Radiative Transfer

6. Radiative transfer in dusty media

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Introduction

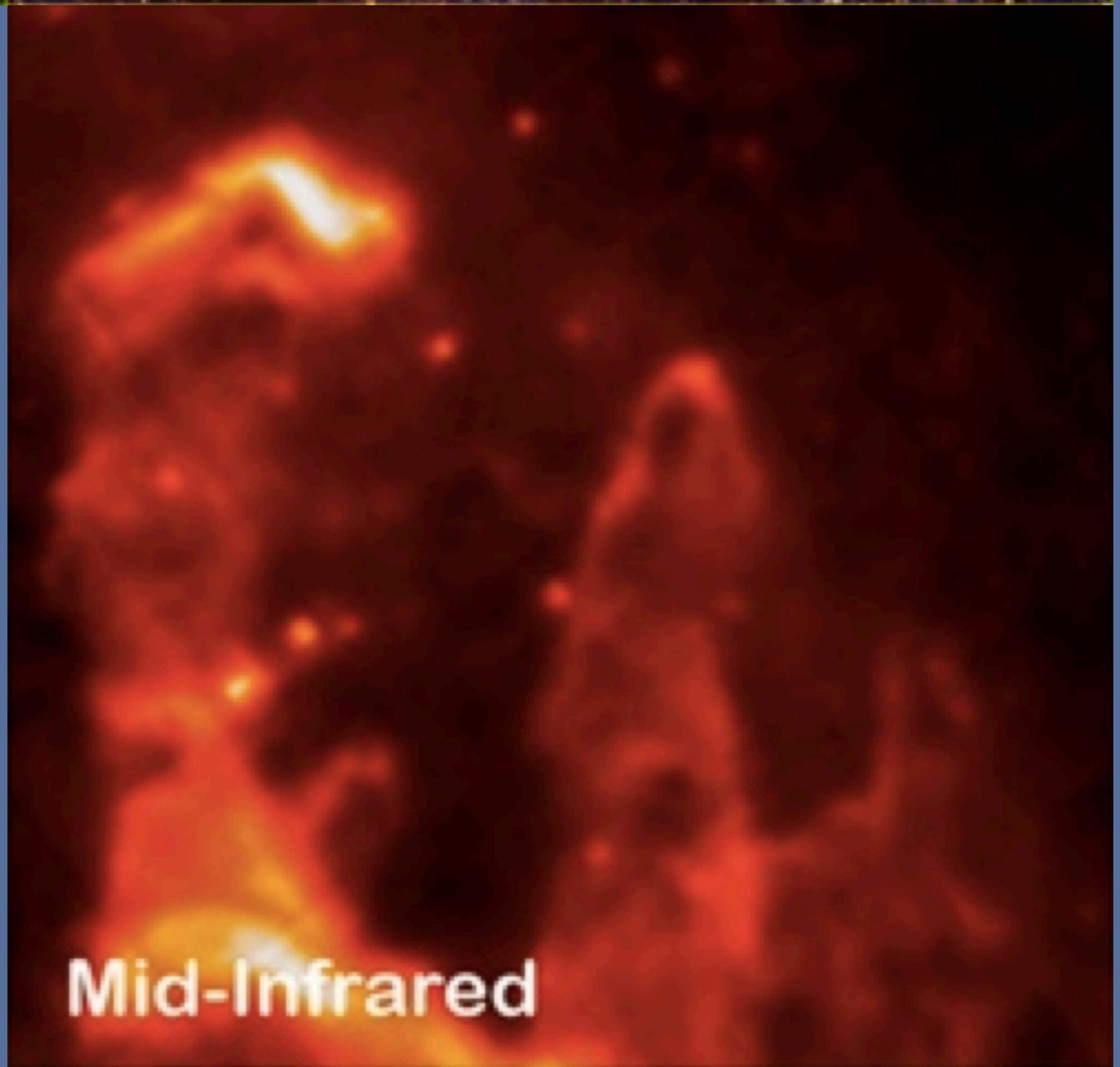
- Now that we have presented the basics of radiative transfer, we will look into a few important radiative processes, highlighting their characteristics and typical difficulties in solving RT
- In this chapter, we will look at thermal continuum emission, typical of media containing interstellar dust
- Interstellar dust is an important constituent of astrophysical environments
- Despite its low abundance (in the molecular ISM, it represents only 1% of the mass), it plays a major role in the regions' thermal equilibrium, by reprocessing the incoming radiation
- Dust is found in a very wide range of environnements: in the ISM, star forming regions and protostars, protoplanetary disks, disks around AGNs, AGB stars, brown dwarf and planetary atmospheres…

Introduction

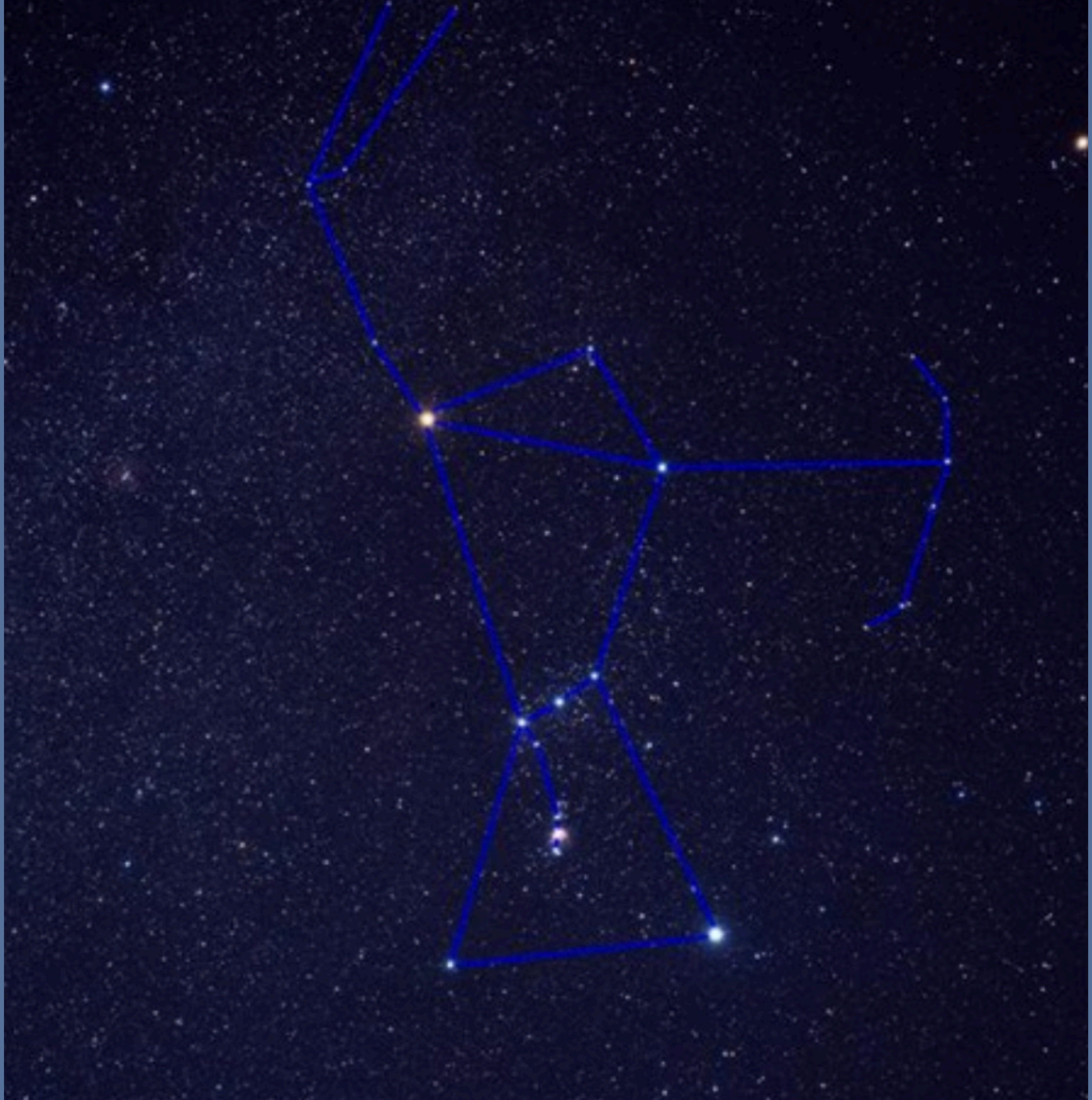
- The reason why dust is important in radiative transfer is due to their high continuum opacities
    - For atomic and molecular gas, the opacities are mostly due to spectral lines, which cover only a small part of the electromagnetic spectrum
    - Dust opacities affect a large portion of the electromagnetic spectrum and can therefore play a major role in energy (heat) transfer.
    - The extinction due to dust also protects the interstellar medium against UV radiation which allows for molecular formation
    - On the other hand, the optical depths arising from dust extinction can be high and prevent us from peering into embedded objects, unless long wavelengths are used
- In the infrared and millimeter domains, the emission is mostly dominated by dust thermal emission
- At shorter wavelengths, dense molecular cores and protoplanetary disks appear as dark silhouettes because of dust extinction
- In the solar system, “zodiacal light” arises from scattering of solar radiation on interplanetary dust grains

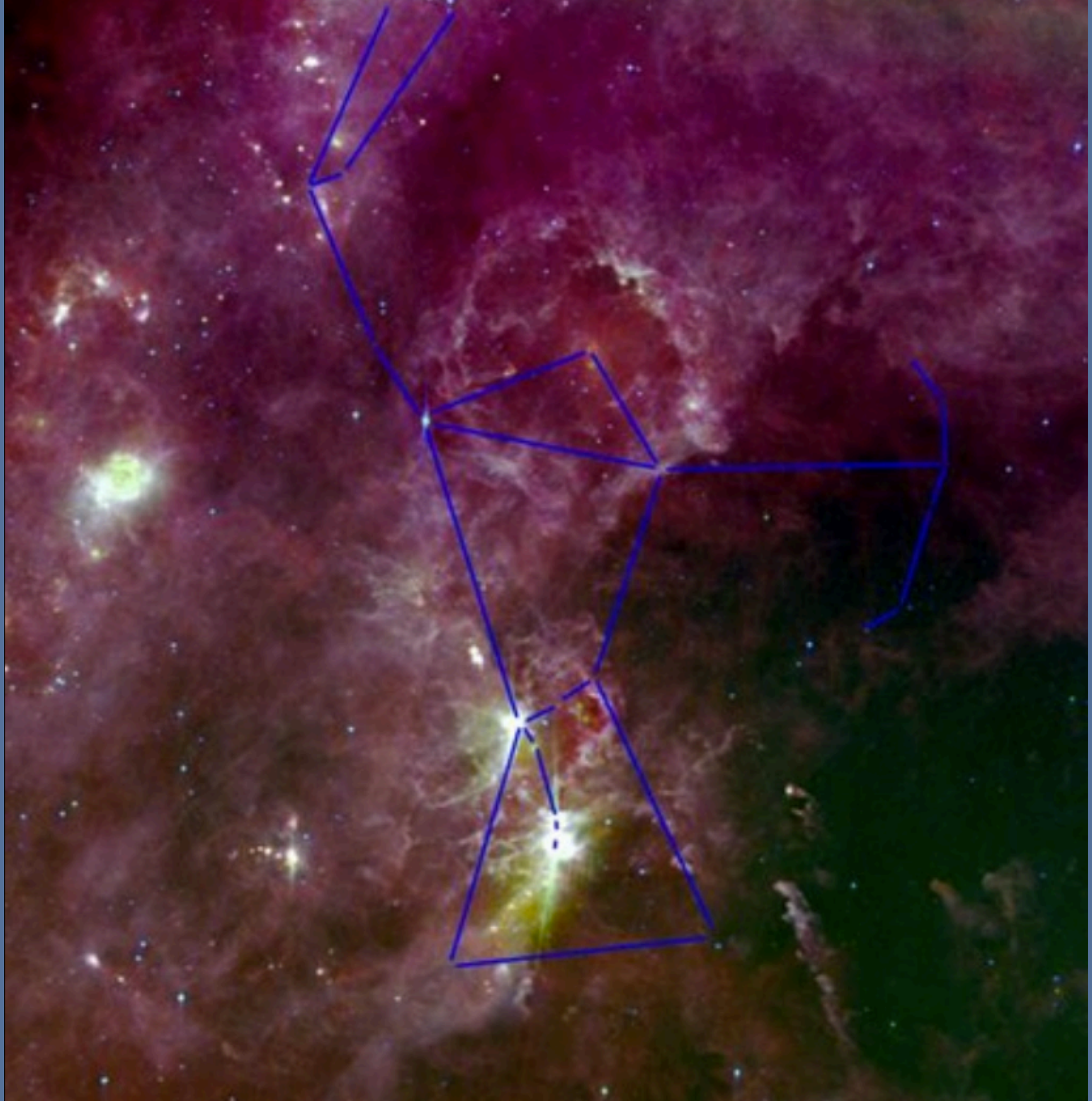

Visible


Near Infrared


Mid-Infrared


Visible
Near Infrared
Mid-Infrared






Visible
Light
HST


Infrared
Light
Spitzer

1. Dust opacities

- Dust grains are small solids mostly made of silicates (species containing a Si-O bond) or carbonaceous material (nanocarbons, PAH, organic matter), sometimes coated in ices (water ice, CO ice, etc.).
- These minerals can be mixed or can coagulate to form agregates of different compositions
- Many uncertainties remain on dust properties: their size distribution, their form, their exact composition
- These characteristics influence their optical properties, which we will discuss in the following

1. Dust opacities

- When dealing with dust, we generally use the (mass) opacity κ_ν
  (linked to the absorption coefficent $\alpha_\nu = \kappa_\nu \, \rho$)
- A photon interacting with a dust grain can be absorbed or scattered.
  - For a drop of water (i.e. transparent sphere) of a given refractive index, hardly any light is absorbed, but because of the refraction at its surface, the light is deflected in another direction. In this case, the opacity κ_ν is almost entirely a scattering opacity
  - On the other hand, for a graphite grain, only a small part of the incident light is scattered, and most of it is absorbed
- It is usual to define the albedo η_ν
  - $\kappa_\nu^{\text{sca}} = \eta_\nu \, \kappa_\nu \qquad\qquad \kappa_\nu^{\text{abs}} = (1 - \eta_\nu) \, \kappa_\nu$
  - With obviously $\kappa_\nu = \kappa_\nu^{\text{abs}} + \kappa_\nu^{\text{sca}}$

1.1 Opacities and related quantities

- Let us consider a spherical dust particle of radius a, with a (mass) density ξ in $\mathrm{g\,cm^{-3}}$. The mass of one particle is $m = \dfrac{4\pi}{3}\,\xi\,a^3$

- If the wavelength at which we observe is much smaller than the grain size $(\lambda \ll 2\pi\, a)$, the approximation of geometric optics can be used. In this case, diffraction effects can be neglected.

