengths

# 4.1.1 Atmospheric extinction



## 4.1.2 Atmospheric refraction

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- Variations of the dielectric constant of the atmospheric gas affect visible radiation
- This is a problem for position astronomy (astrometry) and also for photometry because it affects the objects' positions differently according to wavelength
- Atmospheric turbulence is responsible for stellar scintillation
- Ionospheric refraction affects radio waves

## 4.2 Interplanetary medium

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- Few effects
- Magnetic field affects cosmic rays

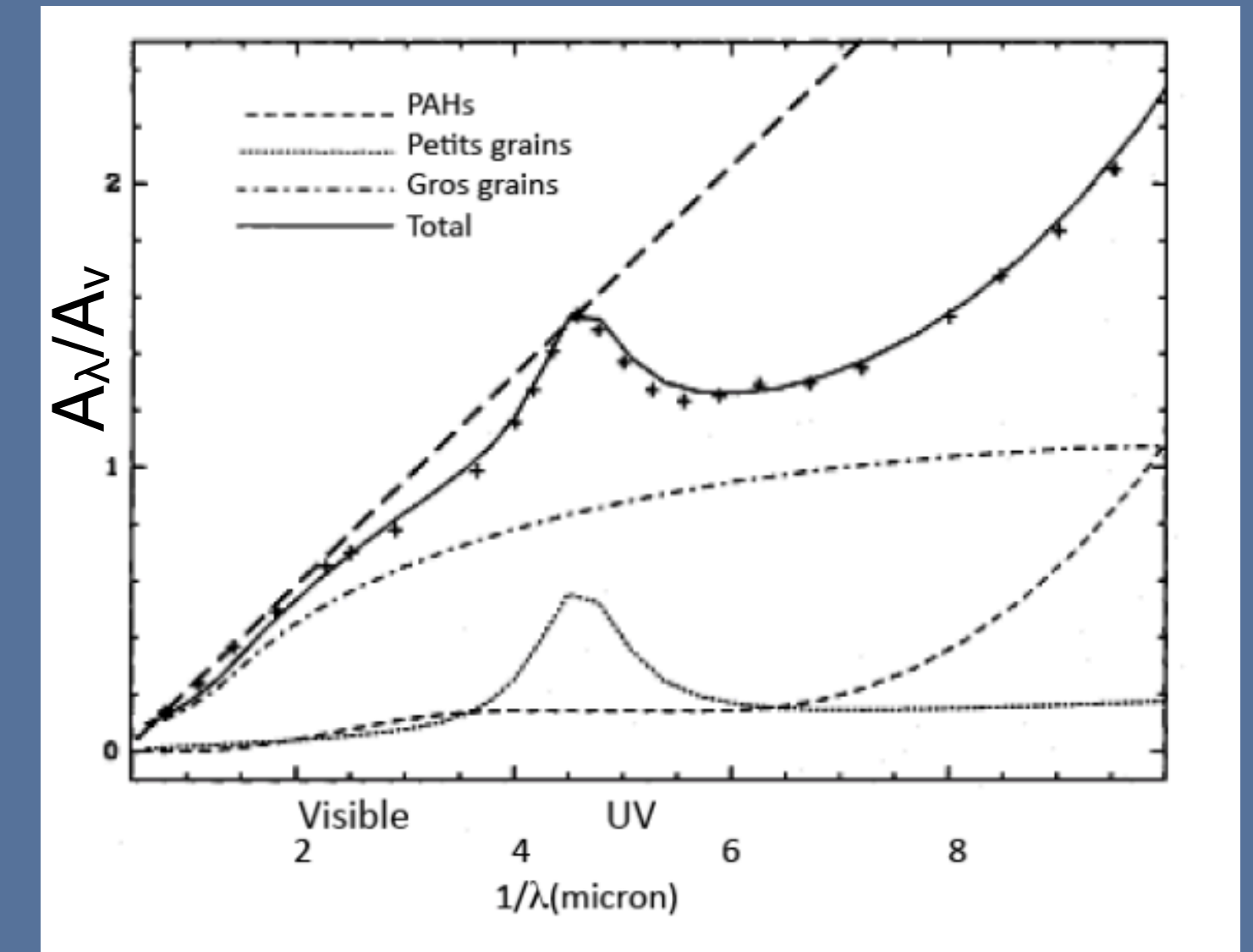


# 4.3 Interstellar medium

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# 4.3.1 Absorption

- Selective
  - Discrete absorption by atoms and molecules
  - Continuous absorption (photoionisation of hydrogen between 100 Å and 912 Å)
- Non selective
  - Absorption by dust particles (extinction coefficient proportional to  $\lambda^{-1}$ )
- Visual extinction  $A_V$  characterises the absorption at 550 nm measured in magnitudes. Mag are a logarithmic (relative) scale

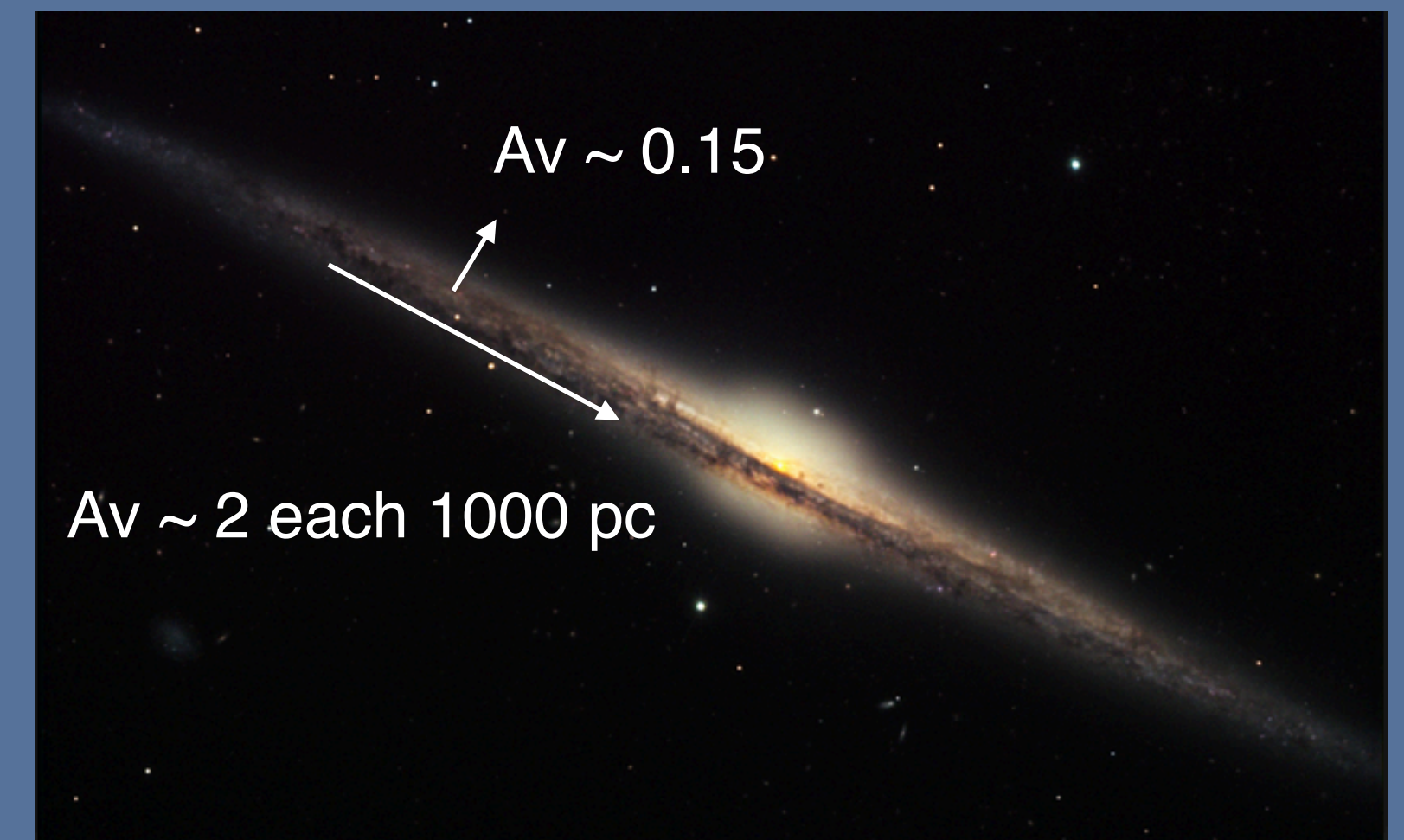


- $$A_V = -2.5 \log \frac{F^{\text{obs}}(550)}{F^{\text{emi}}(550)}$$

- $$A_\lambda = -2.5 \log \frac{F^{\text{obs}}(\lambda)}{F^{\text{emi}}(\lambda)}$$

- Reddening: colour excess  $E_{B-V} = A_B - A_V$  ( $\sim 0.3 A_V$ )

It can also be defined in other colours, when the medium is opaque in the vis



## 4.3.2 Scattering and polarisation

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- Dust grains scatter starlight
- Some nebulae are only visible thanks to the radiation they scatter
- Anisotropic particles with preferential orientation can polarise light

## 4.3.3 Refraction

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- Interstellar gas can behave like weakly ionised plasma and refract radio waves
- This produces an effect similar to scintillation in the visible
- Another consequence is “time scattering” of pulsar signals, as the time it takes them to arrive is proportional to the number of electrons along the line of sight

## 4.3.4 Faraday rotation

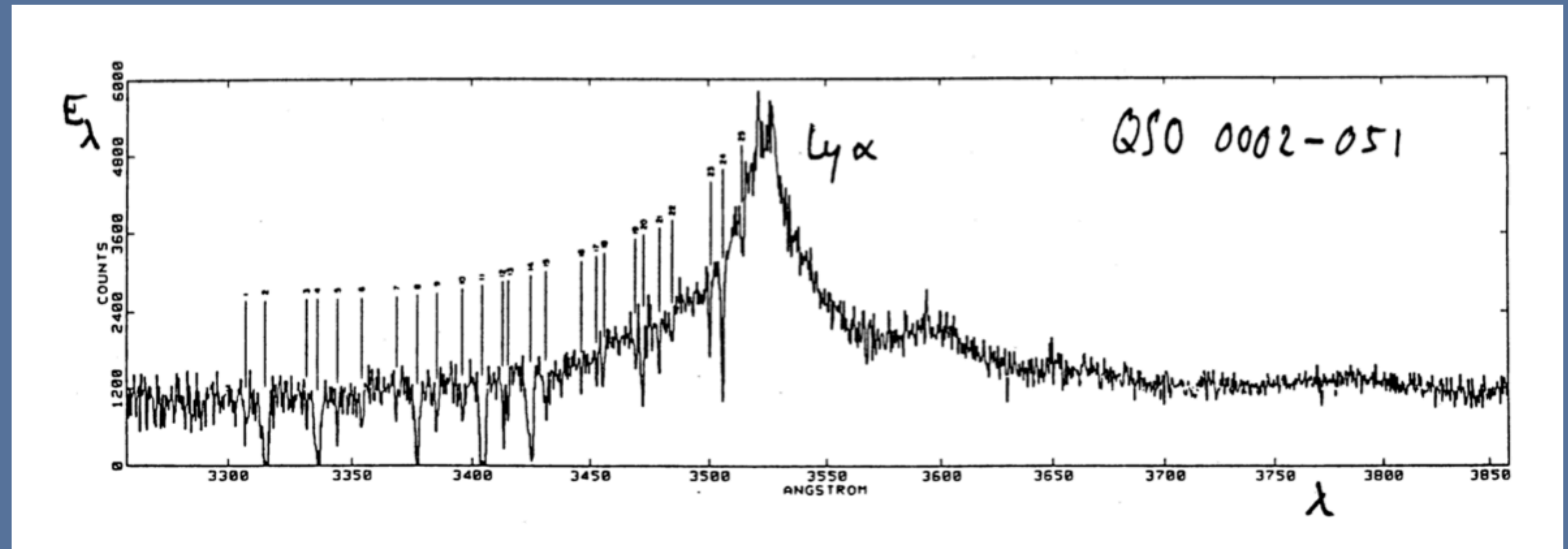
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- For a magnetised medium, the refraction index depends on the magnetic field
- The interstellar medium becomes birefringent, and both polarisations of the incoming light travel at different speeds
- The interstellar medium induces a rotation of the polarisation plane of the radiation, which depends on the wavelength, is proportional to the size of the medium and the value of the magnetic field

# 4.4 Intergalactic medium

- Similar processes to the interstellar medium
- Absorption lines

Ly  $\alpha$  forest

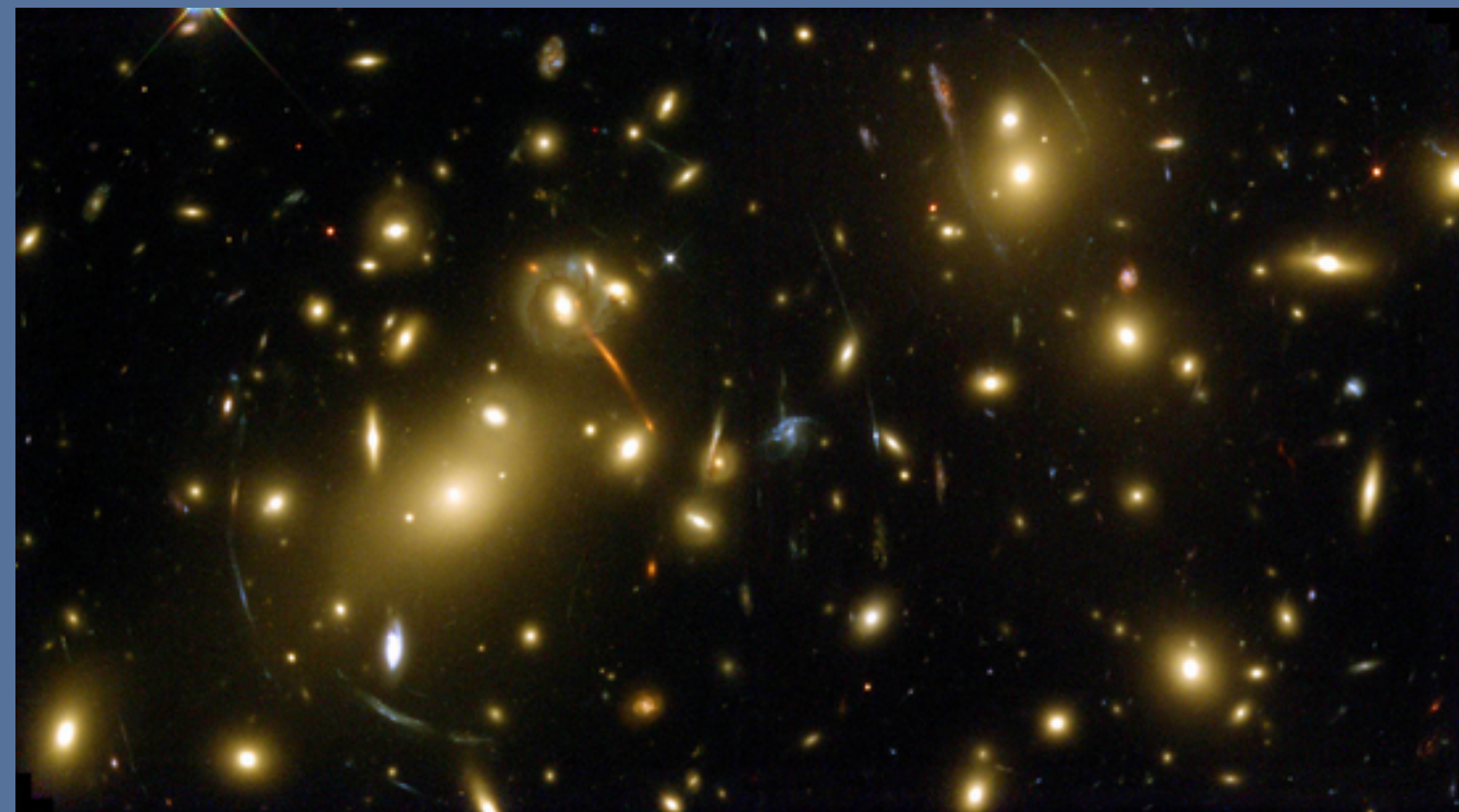


Young et al. (1982)

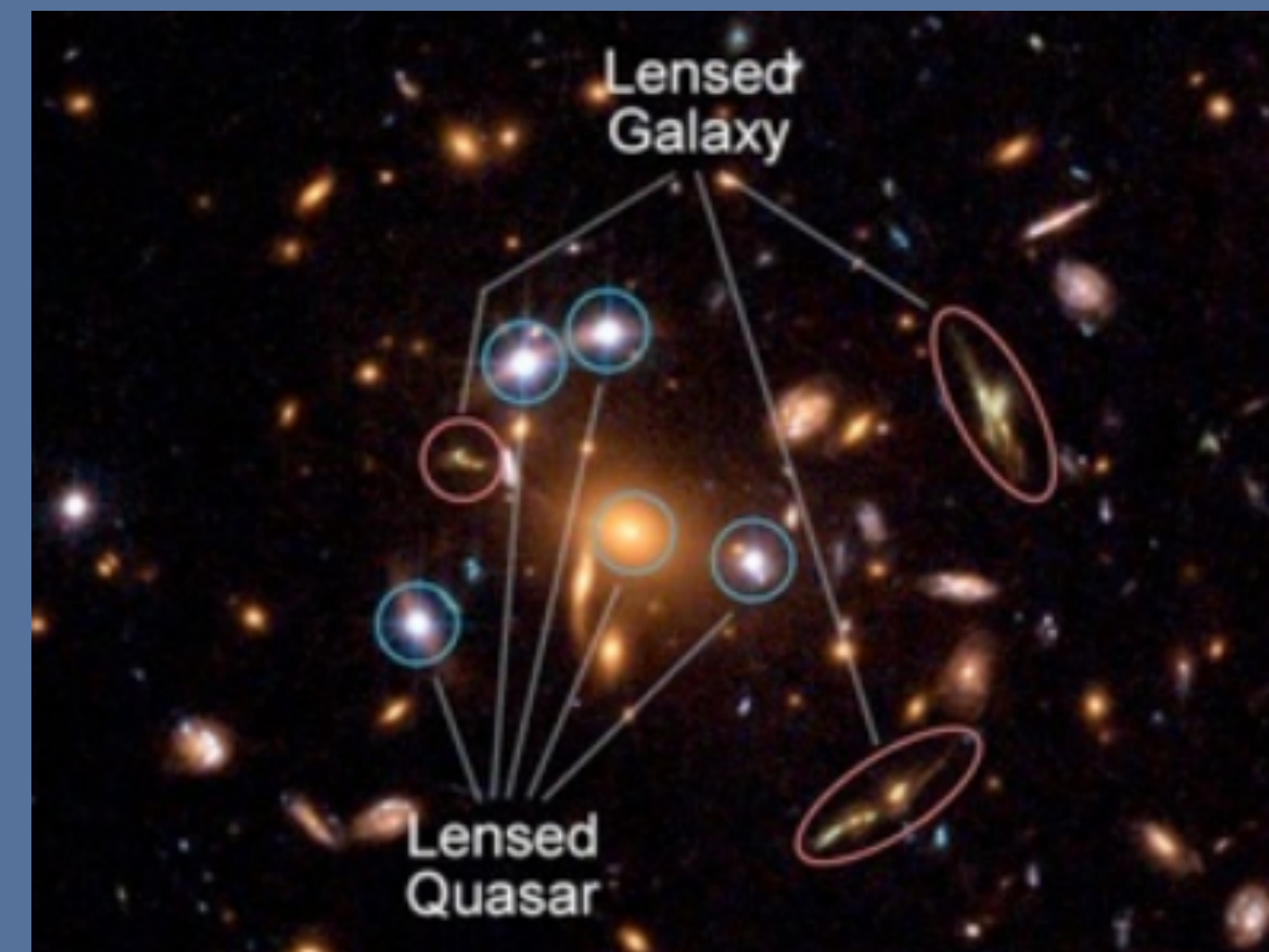
# 4.4 Intergalactic medium

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- Faraday rotation
- Reddening (but small effect because of limited amount of dust)
- Gravitational lensing: massive objects deflect light and amplify it in certain directions, acting as a gravitational lens → multiple amplified images of far-away objects (quasars and galaxies)