  - The particle interaction cross section is equal to the geometric cross section $\sigma_{\mathrm{geo}} = \pi\, a^2$

  - The mass opacity is $\kappa_\nu = \dfrac{\sigma_{\mathrm{geo}}}{m}$

  - In the approximation of geometric optics, the opacity is therefore constant with wavelength

1.1 Opacities and related quantities

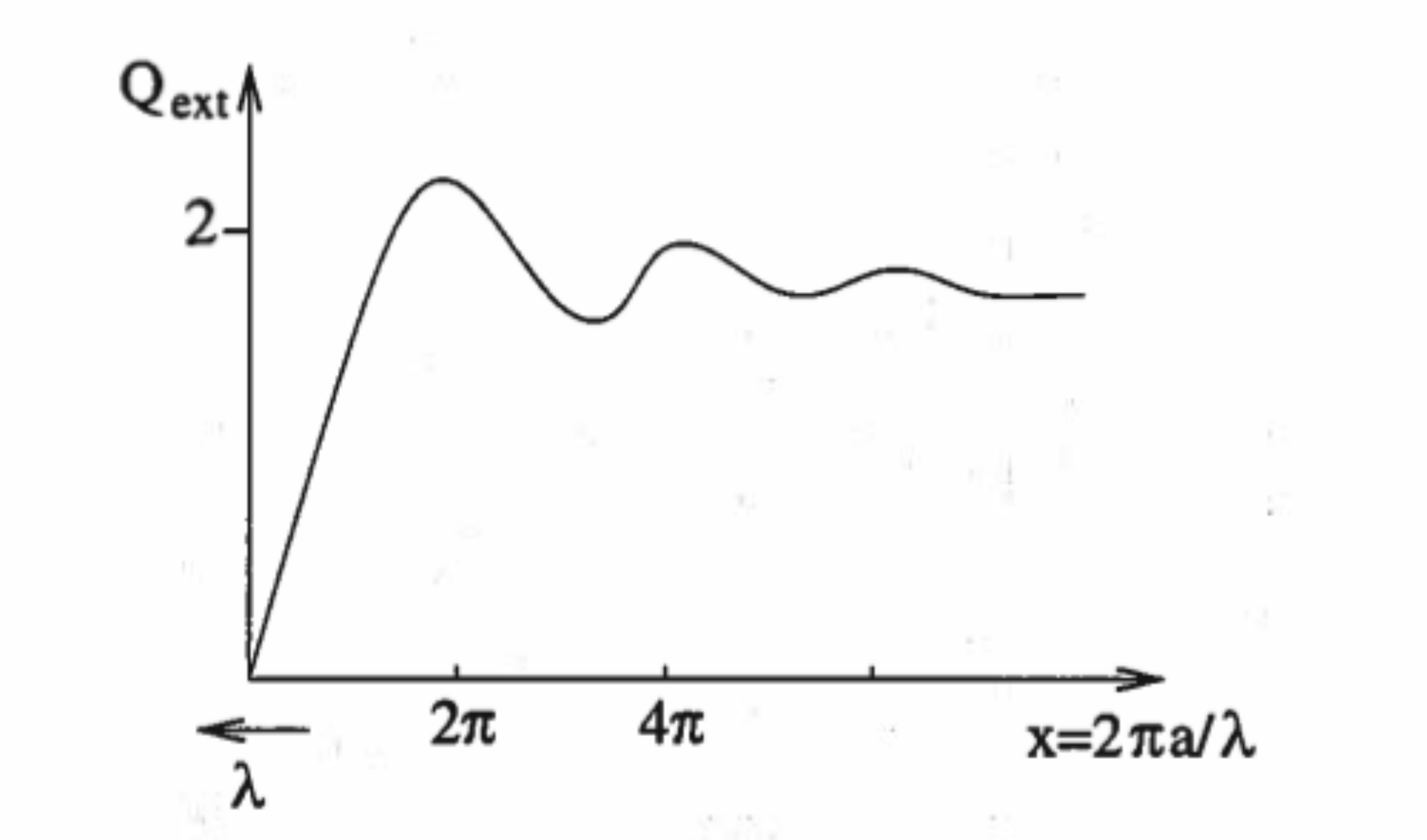
- If the wavelength at which we observe is comparable or greater than the grain size, the approximation of geometric optics is no longer valid. In this case, the opacity κ_ν depends on ν.
  - The extinction factor is defined as the ratio between the interaction cross section and the geometric section: $Q_\nu^{\rm ext} = \dfrac{\sigma_\nu^{\rm ext}}{\sigma_{\rm geo}}$ $\quad Q_\nu^{\rm abs} = \dfrac{\sigma_\nu^{\rm abs}}{\sigma_{\rm geo}}$ $\quad Q_\nu^{\rm sca} = \dfrac{\sigma_\nu^{\rm sca}}{\sigma_{\rm geo}}$
  - For $\lambda \ll 2\pi a$, $Q_\nu = 1$ taking the geometric optics approach.
  - If diffraction effects are taken into account, we have $Q_\nu = 2$. These diffraction effects affect only the far field and slightly deflect the radiation. The cross section of this diffraction is also πa^2, in addition to the geometric section. So in fact, the interaction cross section for a particle is $2\pi a^2$, twice the geometric cross section in case $\lambda \ll 2\pi a$. For an explanation, see Berg et al. (2011, JQSRT, 112, 1170). This effect can be included and treated as highly anisotropic scattering (strongly forward peaked), or ignored, in which case only the geometric cross section is considered.

1.1 Opacities and related quantities

Summary

- $\lambda \ll 2\pi a \Rightarrow Q_\nu = 1$ (or 2)
- $\lambda \gg 2\pi a \Rightarrow Q_\nu \ll 1$

Extinction coefficient as a function of λ, for grains of radius a

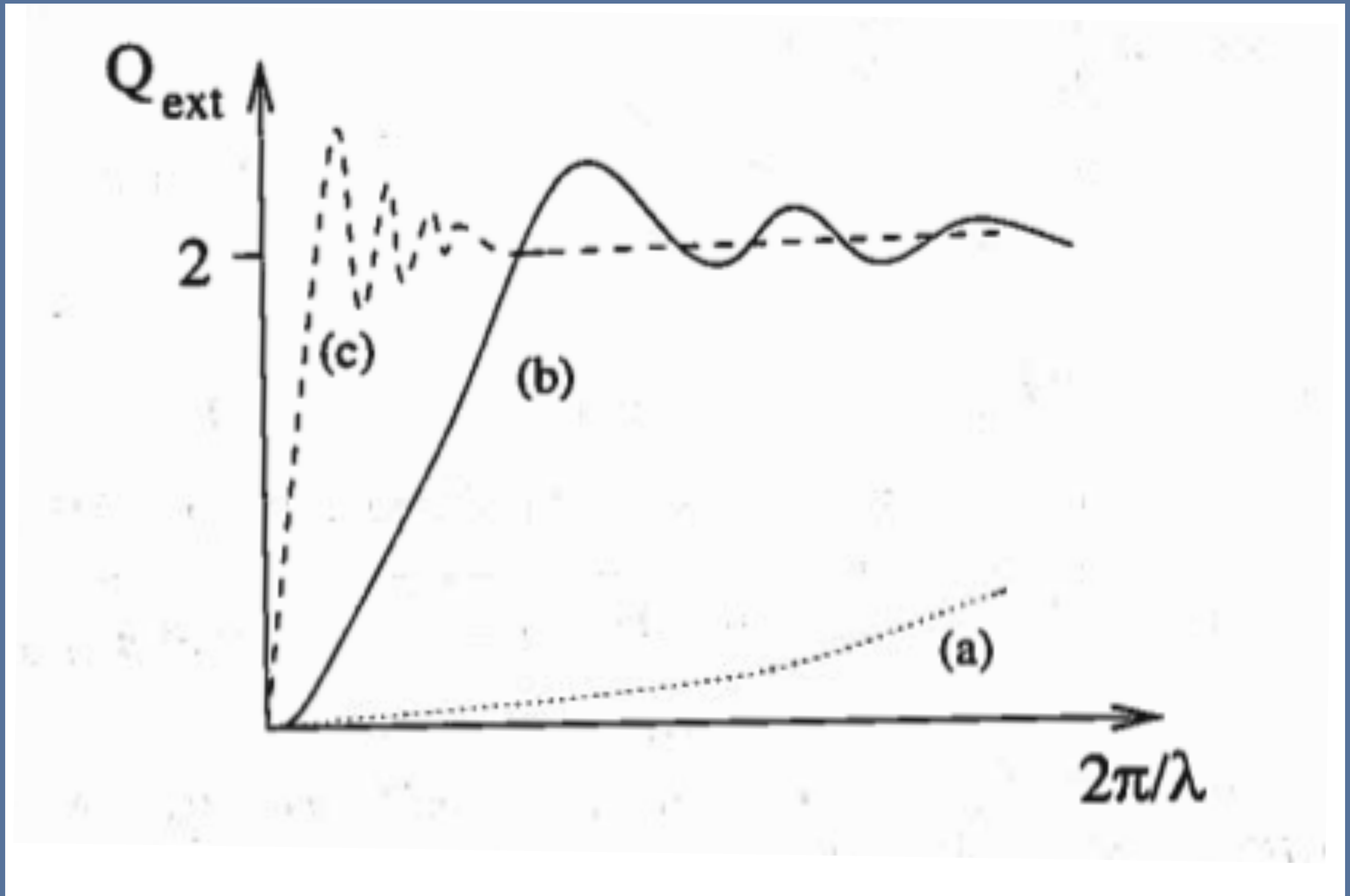


Q^{ext} is maximum when $\lambda \sim a$

1.1 Opacities and related quantities

- At a given wavelength, the value of the extinction coefficient depends on the particle size

Extinction coefficient for several particle sizes:
(a) $0.01\,\mu$m (b) $0.1\,\mu$m (c) $1\,\mu$m