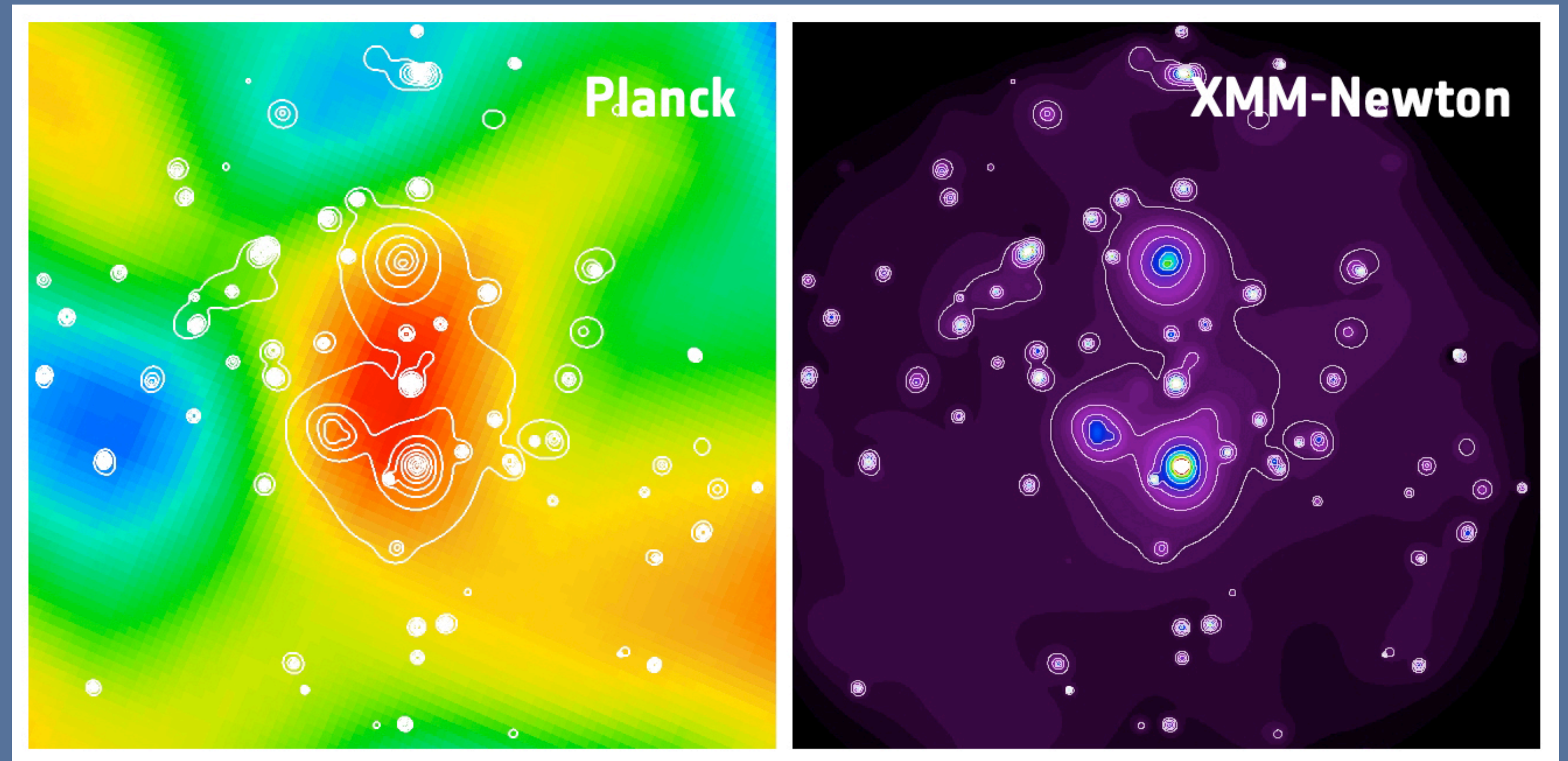
Abell 2218



# 4.4 Intergalactic medium

- Sunyaev-Zel'Dovich effect: deformation of the cosmological blackbody emission by hot electrons in galaxy clusters (inverse Compton scattering)

Galaxy cluster discovered with Sunyaev Zel'dovich effect





# 5. Introduction to radiative processes

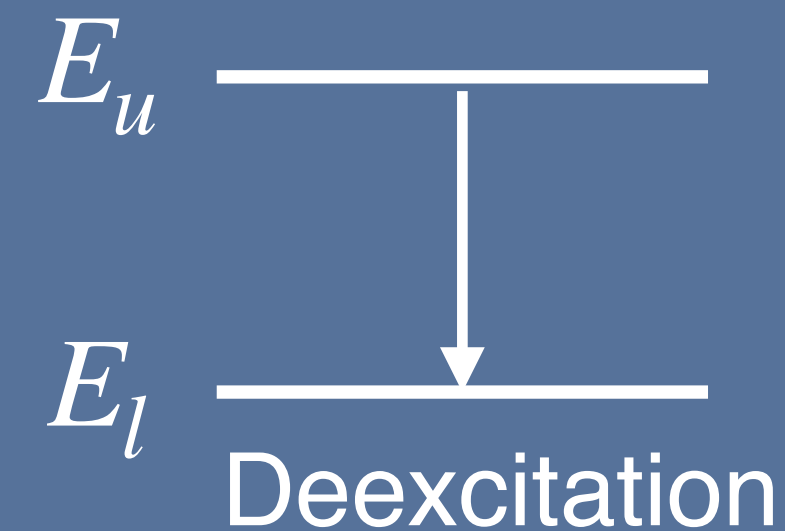
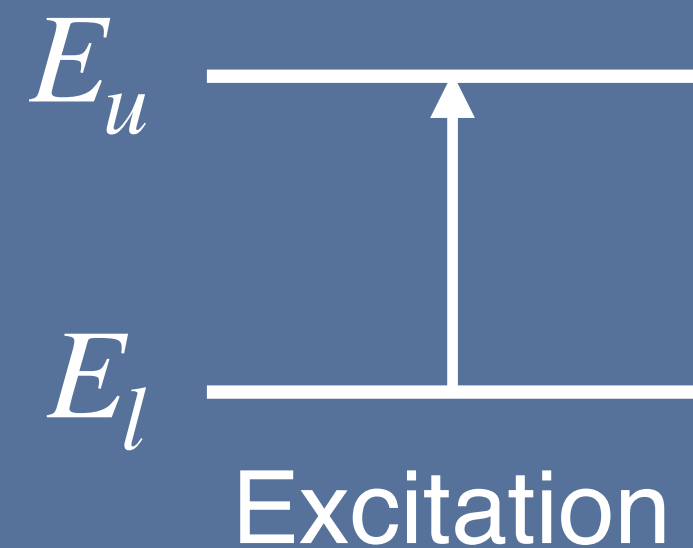
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- Radiative processes are often classified as either discrete processes or continuous processes
- Here we will briefly introduce them, and we will come back to them in later chapters

# 5.1 Discrete processes

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- Spectral lines: arise from radiative transitions between discrete energy levels



- ▶ Radiative excitation

Absorption of a photon of energy

$$\Delta E_{ul} = E_u - E_l = h\nu$$

- ▶ Collisional excitation

Absorption of kinetic energy from a particle

- ▶ Radiative deexcitation

Emission of a photon of energy

$$\Delta E_{ul} = E_u - E_l = h\nu$$

- ▶ Collisional deexcitation

transmission of kinetic energy to a particle

- Bound-bound (bb) processes

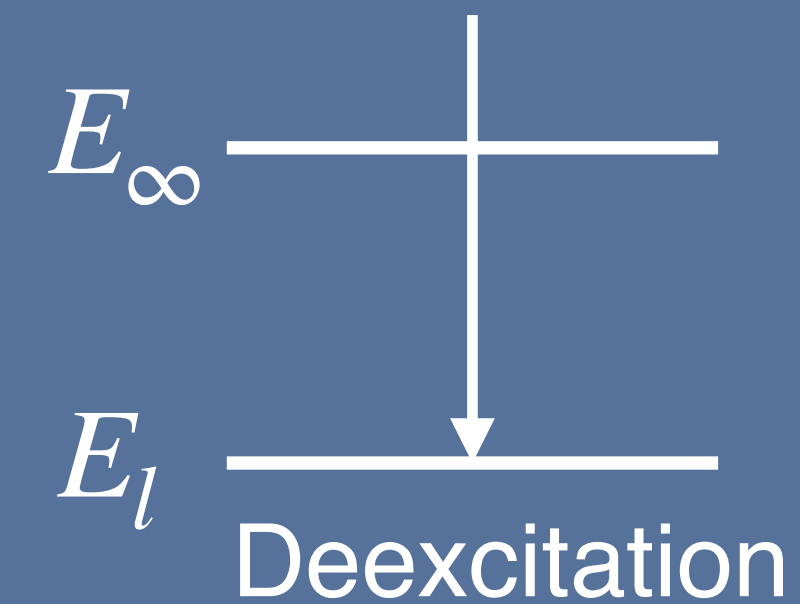
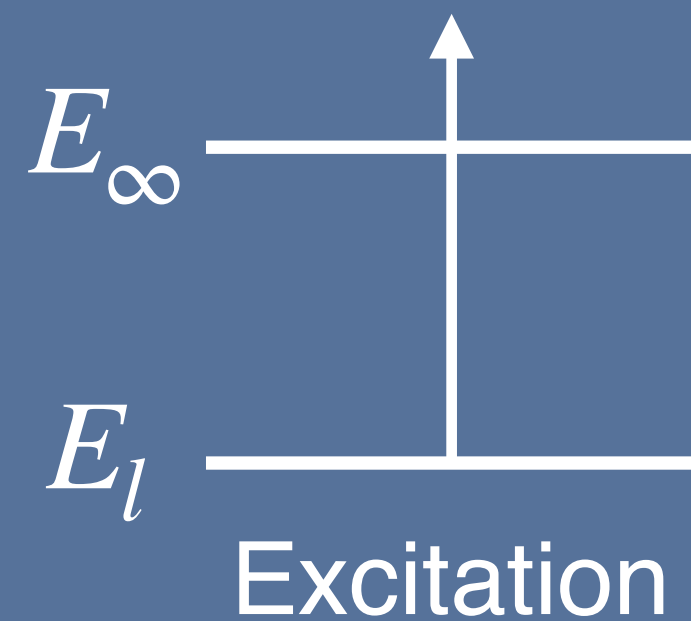
## 5.2 Continuum processes

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- These are non-discrete (non quantised) processes during which photons can be absorbed or emitted

## 5.2.1 Bound-free processes (bf)

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ionisation: liberation of a valence electron

- ▶ **Radiative ionisation**

Absorption of a photon of energy  $h\nu$  greater than  $\Delta E_{\infty l} = E_\infty - E_l$

- ▶ **Collisional ionisation**

Absorption of kinetic energy greater than  $\Delta E_{\infty l}$  from a colliding particle

- The energy of the free electron is not quantized (why?). How much is it ?

recombination: capture of a free electron

- ▶ **Radiative recombination**

Emission of a photon of energy  $h\nu$  greater than  $\Delta E_{\infty l} = E_\infty - E_l$

- ▶ **Collisional recombination**

transmission of kinetic energy to a particle to a colliding particle. In which astrophysical media is this relevant?

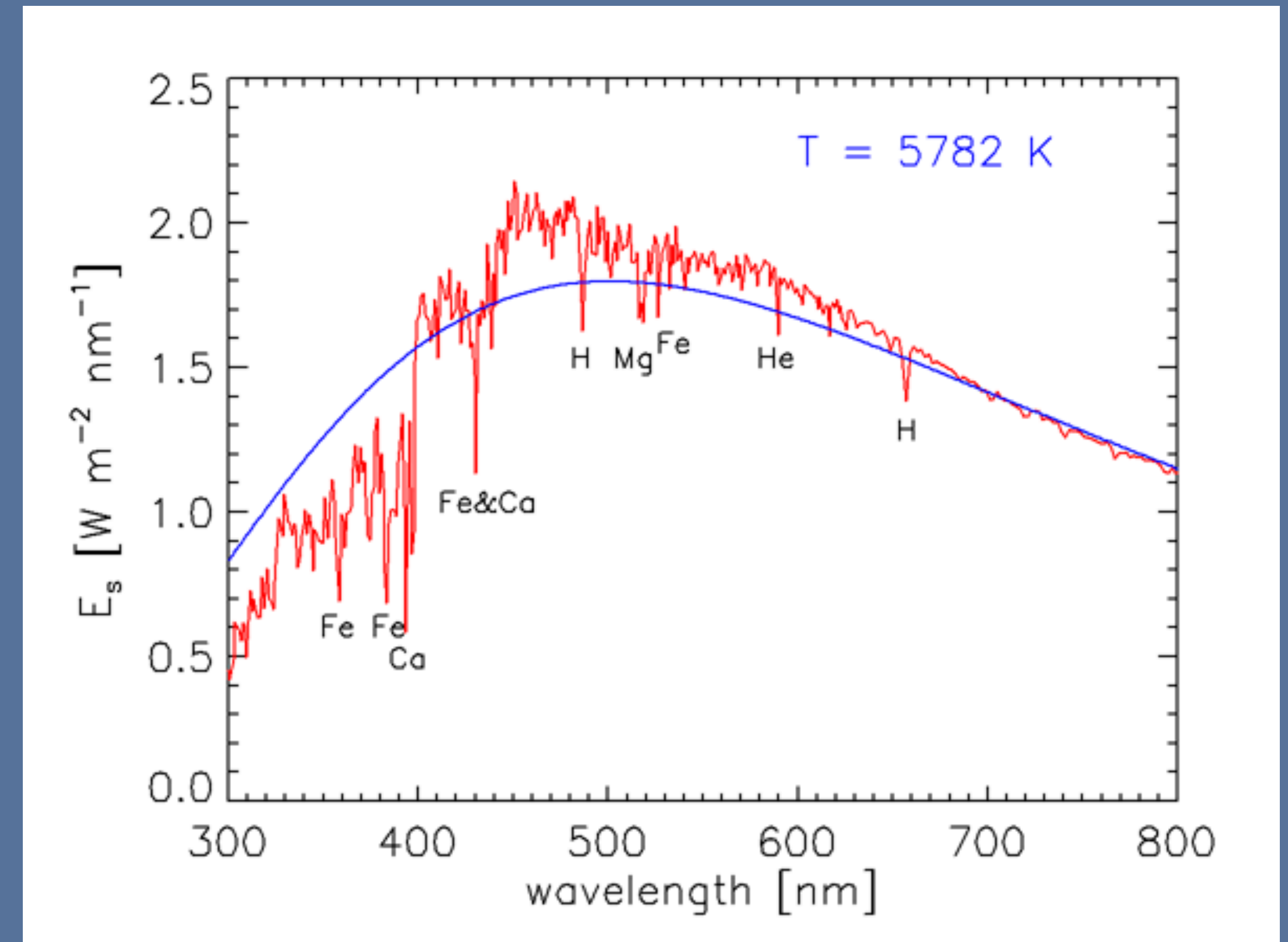
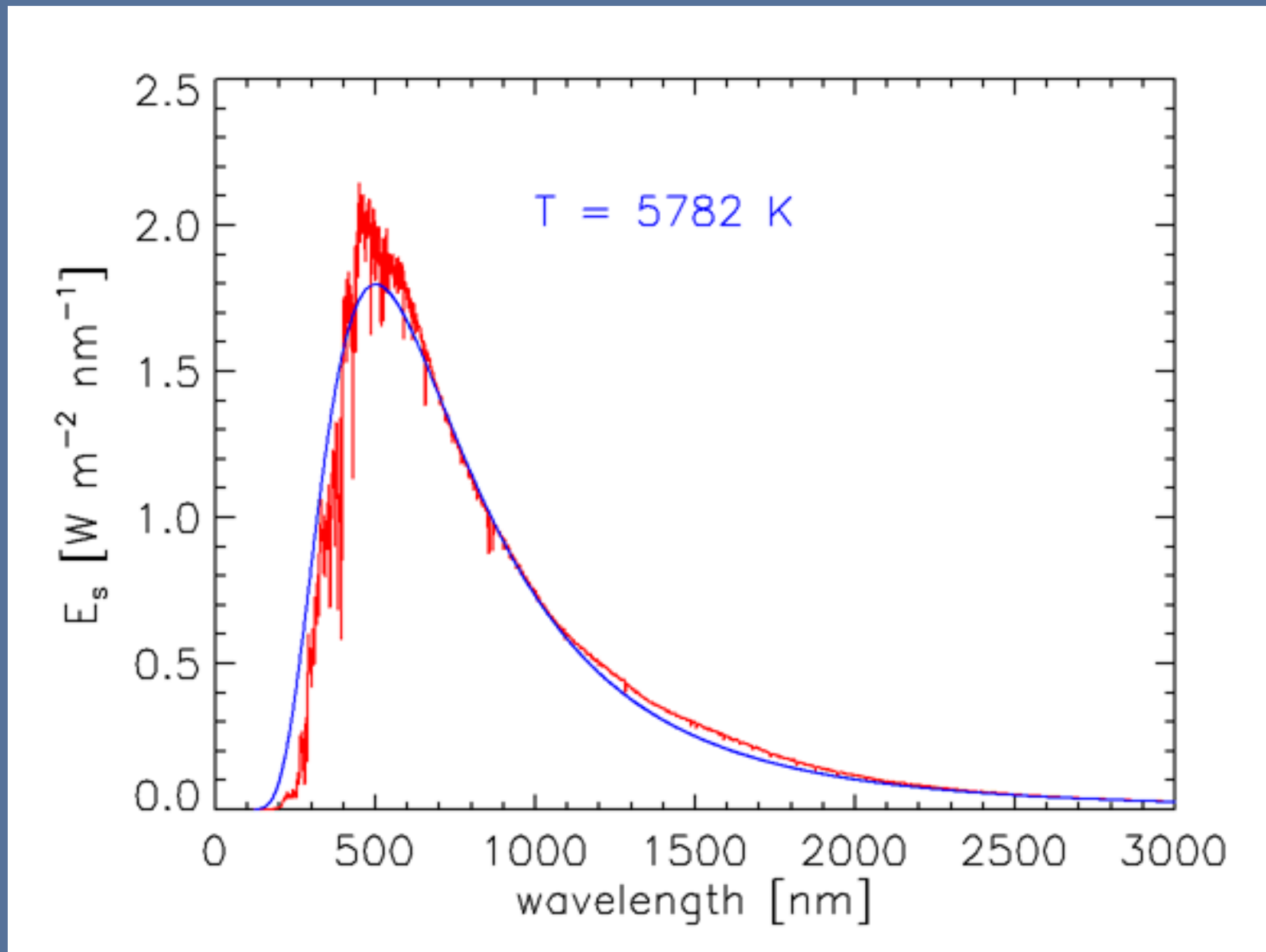
## 5.2.2 Blackbody emission