1.1 Opacities and related quantities

- Simplified model for general opacity of dust grains (Ivezic et al. 1997, MNRAS, 291, 121)

$$Q_\nu^{\rm abs} = 1 \qquad \text{pour } \lambda \leq 2\pi a$$

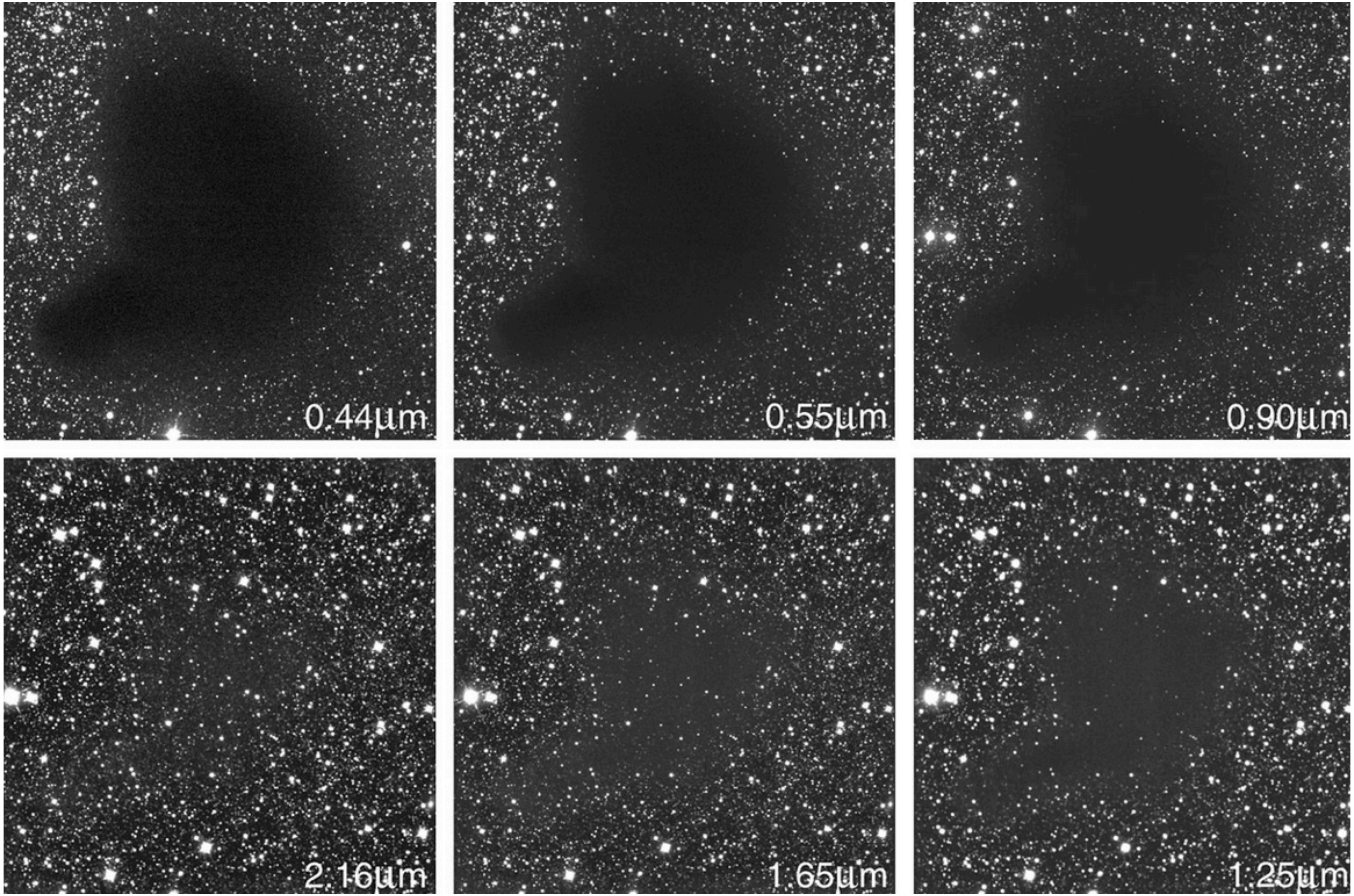
$$Q_\nu^{\rm abs} = \frac{2\pi a}{\lambda} \qquad \text{pour } \lambda > 2\pi a$$

$$Q_\nu^{\rm sca} = 1 \qquad \text{pour } \lambda \leq 2\pi a$$

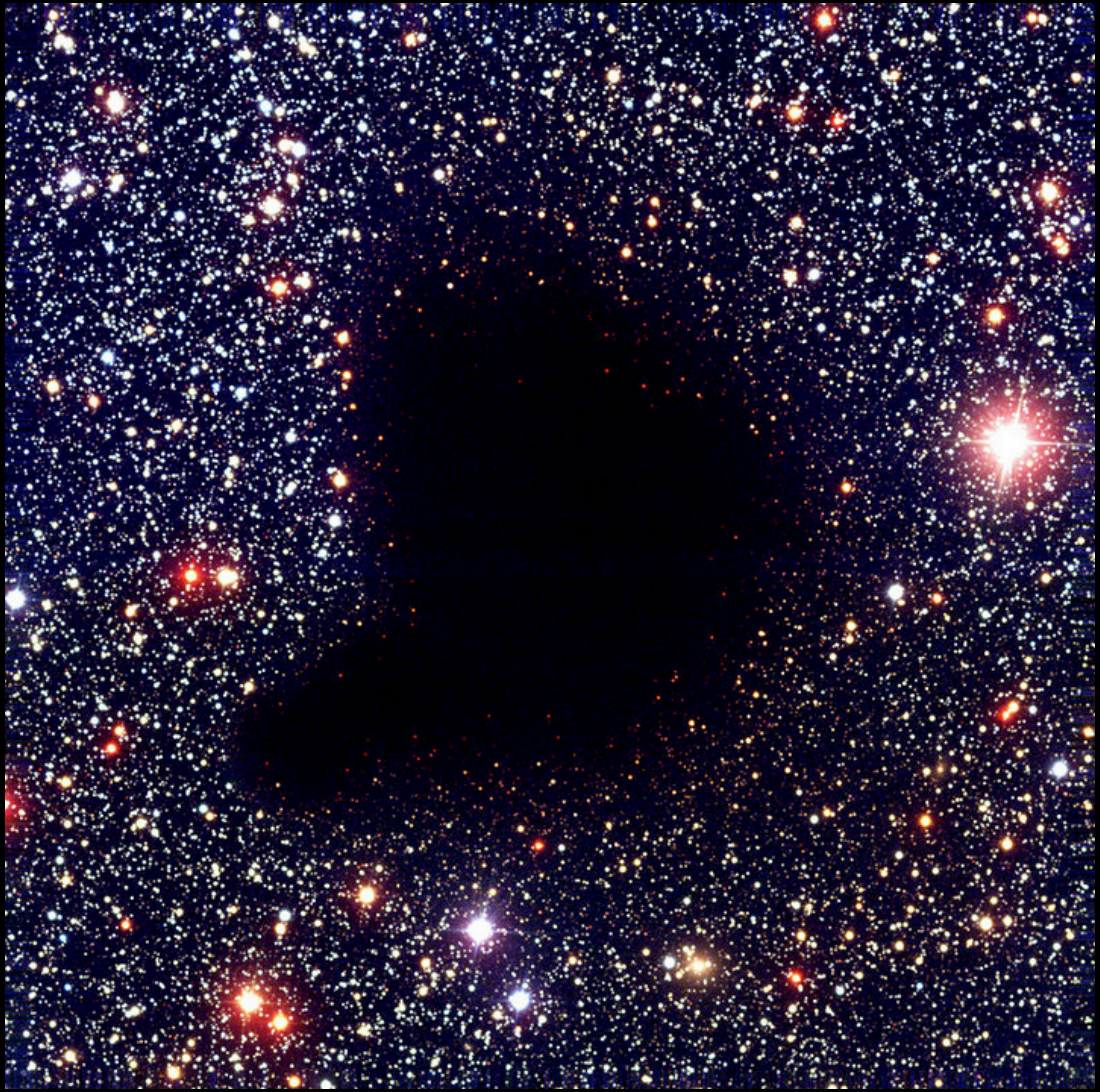
$$Q_\nu^{\rm sca} = \left(\frac{2\pi a}{\lambda}\right)^4 \qquad \text{pour } \lambda > 2\pi a$$

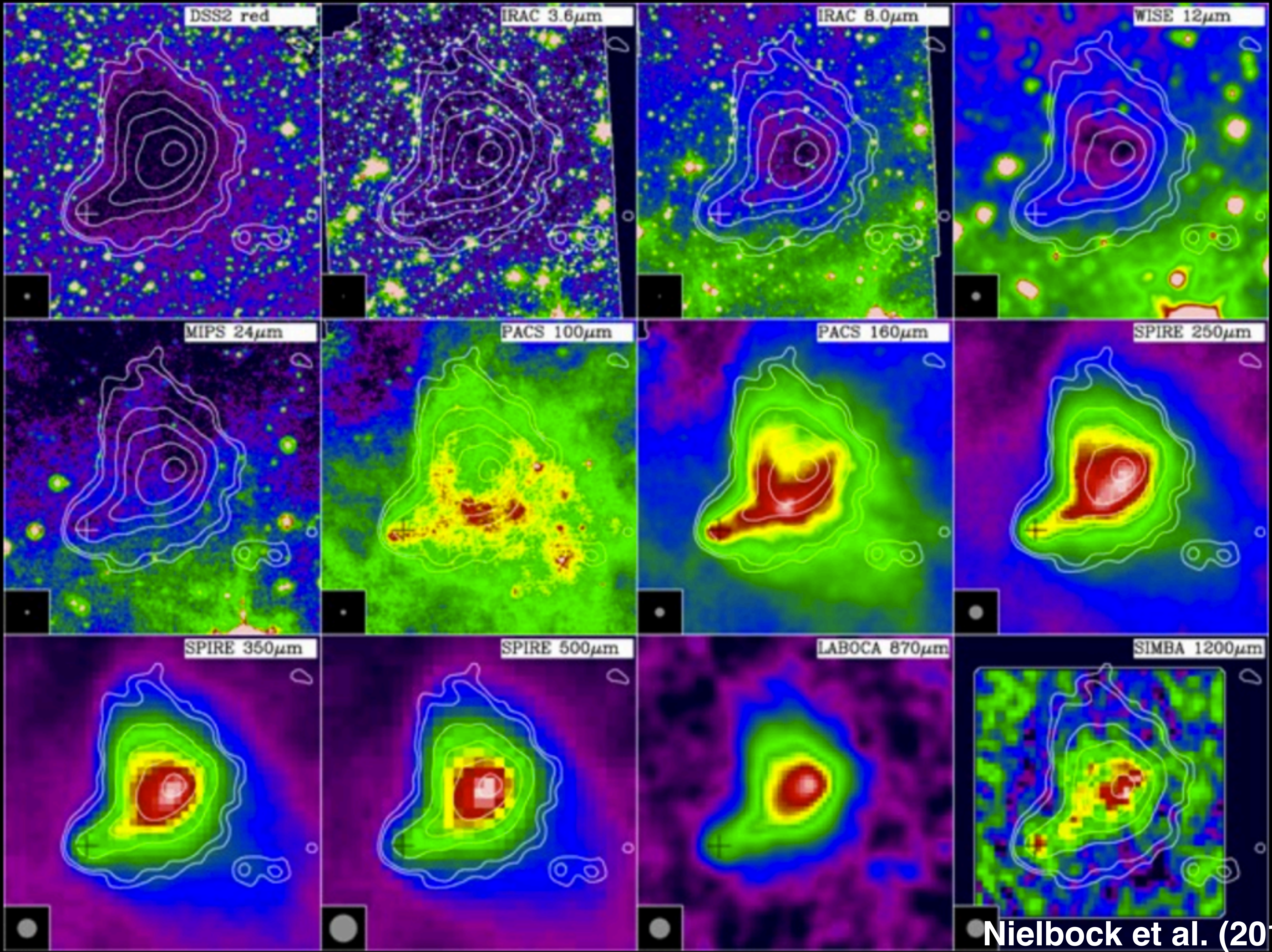
- These opacities should not be used in radiative transfer calculation. They just give orders of magnitude for λ dependencies

Extinction as a function of wavelength



Consequence: reddening

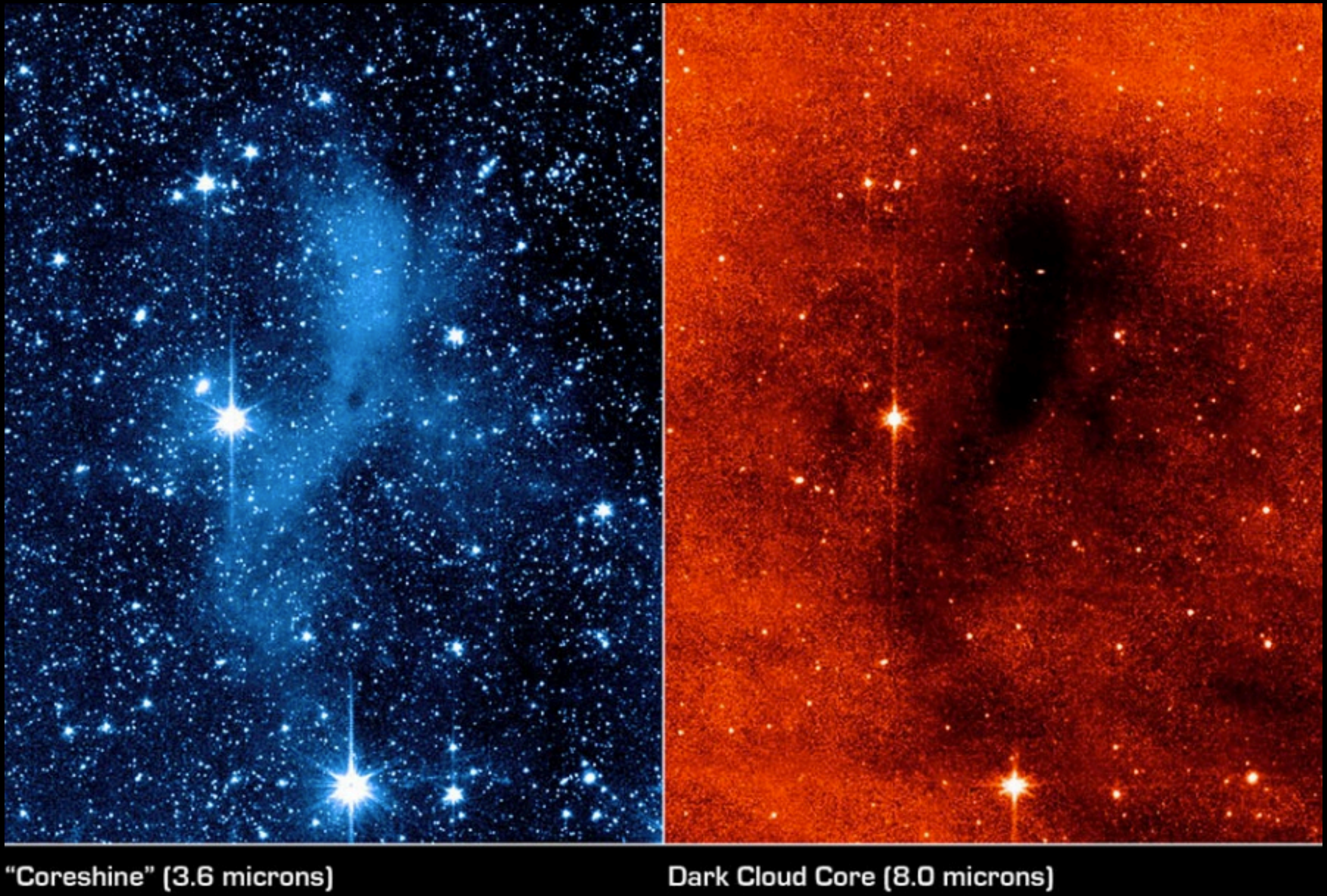




Nielbock et al. (2014)

Coreshine

Scattering by large (~ 1 μm) dust grains in dense cloud cores (Steinacker et al. 2010)



1.1 Opacities and related quantities

- Physical origin of dust opacity:
  The opacity arises from the reaction of the dielectric material to the oscillating electric field of the radiation: the dielectric material emits its own electromagnetic radiation that interacts with the incident radiation field. This interference triggers absorption and scattering

  - If the particles are sufficiently small with respect to the wavelength, the front of the particle cannot shield the interior of the particle from the incident radiation. The entire particle reacts dielectrically to the incident field. For $\lambda \gg 2\pi\, a$, the opacity is a volume effect. This is the Rayleigh limit.

  - On the other hand, if $\lambda \ll 2\pi\, a$, the wave is mostly sensitive to the surface of the particle

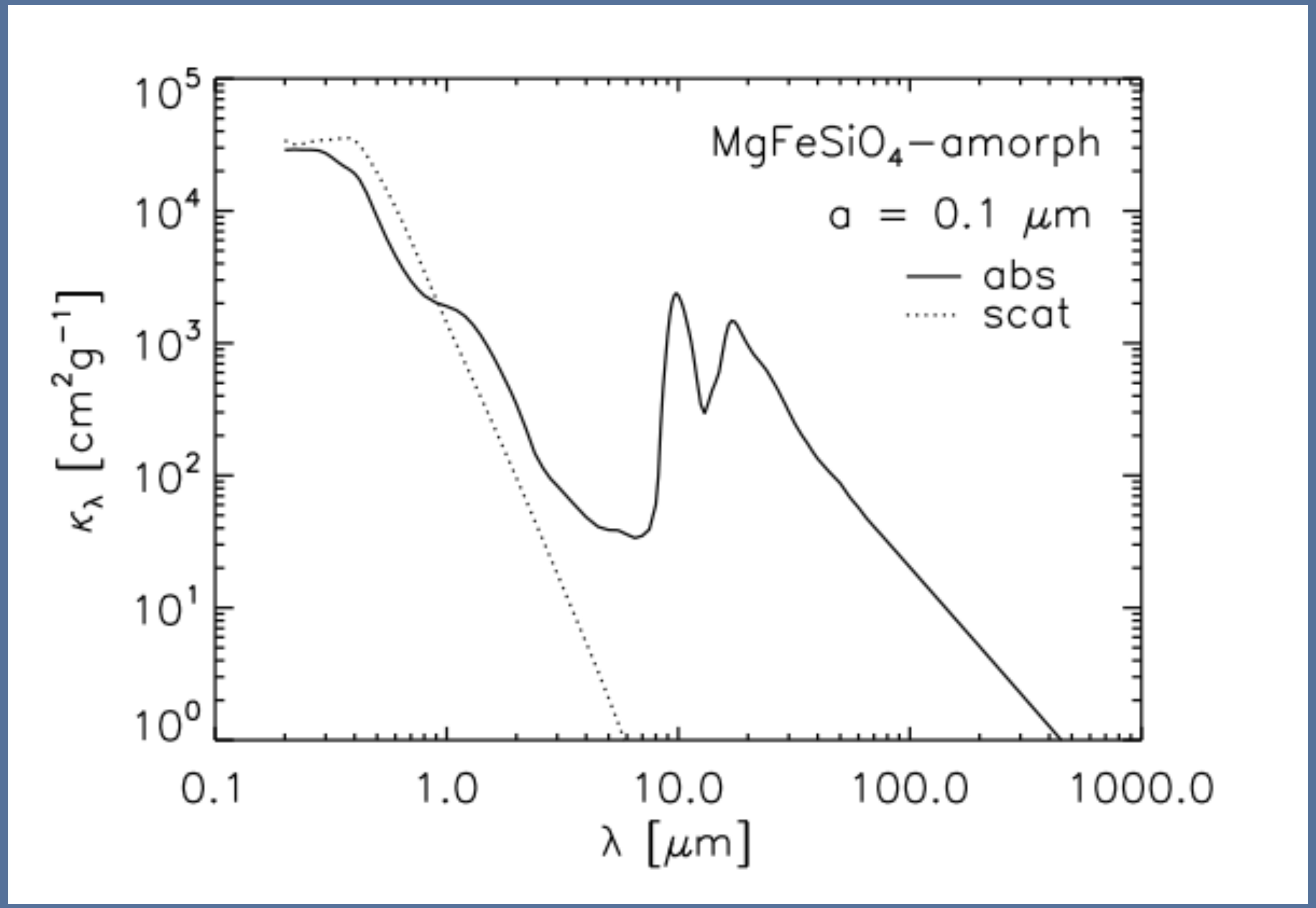
1.2 Opacities of silicates and carbonaceous grains

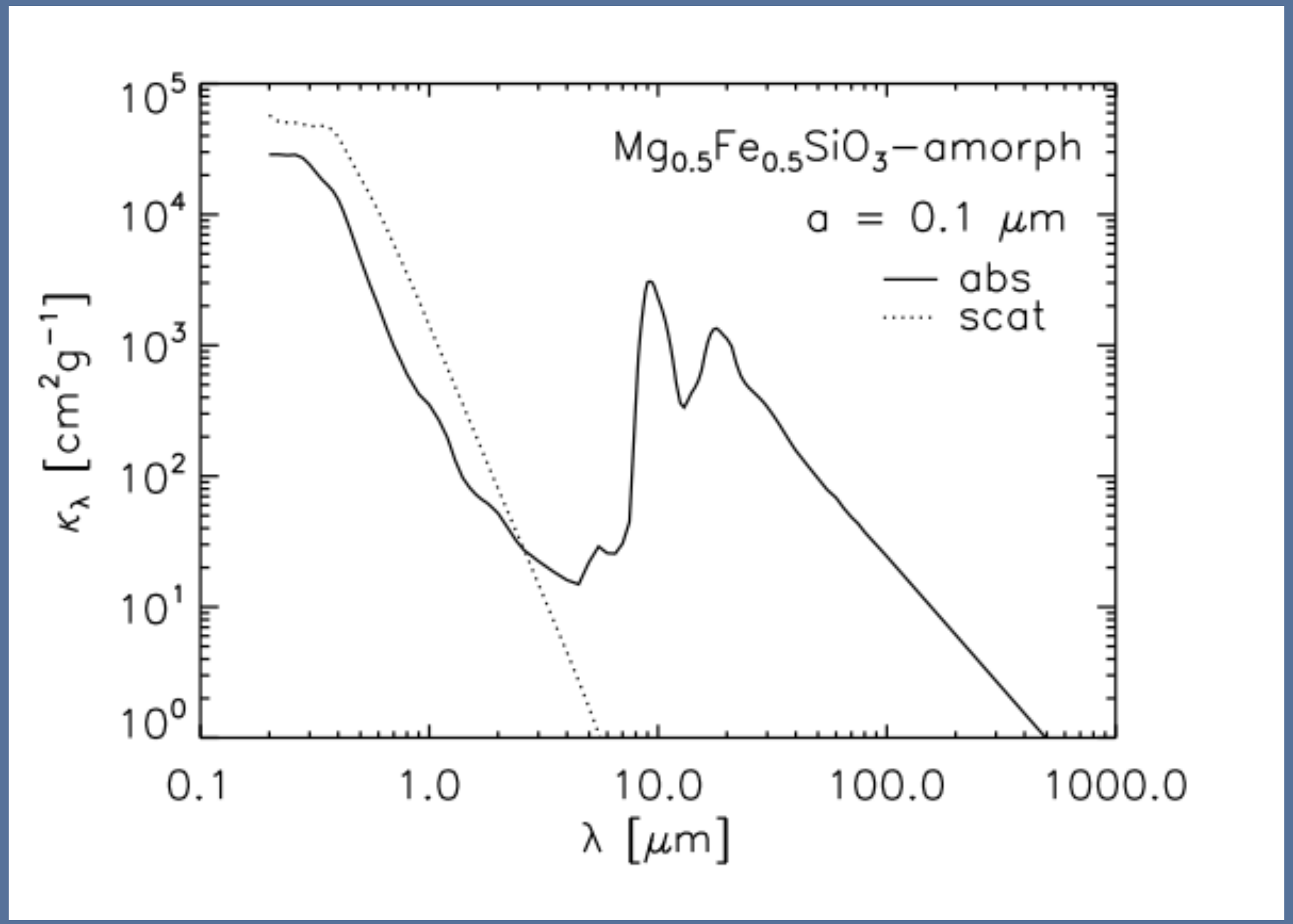
1.2.1 Opacities of silicates

- Silicates are rocky substances characterised by Si-O bonds (e.g. the crust of the Earth). They build up a whole family of materials.
- The simplest one is silica SiO_2 (quartz).
- Other silicates contain in addition to Si and O other metals like Al, Fe, Mg, etc.
- Si-O is negatively charged, and this charge is compensated by the positive metallic ions Al, Fe, Mg.
- In space, the most common silicates are
  - Olivines: $(Mg, Fe)_2SiO_4$
    - Forsterite: Mg_2SiO_4
    - Fayalite: Fe_2SiO_4
    - Any combination of Fe and Mg verifying the charge balance
  - Pyroxenes: $(Mg,Fe)SiO_3$
    - Enstatite: $MgSiO_3$
    - Ferrosilite: $FeSiO_3$
    - Any combination verifying the charge balance

1.2.1 Opacities of silicates

- An important characteristics of silicate opacities is the presence of two strong absorption features towards 10 μm and 20 μm.
- These features are due to the Si-O bond (vibrational transitions)
- They are large peaks, unlike the narrow lines seen in gases, arise because each bond can exchange energy with the rest of the solid. Photons can therefore be absorbed over a wide range of energies (wavelengths)