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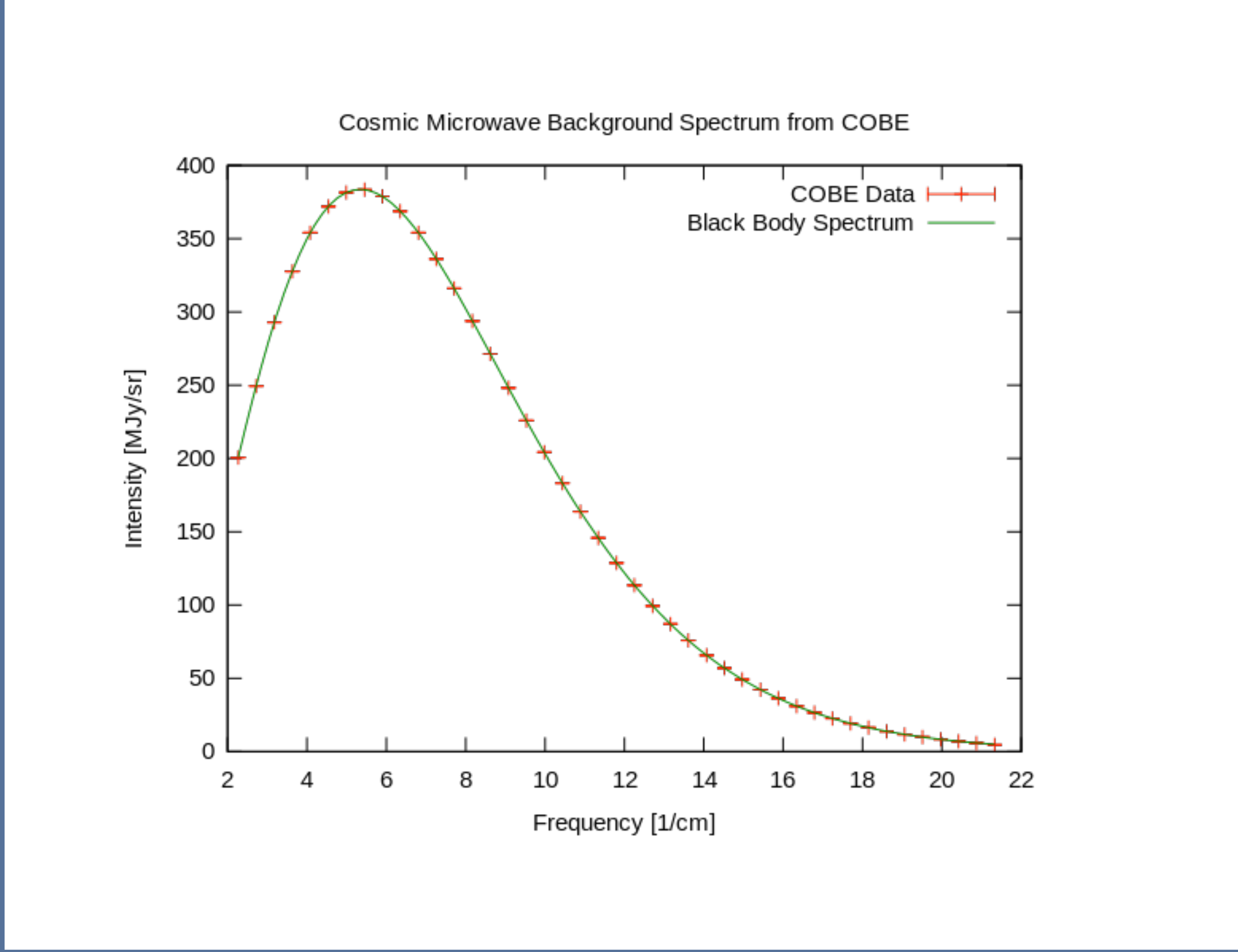
- This is one of the most famous continuum emission processes, the most common
- It characterises equilibrium between radiation and matter
- The emission only depends on temperature
- The continuum emission from stars is close to blackbody emission, though stars are not perfect blackbodies. Note that another characteristic of stellar emission is a wealth of spectral lines (bb processes) seen in absorption against the continuum background
- Cosmic microwave background (though strictly speaking it is not really an emission process): it results from the photons emitted 1 Myr after the Big Bang during the recombination, in the form of a 3000 K blackbody. These photons did not interact with matter but have lost their energy as a consequence of the expansion of the universe. In our epoch, it is a 2.725 K blackbody

# 5.2.2 Blackbody emission

Solar spectrum (red)  
Blackbody fit (blue)



# 5.2.2 Blackbody emission



## 5.2.3 Thermal dust emission

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- This is similar to blackbody emission, but we single it out because we are going to talk in details about it later
- Interstellar dust particles emit like small blackbodies (if they are not too small)
- Dust particles can absorb nearly all radiation, which heats them up
  - Temperatures of  $\sim 10$  to  $1000$  K
  - Most of the radiation is emitted from the MIR (hot particles) and FIR to submillimetre (cold particles)
- Dust is found throughout the Universe so this is an important mechanism



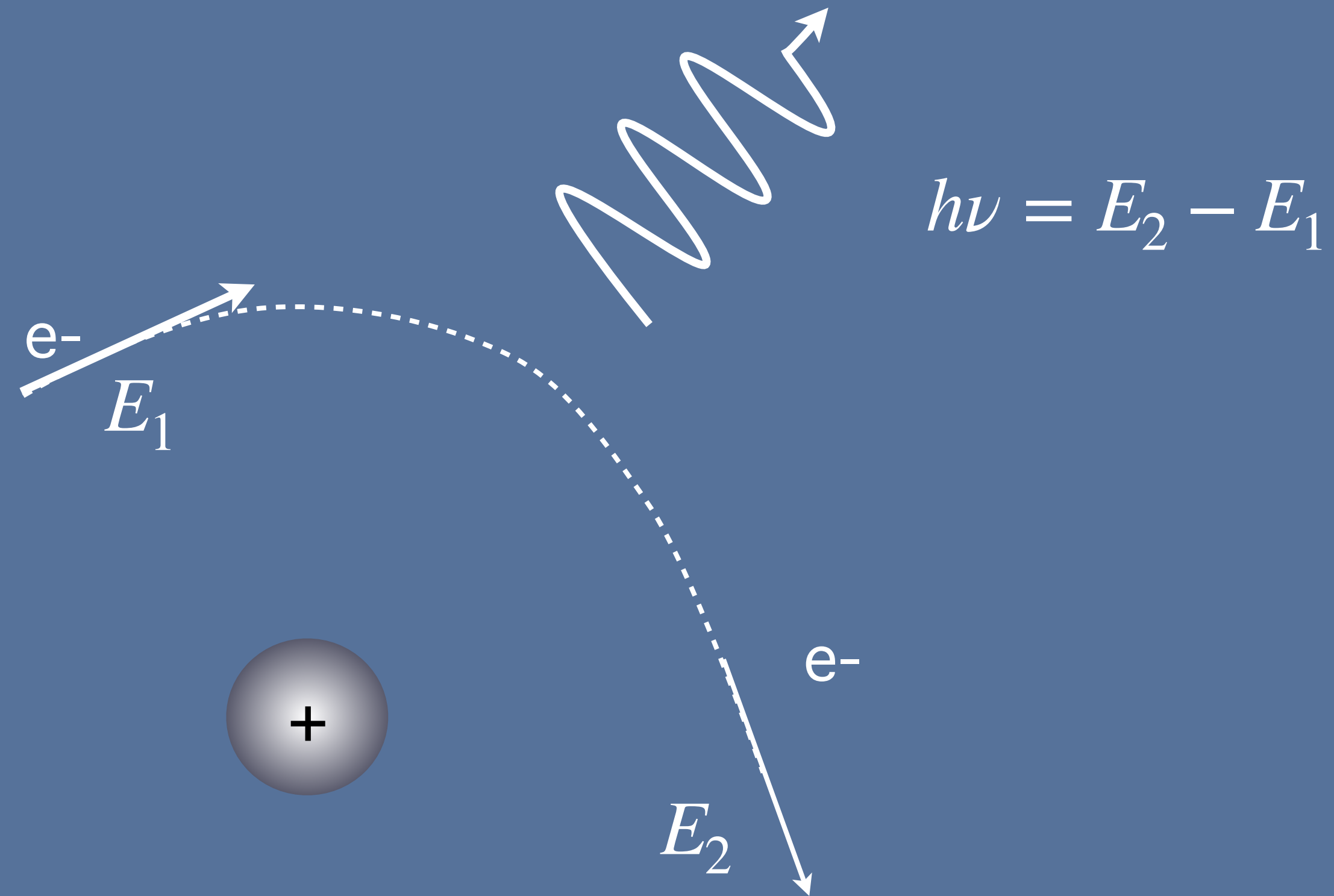
## 5.2.4 Free-free emission

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- Free-free emission is a very important mechanism in astrophysics
- It is the radiation of charged particles which are accelerated (from the Maxwell equations)
- Example: in a collision between an electron and an ion, the electron loses energy in the form of photon emission
- The energy between the two states is not quantised, hence the name “free-free” (abbreviated ff)
- Sometimes the German term is used: Bremsstrahlung (braking radiation)
- Can be thermal (hot nebulae, SFR) or non-thermal if the velocity distribution of the particles is not Maxwellian