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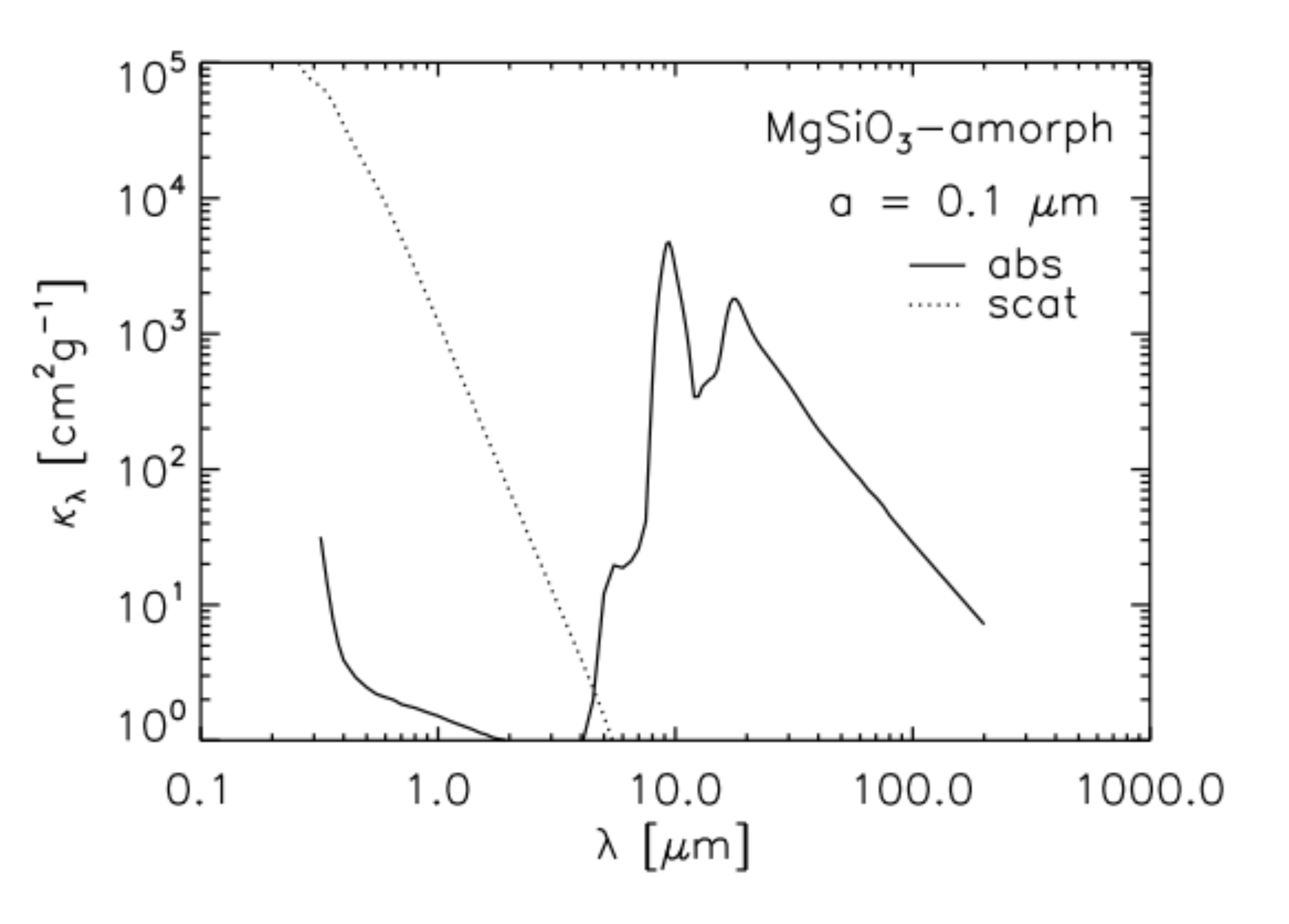
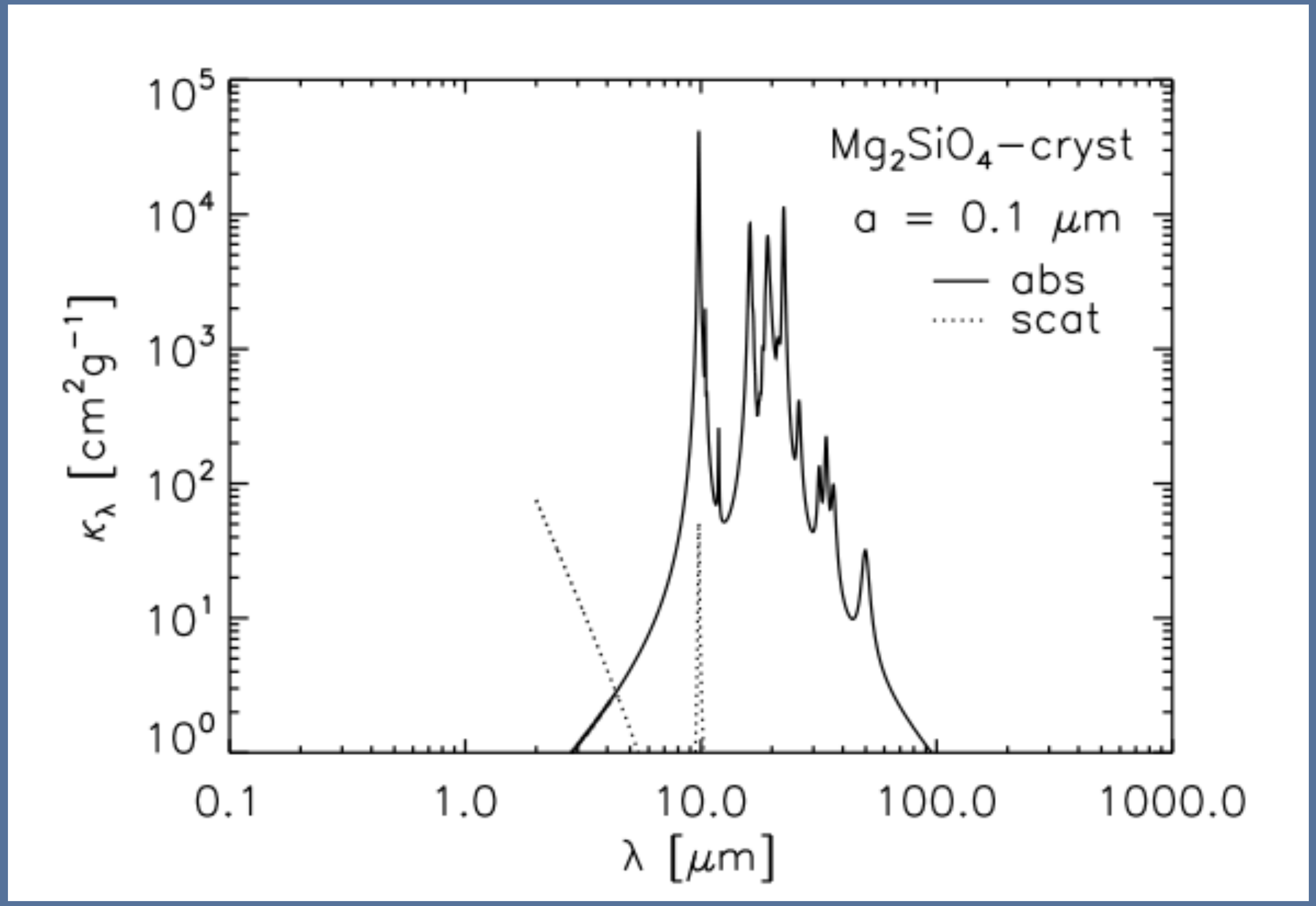
1.2.1 Opacities of silicates

- Silicates on Earth are usually crystalline, even if the cristals are not always macroscopic
- In space, silicates are generally amorphous.
- This difference is very important because the opacities are very different depending on whether they are amorphous or crystalline
- For crystalline silicates, the exchange of energy with the rest of the solid is more limited





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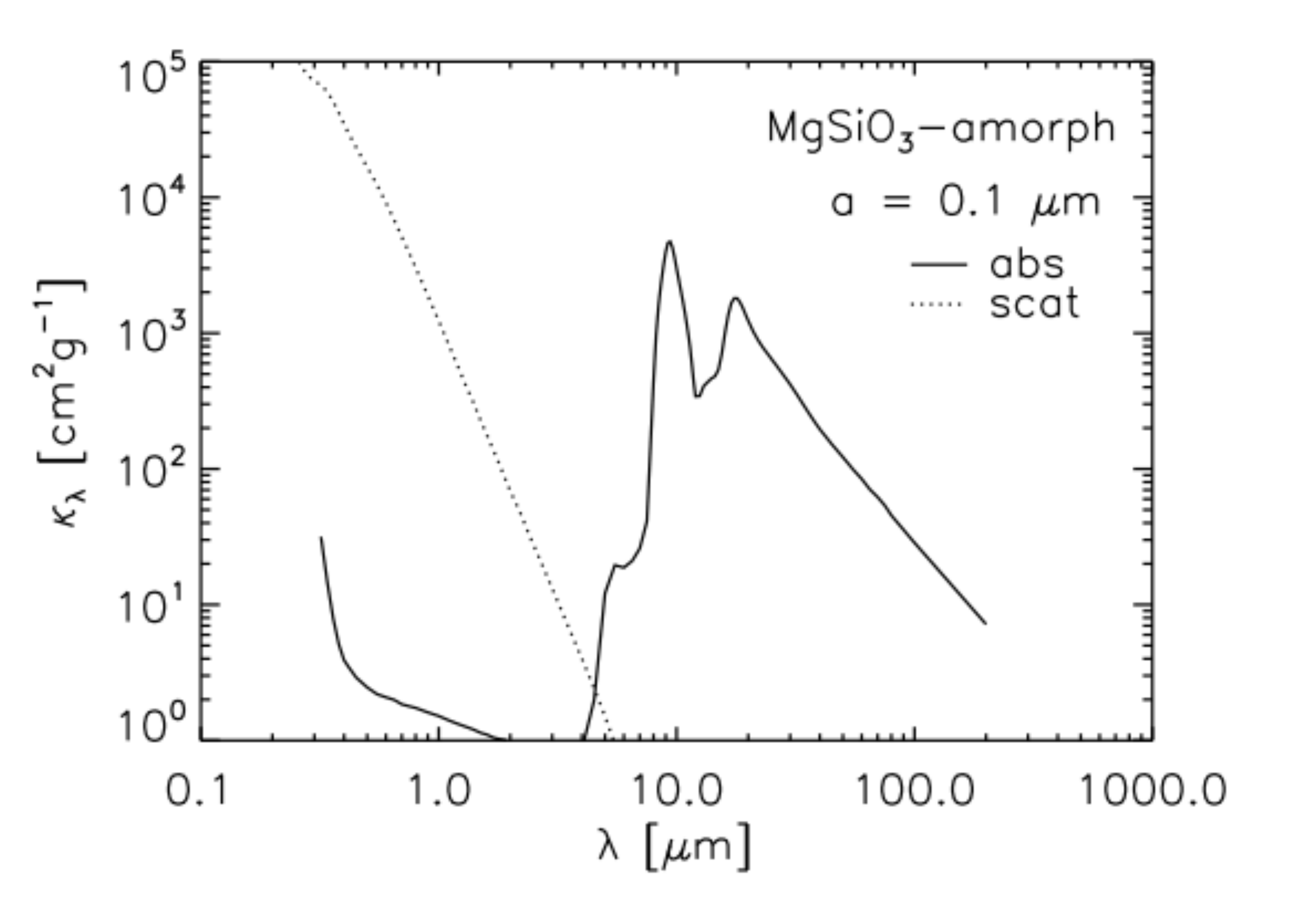
1.2.1 Opacities of silicates

- Silicates in space are believed to be amorphous because they are regularly hit by cosmic rays that destroy possible crystalline structures
- Sometimes crystalline silicates are found in some sources. This is interpreted as evidence for recent heating which annealed the particles and made them crystalline.
- This remains however debated because crystalline silicates have been spectroscopically detected in the outer cold regions of protoplanetary disks
- Or this could be an indication of radial mixing?

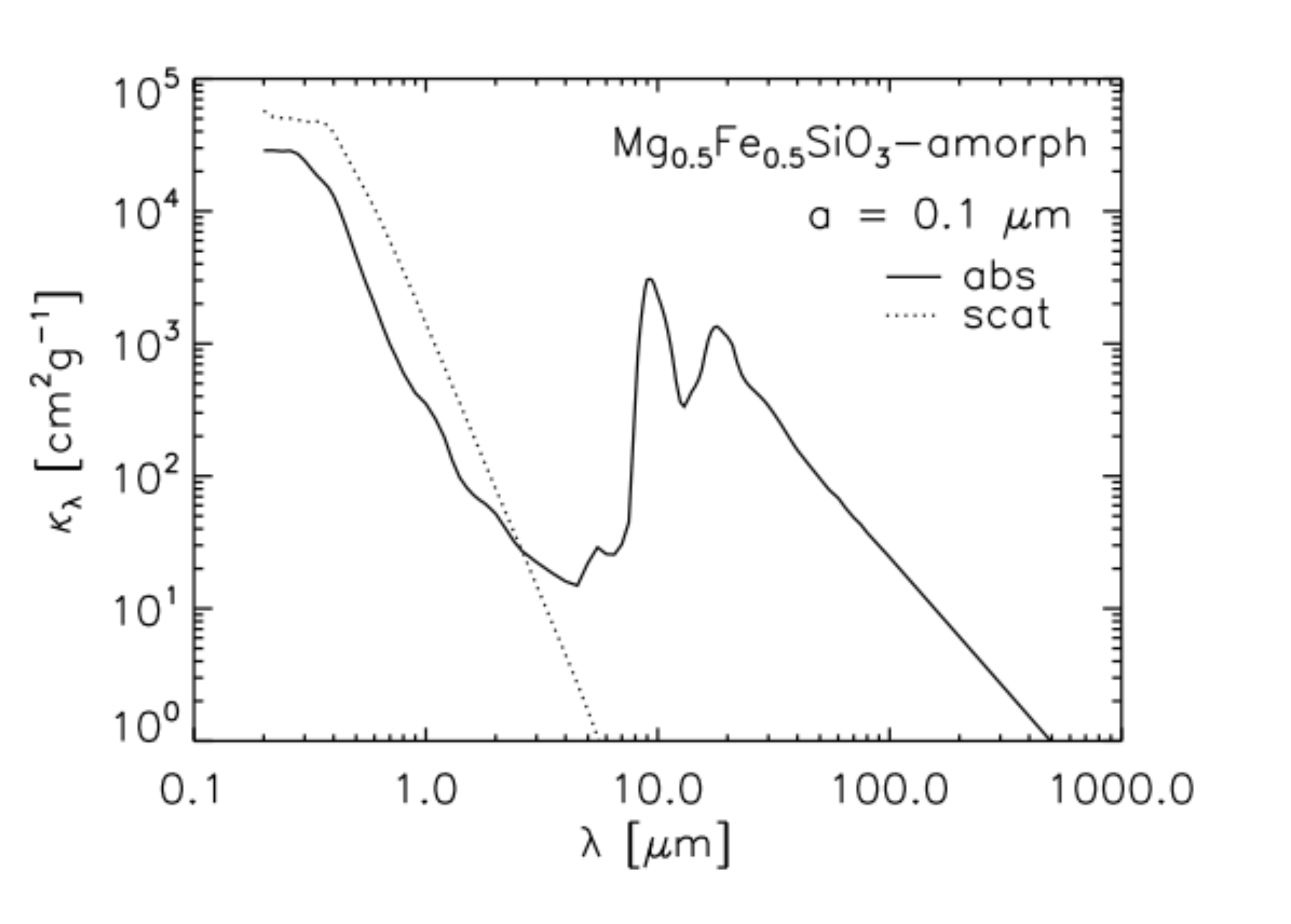
1.2.1 Opacities of silicates

- Iron plays an important role in absorption observed in the visible and NIR
- Without iron, the absorption is low in this spectral regions (though the scattering is less affected)

Without iron



With iron



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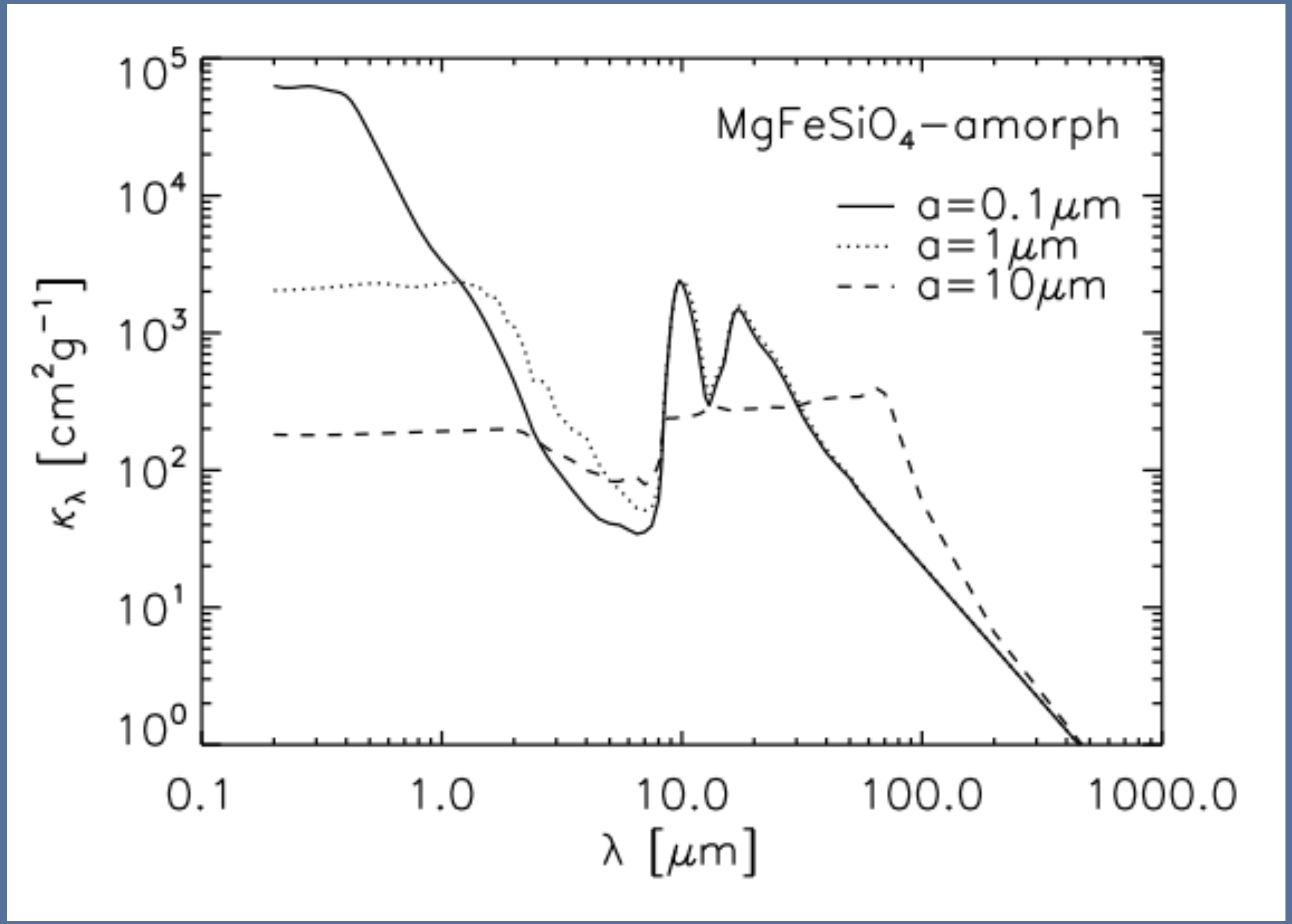
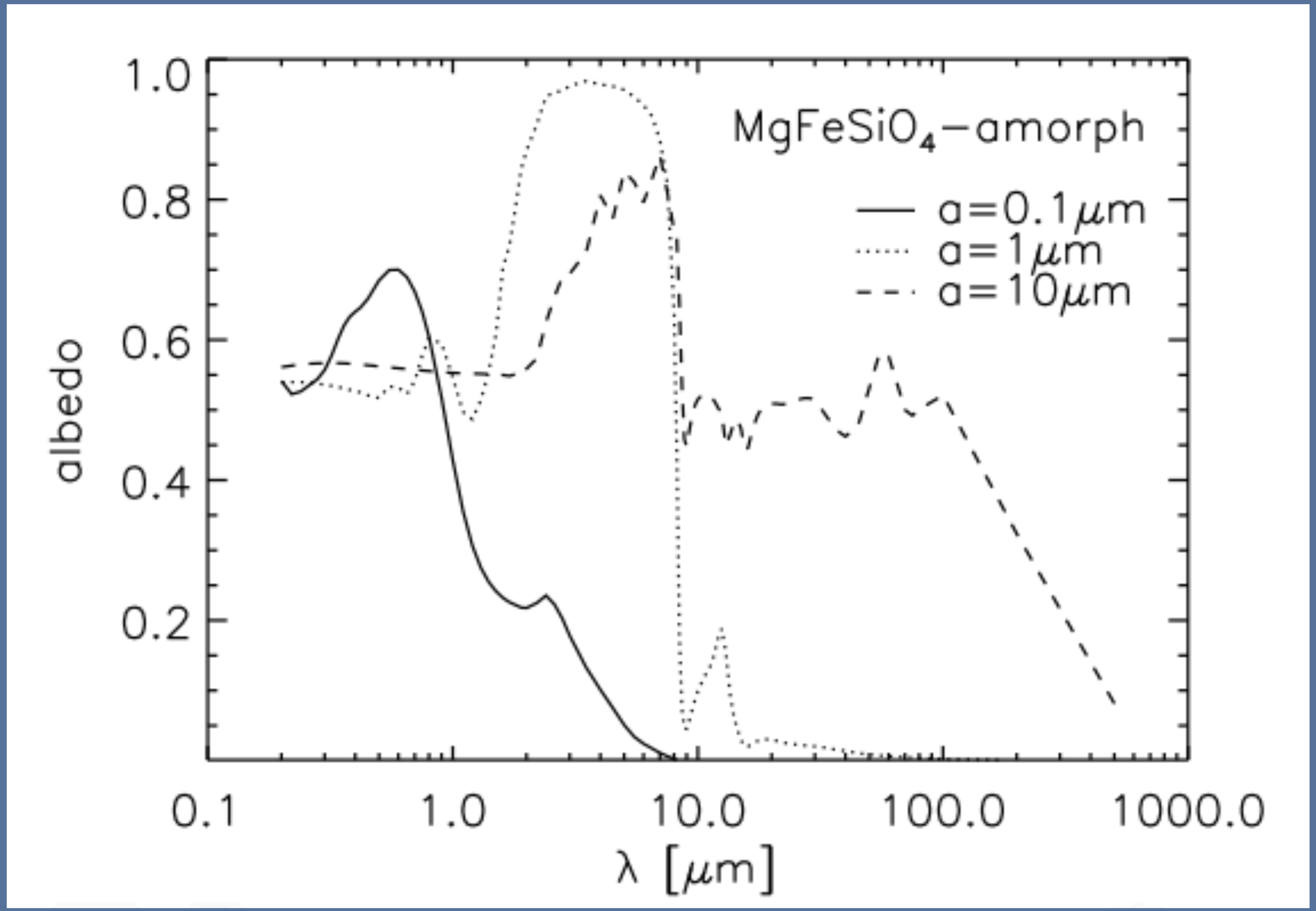
1.2.1 Opacities of silicates

Dependence on grain sizes of absorption and albedo of silicates





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- The overall shape of the absorption and albedo are reminiscent of the Ivezic model at large wavelengths

1.2.1 Opacities of silicates

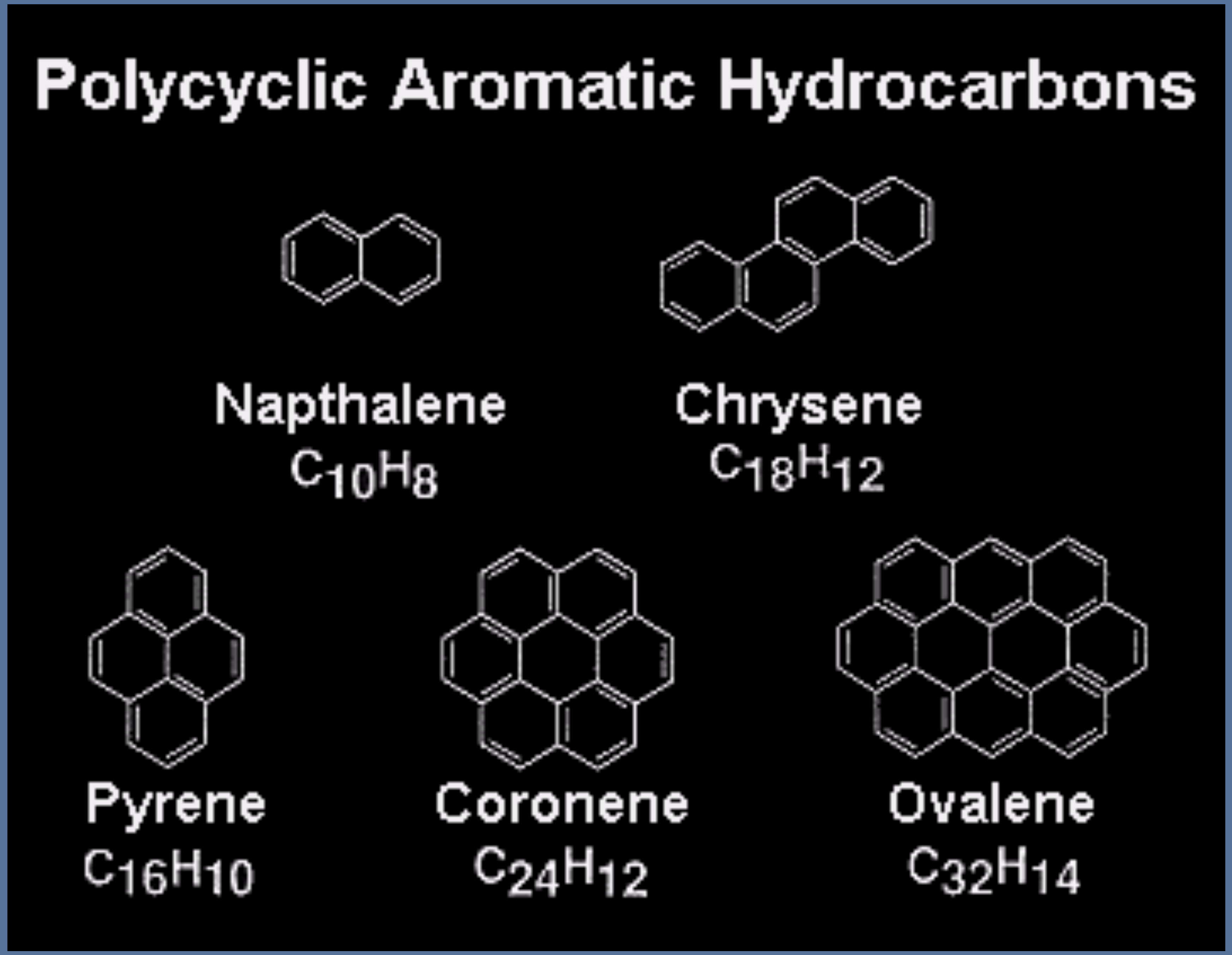
Dependence on grain sizes of absorption and albedo of silicates

- For small grains (0.1 and 1 μm), the opacities beyond 10 μm do not depend on grain size
- In this case, we have $\lambda > 2\pi a : Q_\nu \sim \dfrac{2\pi a}{\lambda}$
- $\sigma_\nu = \sigma_{\text{geo}} \, Q_\nu \;\Rightarrow\; \sigma_\nu = \dfrac{2\pi a}{\lambda} \cdot \pi \, a^2$
- Because $\kappa_\nu = \dfrac{\sigma_\nu}{m}$ and $m \propto a^3$, the dependance on the grain size a disappears.
- For large grains, the opacity depends on grain size but does not vary with wavelength
- The albedo is flat up to $\sim \lambda = 2\pi \, a$ and then decreases very fast with λ