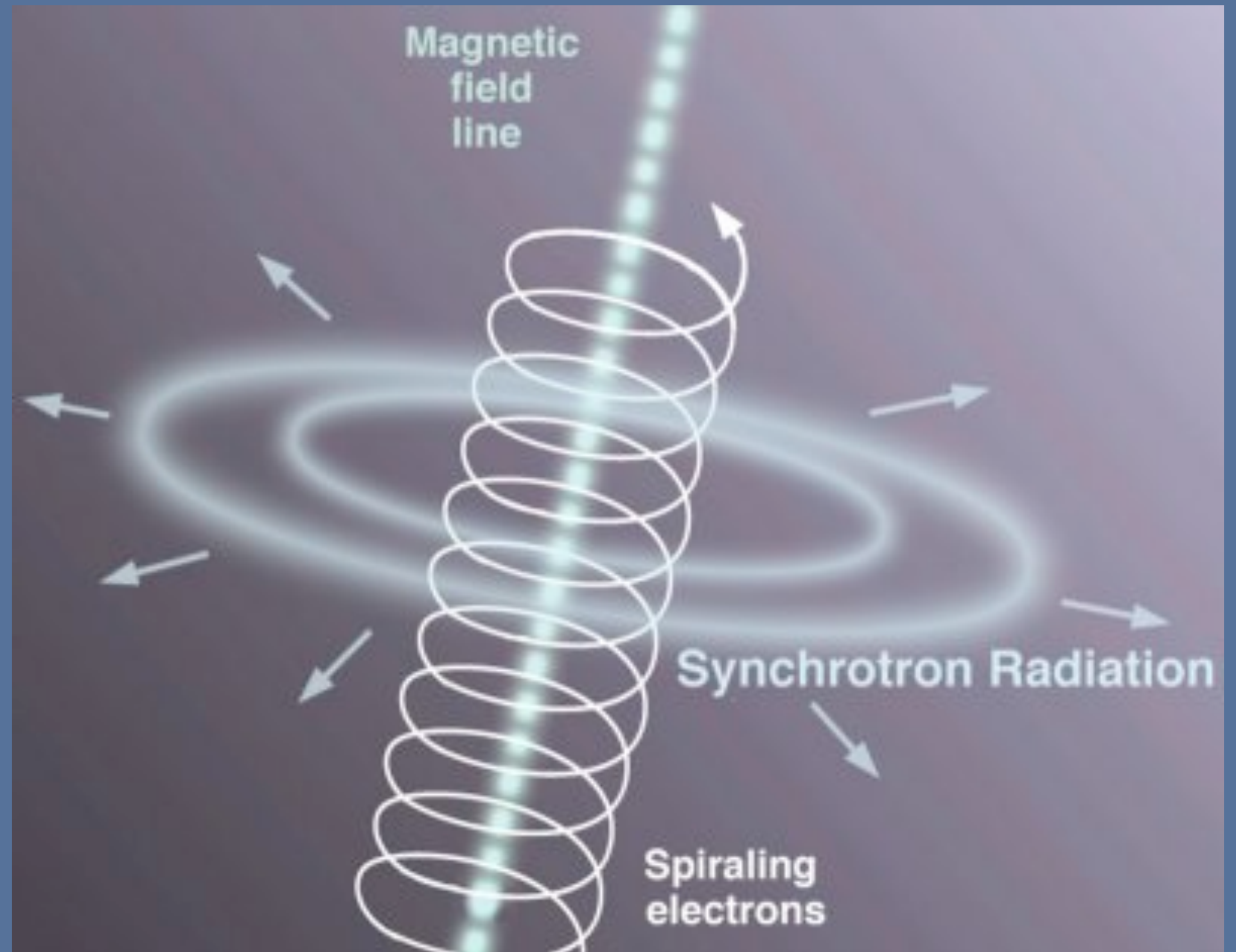
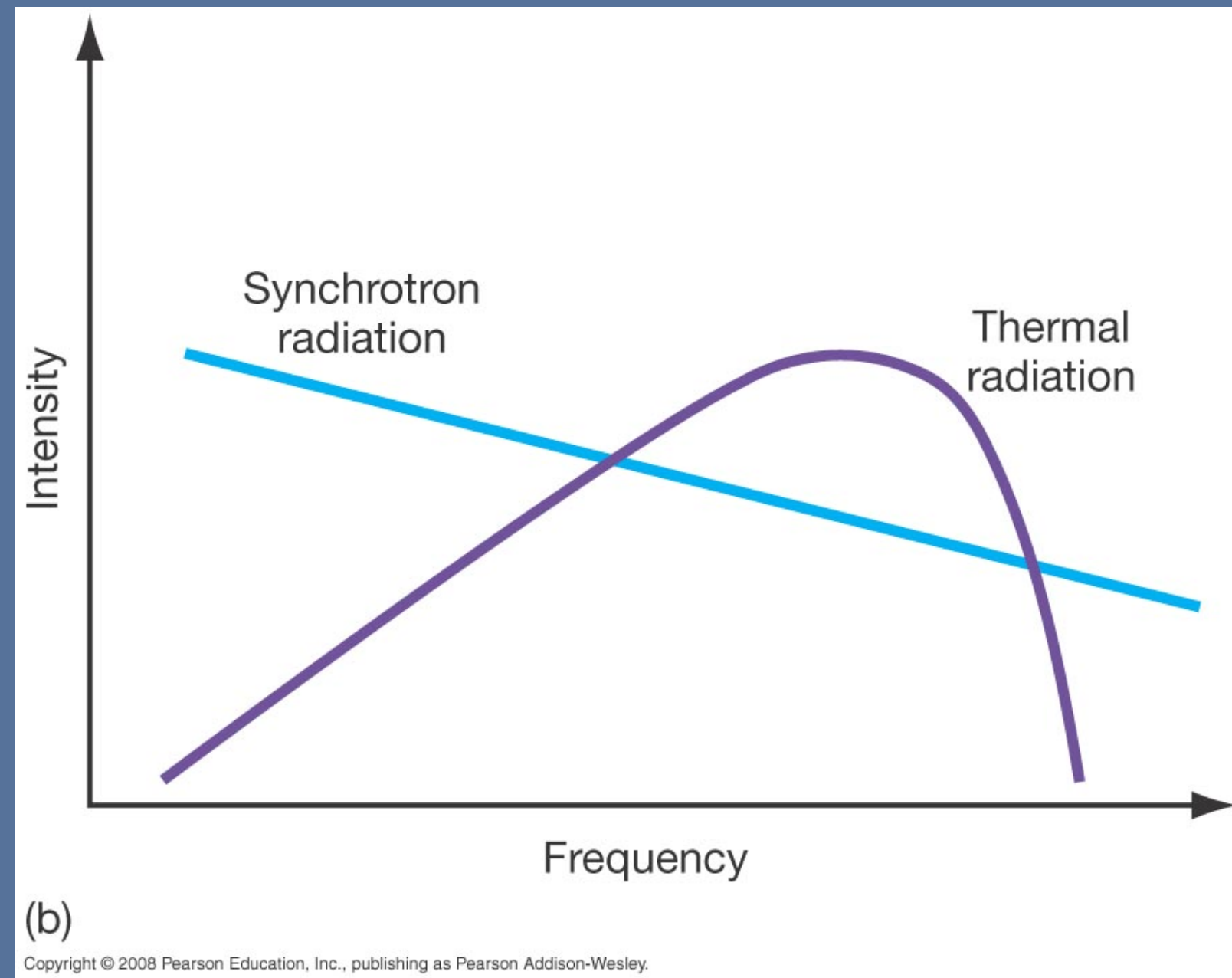
## 5.2.4 Free-free emission

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# 5.2.5 Cyclotron and synchrotron emission

- These emissions result from the acceleration of a charge in a magnetic field
  - cyclotron: non relativistic
  - synchrotron: relativistic



## 5.2.6 Cherenkov radiation

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- Important mechanism for cosmic rays: it results from the bow shock of a charged particle moving at a speed greater than the speed of light in the medium

## 5.3 Scattering

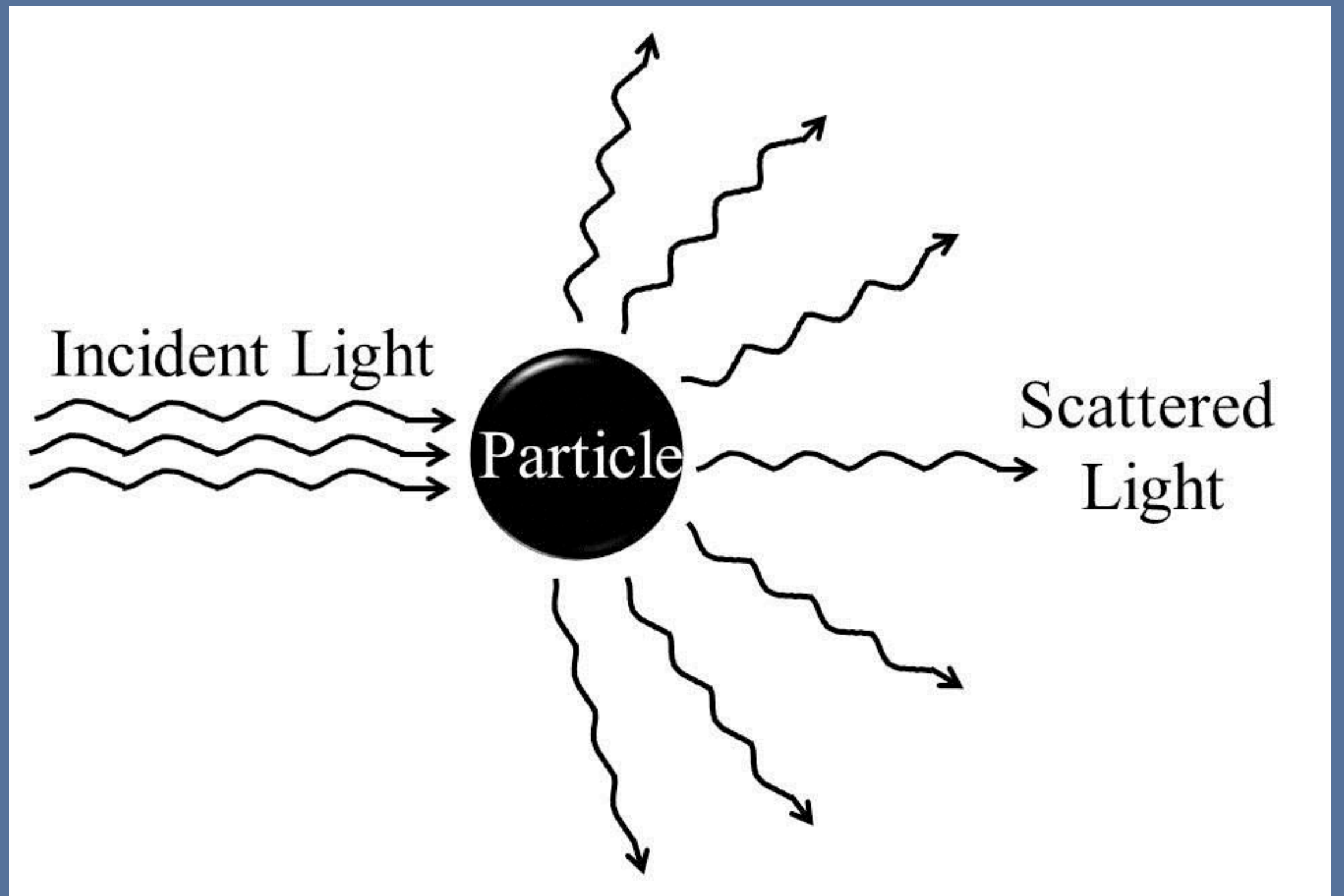
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- Scattering is not exactly an emission process, as it deals with photons that were emitted elsewhere.
- However, it is worth mentioning here as it may appear as photons that arise locally at the scattering particle.
- It is the change of direction undergone by photons impinging on a particle.
- It can be elastic (without change of photon energy) or inelastic (with change of photon energy).