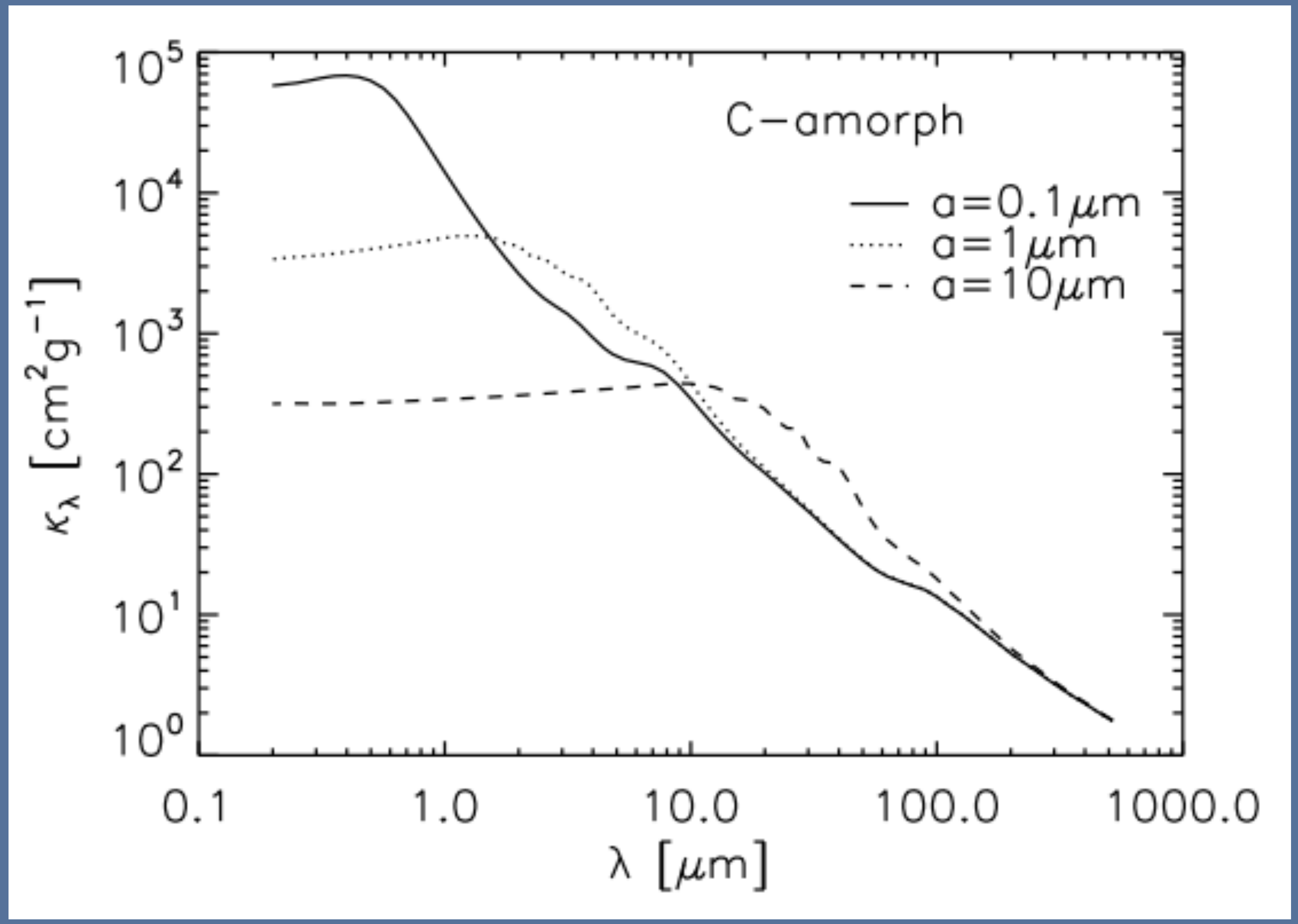
1.2.2 Opacities of carbonaceous grains

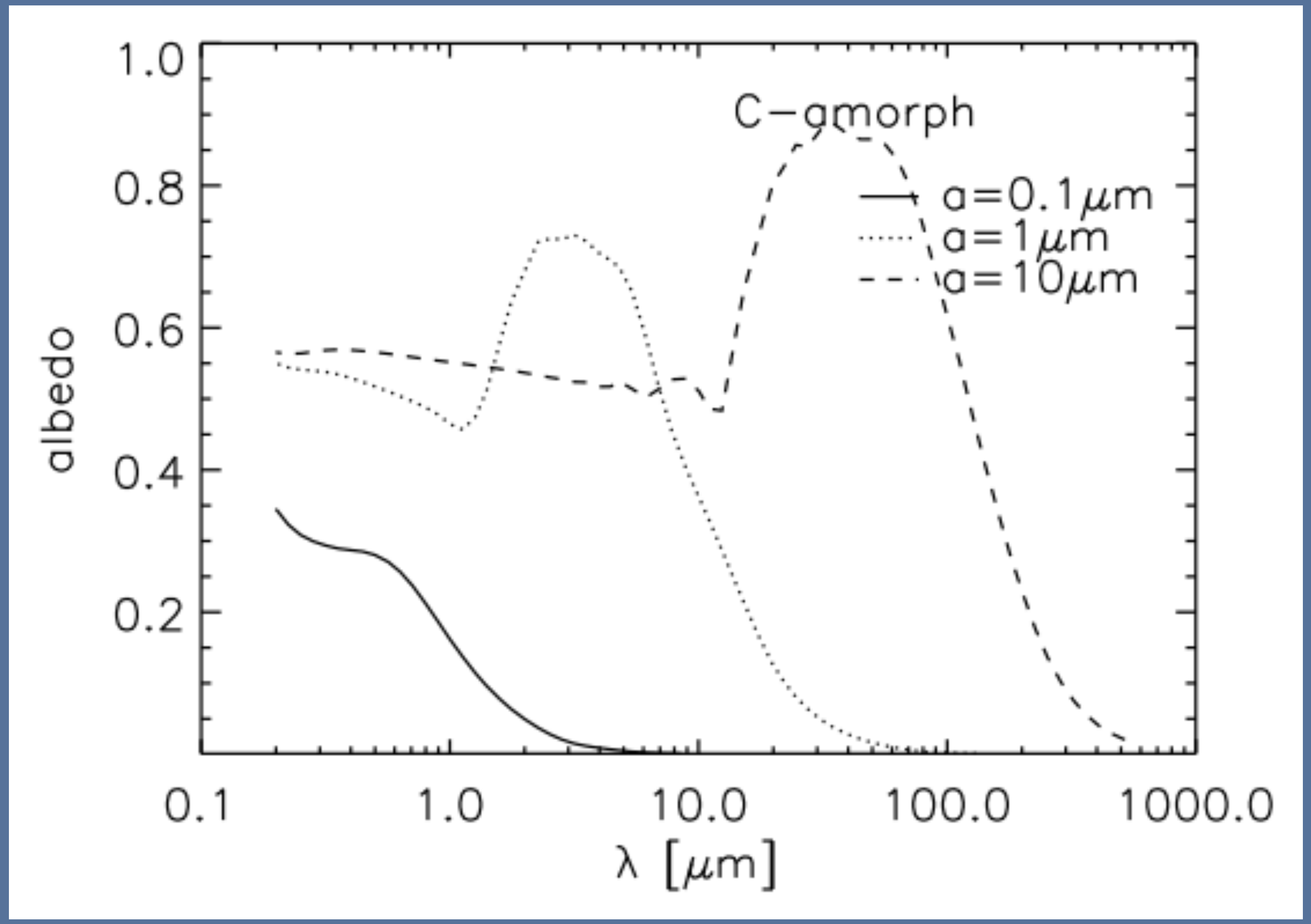
- Solid carbon is another major component of interstellar dust.
- Carbonaceous grains are found in several different structures: PAH (Polycyclic aromatic hydrocarbons), graphite, nanodiamonds, small amorphous grains, or complex organic compounds

Examples of PAH



1.2.2 Opacities of carbonaceous grains





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- The opacities of amrophous carbon are much more “simple” than those of silicates: there is no spectral band. The opacity curves follow the Ivezic model.
- The albedo peaks around $\lambda = 2\pi a$ and decreases very fast with λ
- Carbon opacities do not show a dip in the NIR, contrary to the case of silicates. This may indicate that the opacity observed in the NIR is dominated by carbon
- This has important consequences on thermal equilibrium of dust particles

1.3 Models of astrophysical dust mixtures

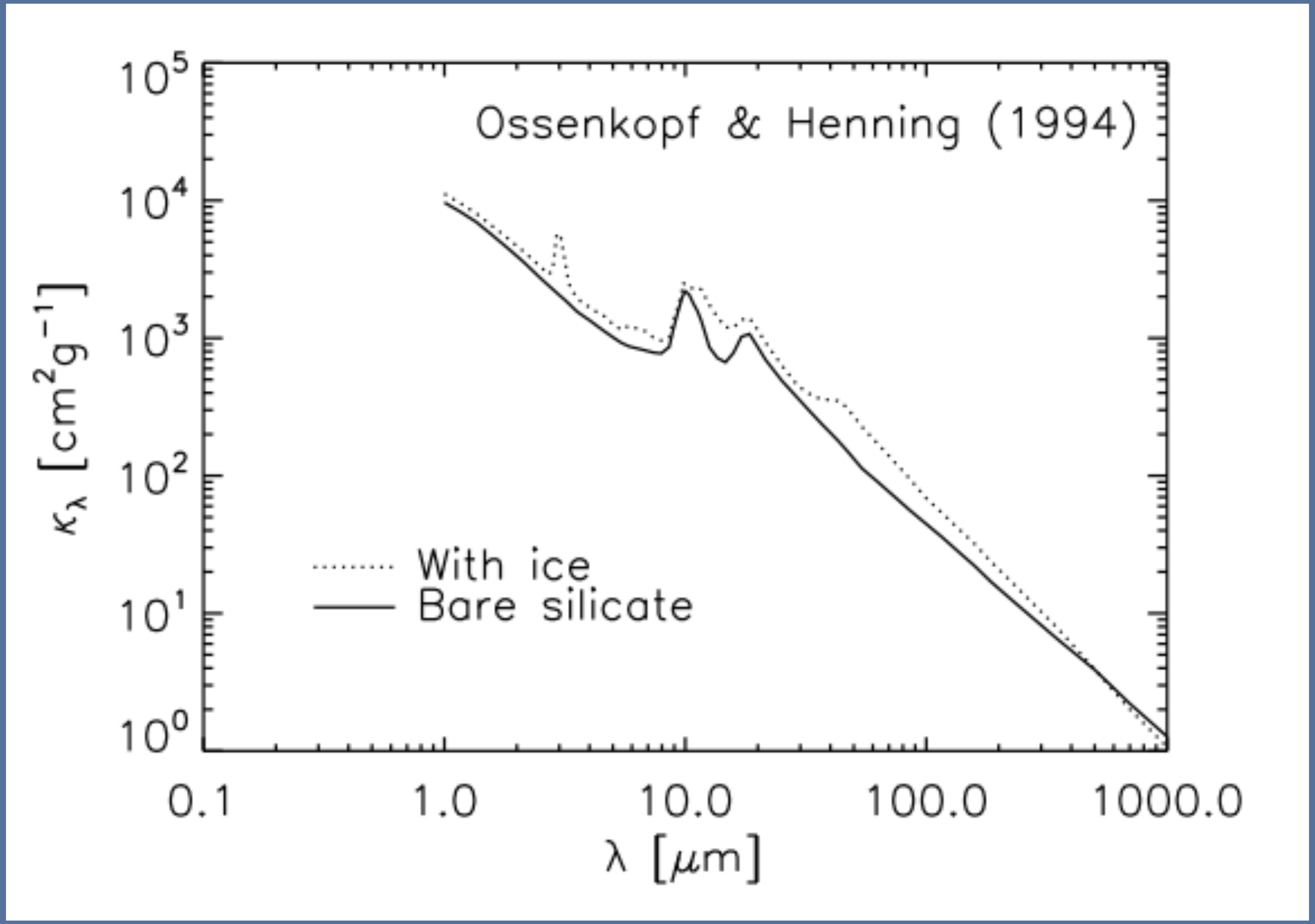
- Up to now we have only considered particles of one type at a time
- In astrophysical environments, this is not the case
  - Grains can have a mixed composition, either because they were formed so, or because they have coagulated to form a small agregate.
  - Moreover in dense regions of molecular clouds, molecules can condensate on grain surfaces to form ice mantles. The most abundant species are H_2O, CO, CO_2, NH_3, CH_4 and organic matter.
  - Several types of grain populations can be found, with different compositions or sizes
- The lack of knowledge of the properties of grain mixtures is a major difficulty in the analysis of thermal dust continuum emission

1.3 Models of astrophysical dust mixtures

- After assuming properties for a grain mixture, the calculation of the opacities for such grains is a difficult task
- Several studies discuss models of more realistic grains, the most widely used are
  - Draine & Lee 1984, ApJ, 285, 89
  - Ossenkopf & Henning 1994, A&A, 291, 943
  - Jones et al. 2017, A&A, 602, A46: THEMIS model

1.3 Models of astrophysical dust mixtures

- The model of Ossenkopf & Henning calculates the optical properties of fractal agregates for dense star forming regions
- The silicate absorption bands are much less apparent than for pure silicates
- Many models of dense cores have confirmed this characteristics of the Ossenkopf & Henning model, which agrees better with observations than models of pure silicates