## 5.3.1 Scattering of small (dust) particles

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- Small dust particles like those present in the ISM can scatter radiation
- After interaction with the particle, the photon is sent in another direction
- Only the direction of the photon changes (elastic scattering)
- Mie scattering or Rayleigh scattering

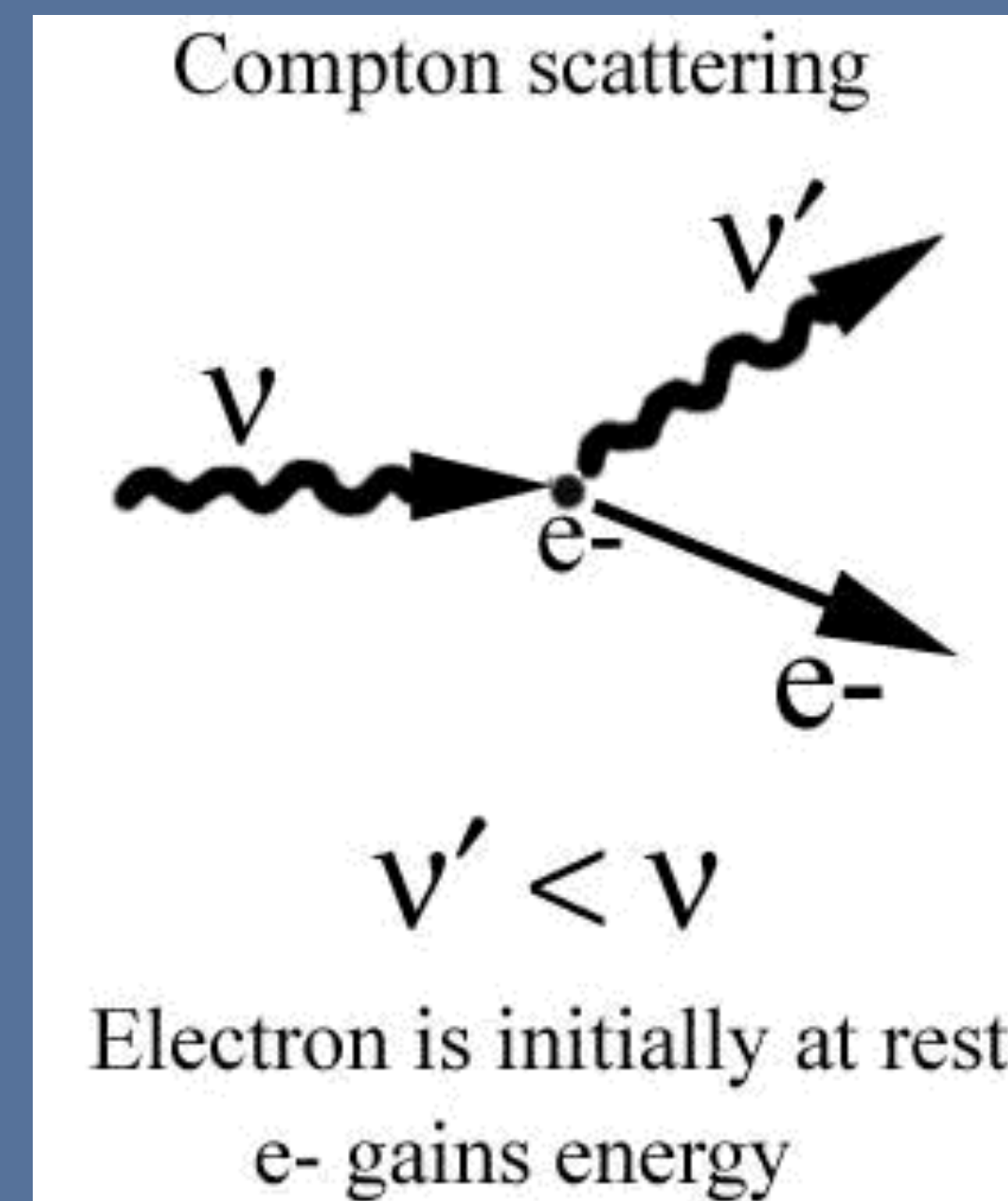


## 5.3.2 Compton and inverse Compton scattering

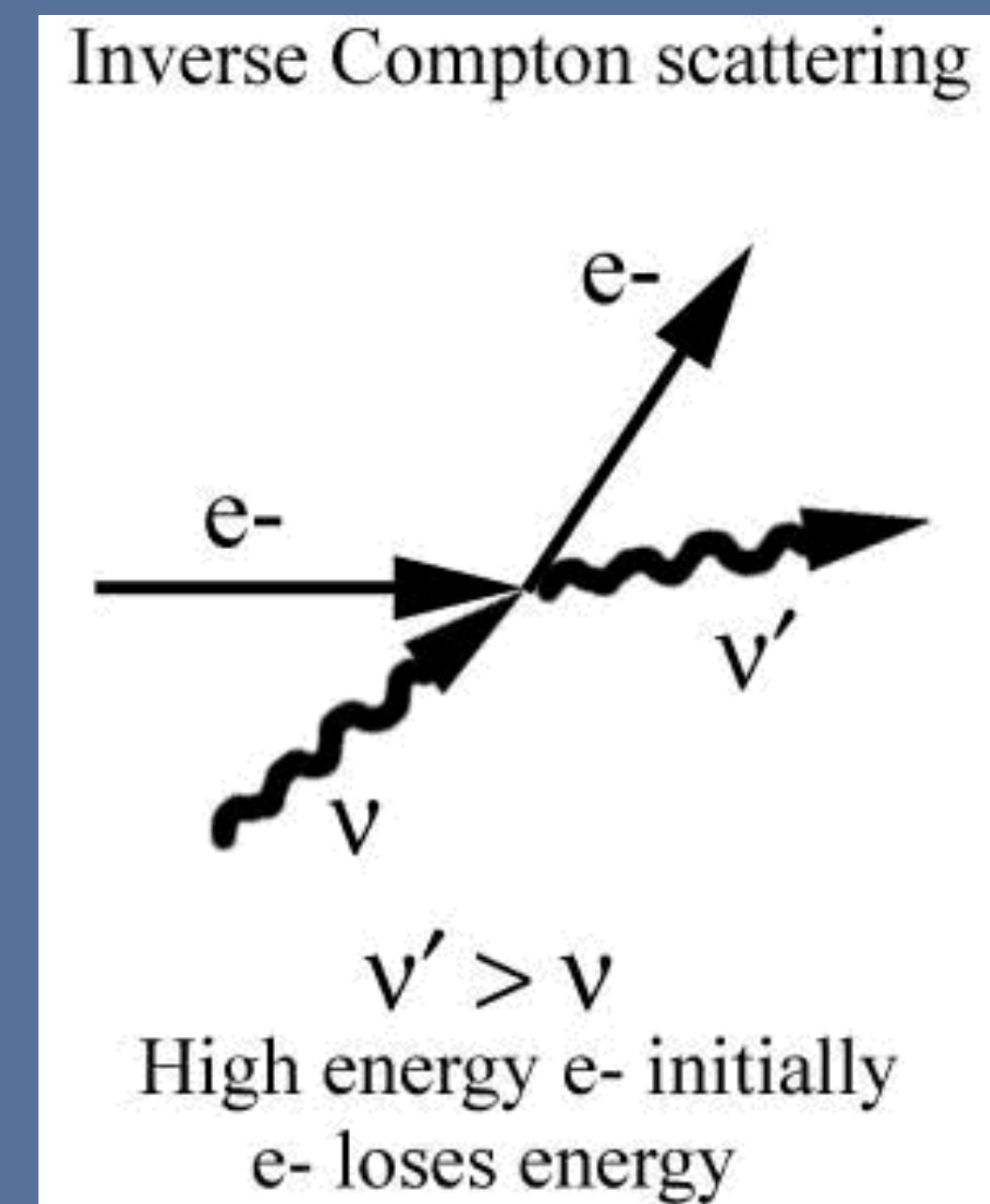
- Scattering of a photon on an electron
- Non-elastic interaction: there is a change in the energy of the photon (ie a change in its wavelength)

- This can be very efficient if the photons can be scattered many times
- Inverse Compton scattering can account for the Sunyaev Zel'dovich effect

High energy photons can be scattered by electrons at rest



Low energy photons can be scattered by energetic electrons

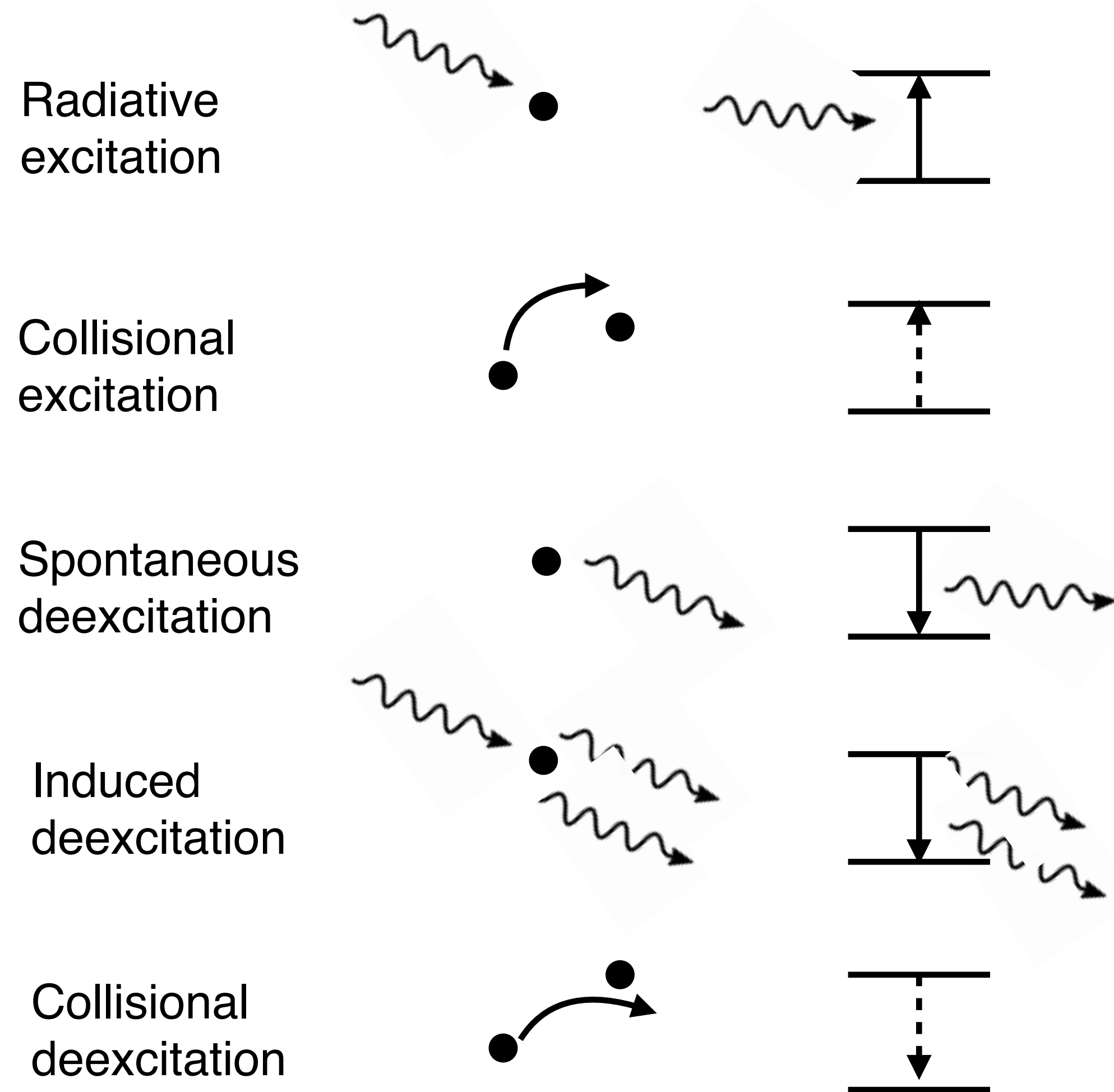


	Radio	Micro-wave	Infra-Red	Visual	Ultra-Violet	X-ray	$\gamma$ -ray
Jargon $\nu/\lambda$ Jargon $E$	MHz mJansky	GHz mJansky	$\mu\text{m}$ $T_B$	$\text{\AA}$ , nm erg, $I/I_0$	$\text{\AA}$ , nm erg	keV	MeV/GeV
Facilities: first, notable	radar VLA, WSRT, LOFAR, SKA	COBE WMAP, Planck	IRAS ISO, Spitzer, Herschel, SOFIA	Galilei ESO Chile, Keck Hawaii, La Palma	IUE Hubble, GALEX	Uhuru Chandra, XMM Newton	balloons/ SAS-2/COS-B Compton Swift
Imaging Dispersion Detection	Aperture synthesis filters amplitude +phase	mirror filters energy	mirror filters energy	mirror, lens filters, grating photons collective	mirror filters, grating some photons	mask, mirror filters, grating individual photons	mask, mirror filters, grating individual photons
Continua Lines	Bremsstrahlung	free-free molecules	free-free molecules	bound-free atoms	Thomson ions	Compton nuclei	Compton nuclei
Characteri- stic object	galaxy, quasar	CMB	IM, protostar	cool star	hot star	accretion disk	GRB

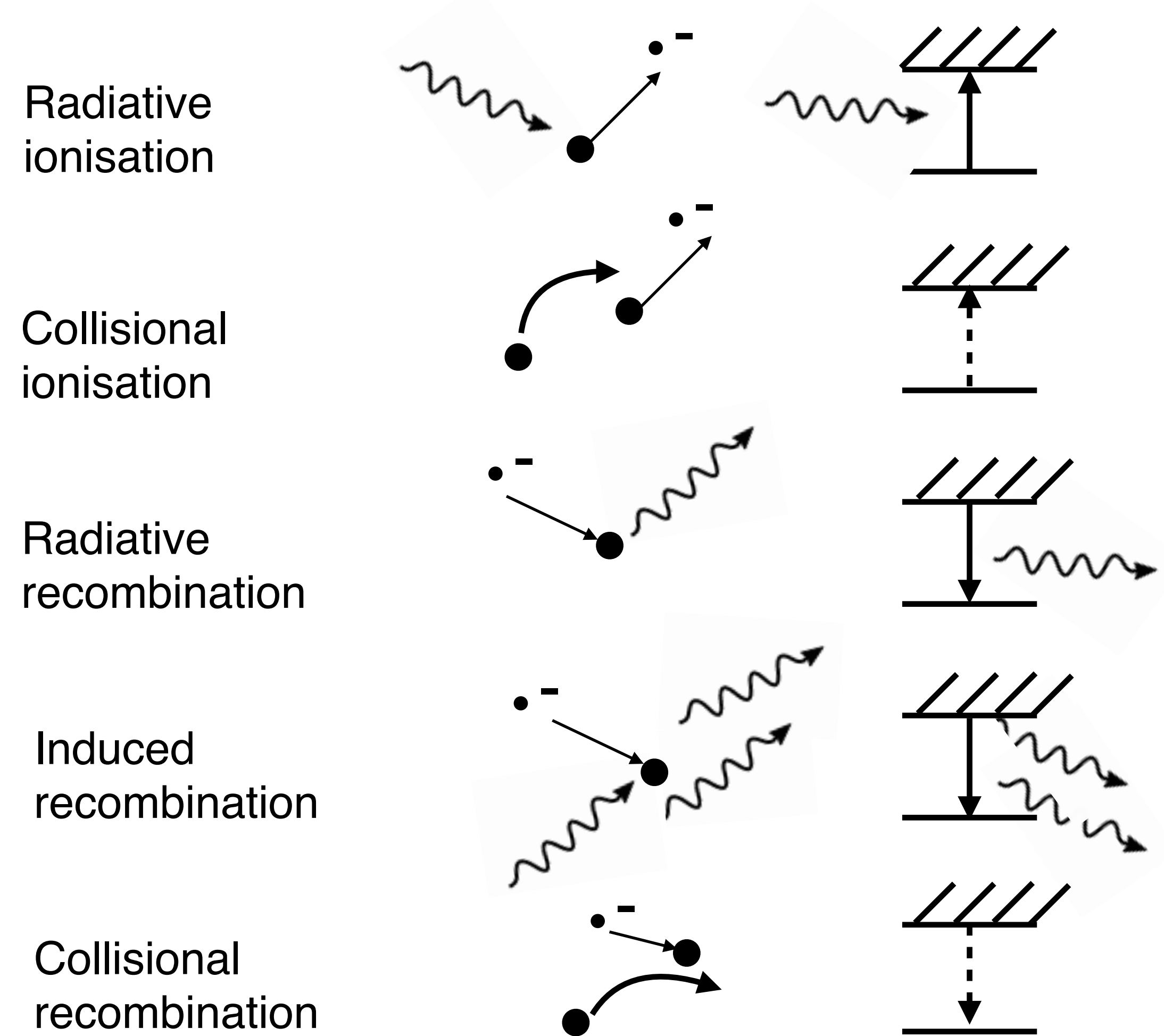


# 5.4 Summary of processes involving bound levels

## Bound-bound process



## Bound-free process



## 5.4.1 Photon creation / destruction

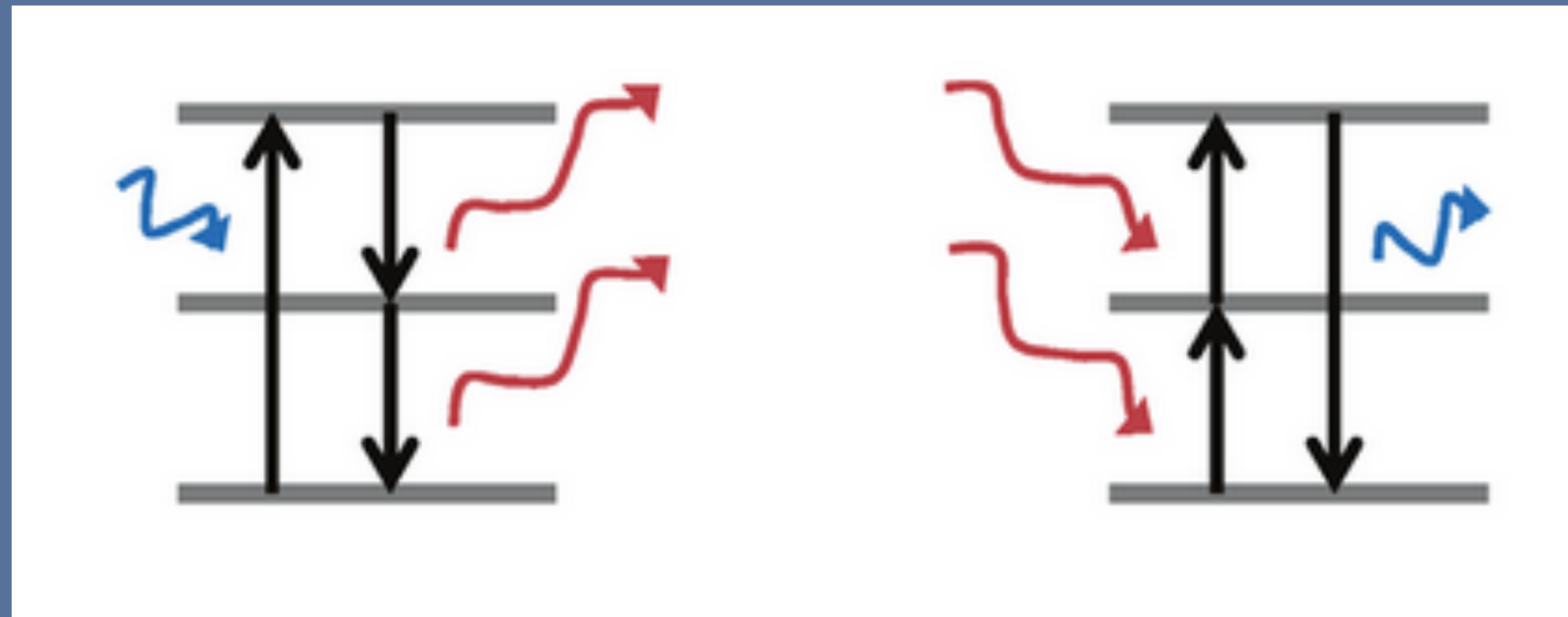
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- **Photon creation**: collisional excitation followed by radiative deexcitation
  - In this process, kinetic energy is converted into radiative energy
- **Photon destruction**: radiative excitation followed by collisional deexcitation
  - In this process, radiative energy is converted into collisional energy
- A radiative excitation followed by a radiative deexcitation between the same levels changes the propagation direction of the photons. This is sometimes called “**photon scattering**” although it is strictly valid only for a two-level system
- If the frequency remains unchanged, it is called “monochromatic” or “coherent”

## 5.4.2 Photon conversion

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- For a system with at least three levels: an energetic photon can be converted into several photons of greater wavelengths



M. Balestrieri

## 5.4.3 Coupling between radiation and matter

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- We have seen that photon creation and destruction allow for the conversion of kinetic energy into radiative energy, and vice versa
- This means that the local radiation field is coupled with the local conditions in the medium
  - ▶ High densities (many collisions): strong coupling
  - ▶ Low densities or radiation dominated by scattering: weak coupling, the radiation field can be independent of the local energies of the particles
  - ▶ In the case of dominant scattering, the radiation does not give us information about local conditions where photons were created. The photon provided by the scattering atom comes from a different place and was created in a completely different region. This photon carries information related to its creation in particular the kinetic energies of the particles that created it. The photon keeps its own identity during the scattering events
- The non-local aspect of scattered radiation is central in radiative transfer