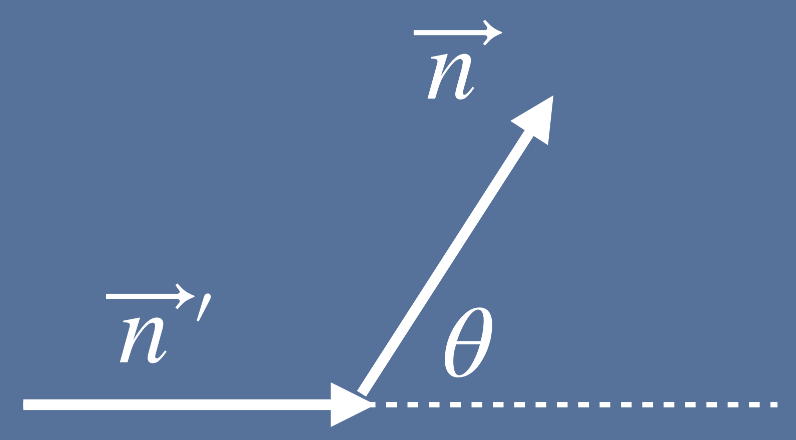


1.4 Scattering phase function

- The previous chapter considered isotropic scattering
- For dust grains, scattering is generally anisotropic
- To describe the scattering in such a case, a scattering phase function is defined: $\phi(\vec{n}, \vec{n}', \vec{x}, \lambda)$
- The scattering phase function gives the probability for a photon originally propagating in direction $\vec{n}'$ to be scattered at position $\vec{x}$ in the direction $\vec{n}$
- This function is normalised: $\displaystyle\int_{4\pi} \phi(\vec{n}, \vec{n}', \vec{x}, \lambda)\, d\Omega = 1$
- In the isotropic case, $\phi(\vec{n}, \vec{n}', \vec{x}, \lambda) = \dfrac{1}{4\pi}$
- It is also possible to define $\phi(\vec{n}, \vec{n}', \vec{x}, \lambda)$ per steradian, in which case we have $\phi(\vec{n}, \vec{n}', \vec{x}, \lambda) = 1$ in the isotropic case

1.4 Scattering phase function

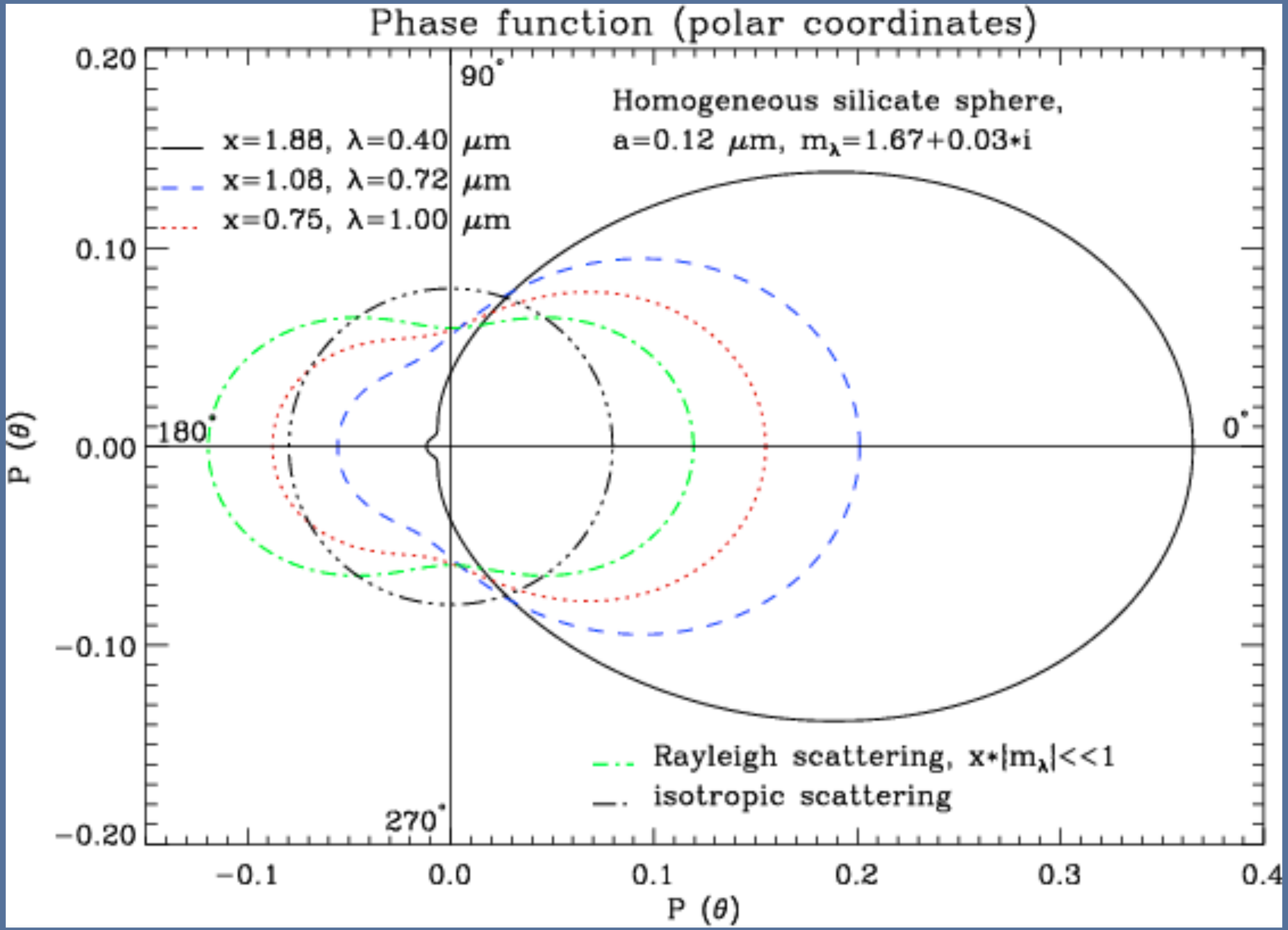
- We can also define $p(\mu)$ the scattering probability in the direction $\mu = \cos\theta$, where θ is the deflection angle with respects to the direction of the incident photon.



- If a photon moves in a direction $\vec{n}'$ and is scattered in a direction $\vec{n}$, $\mu = \cos\theta = \vec{n}' \cdot \vec{n}$
- The normalisation is written $\int_{-1}^{+1} p(\mu)\, d\mu = 1$
- In the isotropic case, $p(\mu) = \dfrac{1}{2}$

1.4 Scattering phase function

- The scattering phase function for realistic particles can have a complex form
- In the visible wavelength range, the scattering is peaked in the forward direction, but approaches the Rayleigh scattering limit where the ratio between forward and perpendicular scattered intensities becomes 2, for longer wavelengths.
- Generally when $\lambda < 2\pi a$, the scattering is peaked forwards



Steinacker et al. 2002

m_λ: mean complex refractive index for all 3 wavelengths

$x = 2\pi a/\lambda$: size parameter

1.4 Scattering phase function

- The parameter g is used to characterise the shape of the phase function:

$$g = < \mu > = \int_{-1}^{+1} p(\mu)\mu \, d\mu$$

- To solve the transfer with anisotropic scattering, we would have to calculate the function $p(\mu)$ at each frequency.
- To make this easier, an approximation is sometimes used with the phase function of Henyey-Greenstein

$$p_g(\mu) = \frac{1}{2} \frac{1 - g^2}{(1 + g^2 - 2g\mu)^{3/2}}$$

- This means that when we calculate the opacity tables, we need at each frequency $\kappa_\nu^{\rm abs}$, $\kappa_\nu^{\rm sca}$, and g_ν.
- g_ν is calculated with the exact phase function from $g = < \mu >$. In the RT calculation, $p_g(\mu)$ is used.
- This is still an approximation, but it is better than the isotropic approximation.

2. Dust thermal emission in the RT equation

We are now going to see the specificities of radiative transfer in the presence of interstellar dust, how to write the transfer equation, how to solve it (and where the main difficulties are)

2.1 Transfer equation

- We have already seen the transfer equation in the presence of scattering. We had made some assumptions:
  - monochromatic case (no frequency redistribution)
  - Isotropic scattering
- For radiative transfer with dust, these two hypotheses are no longer valid (but the overall treatment of scattering remains correct)
- We are going to describe in detail each term of the radiative transfer equation, first in the case of one single type of dust particles, then in the case of a mixture

2.1.1 For one type of dust

- In what follows, we are going to use κ_ν, as is often the case for radiative transfer in dusty media. Switching to $\alpha_\nu = \rho\,\kappa_\nu$ is trivial.
- The general transfer equation is

$$\frac{dI_\nu}{ds} = -\,\kappa_\nu(s)\,\rho\,I_\nu(s) + j_\nu(s)$$

where we have not given more details about the different terms. This is what we are going to do now

2.1.1 For one type of dust

- Primary absorption and emission: these are two important and obvious processes that have to be taken into account for dusty media.
    - Primary emission takes into account the radiative energy added to the radiation field, often stellar emission, but also the radiation from an AGN, spectral lines from ionised gas, Bremsstrahlung, i.e. everything that can inject radiation in the medium
    - It can be described by the function $j^*_\nu(\overrightarrow{x}, \overrightarrow{n})$
    - Absorption is the process we talked about previously, for which radiation is turned into internal energy by the dust grains, and is characterised by the (mass) absorption coefficient $\kappa_\nu^{\rm abs}$ (or $\alpha_\nu^{\rm abs}$)
    - Taking into account both processes (primary emission and absorption), the transfer equation becomes:

$$\frac{dI_\nu(\overrightarrow{x}, \overrightarrow{n})}{ds} = -\kappa_\nu^{\rm abs}(\overrightarrow{x})\,\rho(\overrightarrow{x})\,I_\nu(\overrightarrow{x}, \overrightarrow{n}) + j^*_\nu(\overrightarrow{x}, \overrightarrow{n})$$

Simple first order differential equation

2.1.1 For one type of dust

- Scattering:
  - scattering, just like absorption, removes photons from the beam and is considered as an additional sink term in the transfer equation, with an efficiency given by the scattering coefficient κ_ν^{sca} (ou α_ν^{sca}).
  - In this case, radiation is not converted into internal energy but is reemitted in another direction
  - Scattering is therefore not only described by a second sink term, but also by a second source term
  - The scattering phase function $\phi(\vec{n}, \vec{n}', \vec{x}, \lambda)$ gives the probability for photons initially propagating in direction $\vec{n}'$ and scattered at position $\vec{x}$ to propagate in a new direction $\vec{n}$ after scattering. Normalisation yields: $\int_{4\pi} \phi(\vec{n}', \vec{n}, \vec{x}, \nu) d\Omega = 1$

$$\frac{dI_\nu(\vec{x}, \vec{n})}{ds} = -\kappa_\nu^{\text{abs}}(\vec{x})\,\rho(\vec{x})\,I_\nu(\vec{x}, \vec{n}) + j_\nu^*(\vec{x}, \vec{n})$$

Simple first order differential equation

2.1.1 For one type of dust

- With those two additional terms, the radiative transfer equation becomes

$$\frac{dI_\nu(\vec{x},\vec{n})}{ds} = -\kappa_\nu^{\rm ext}(\vec{x})\,\rho(\vec{x})\,I_\nu(\vec{x},\vec{n}) + j_\nu^*(\vec{x},\vec{n}) + \kappa_\nu^{sca}(\vec{x})\,\rho(\vec{x})\int_{4\pi}\phi(\vec{n},\vec{n}',\vec{x},\lambda)\,I_\nu(\vec{x},\vec{n}')\,d\Omega'$$

with $\kappa_\nu^{\rm ext} = \kappa_\nu^{\rm abs} + \kappa_\nu^{\rm sca}$

- The transfer equation has now become an equation where the radiation fields at all positions and in all directions are coupled. This equation is even more complex than that seen in the previous chapter because scattering by dust is anisotropic

- For wavelengths from the UV to the NIR, the albedo is at least 50% and scattering by dust is highly anisotropic

- In general, even in the MIR for which scattering by "classical" interstellar grains is small, it is important to take scattering into account to calculate correctly the heating by dust grains

2.1.1 For one type of dust

- Dust emission:
  - In addition to primary emission, absorption and scattering, a 4th process has to be taken into account, the thermal emission of the dust
  - Dust grains that absorb the radiation can reemit the stored radiative energy at wavelengths usually larger than 1 μm. It is therefore necessary to take this term $j_\nu^{\mathrm{dust}}(\vec{x})$ into account in the radiative transfer equation

$$\frac{dI_\nu(\vec{x},\vec{n})}{ds} = -\kappa_\nu^{\mathrm{ext}}(\vec{x})\,\rho(\vec{x})\,I_\nu(\vec{x},\vec{n}) + j_\nu^{*}(\vec{x},\vec{n}) + j_\nu^{\mathrm{dust}}(\vec{x},\vec{n}) + \kappa_\nu^{sca}(\vec{x})\,\rho(\vec{x})\int_{4\pi}\phi(\vec{n},\vec{n}',\vec{x},\lambda)\,I_\nu(\vec{x},\vec{n}')\,d\Omega'$$

  - Dust emission can simply be considered as an additional source term with respects to primary emission
  - Its exact form depends on the emission process and this term often depends on the intensity of the radiation field, in a non linear and non trivial way

2.1.1 For one type of dust

- One common hypothesis is that dust grains are in thermal equilibrium with the local radiation field
- The grain emissivity can then be described by a modified blackbody emission at temperature $T(\overrightarrow{x})$
- $j_\nu^{\rm dust}(\overrightarrow{x}) = \kappa_\nu^{\rm abs}\,\rho(\overrightarrow{x})\,B_\nu(T(\overrightarrow{x}))$
- The name “modified black body” comes from the presence of the absorption coefficient (often dependent on ν) in front of the Planck function
- The equilibrium temperature is determined by the condition that the absorbed energy is equal to the emitted energy

2.1.1 For one type of dust

$$\int_0^\infty \kappa_\nu^{\text{abs}} J_\nu(\overrightarrow{x})\, d\nu = \int_0^\infty \kappa_\nu^{\text{abs}} B_\nu(T(\overrightarrow{x}))\, d\nu$$ with J_ν the mean intensity

Absorbed energy reemitted energy

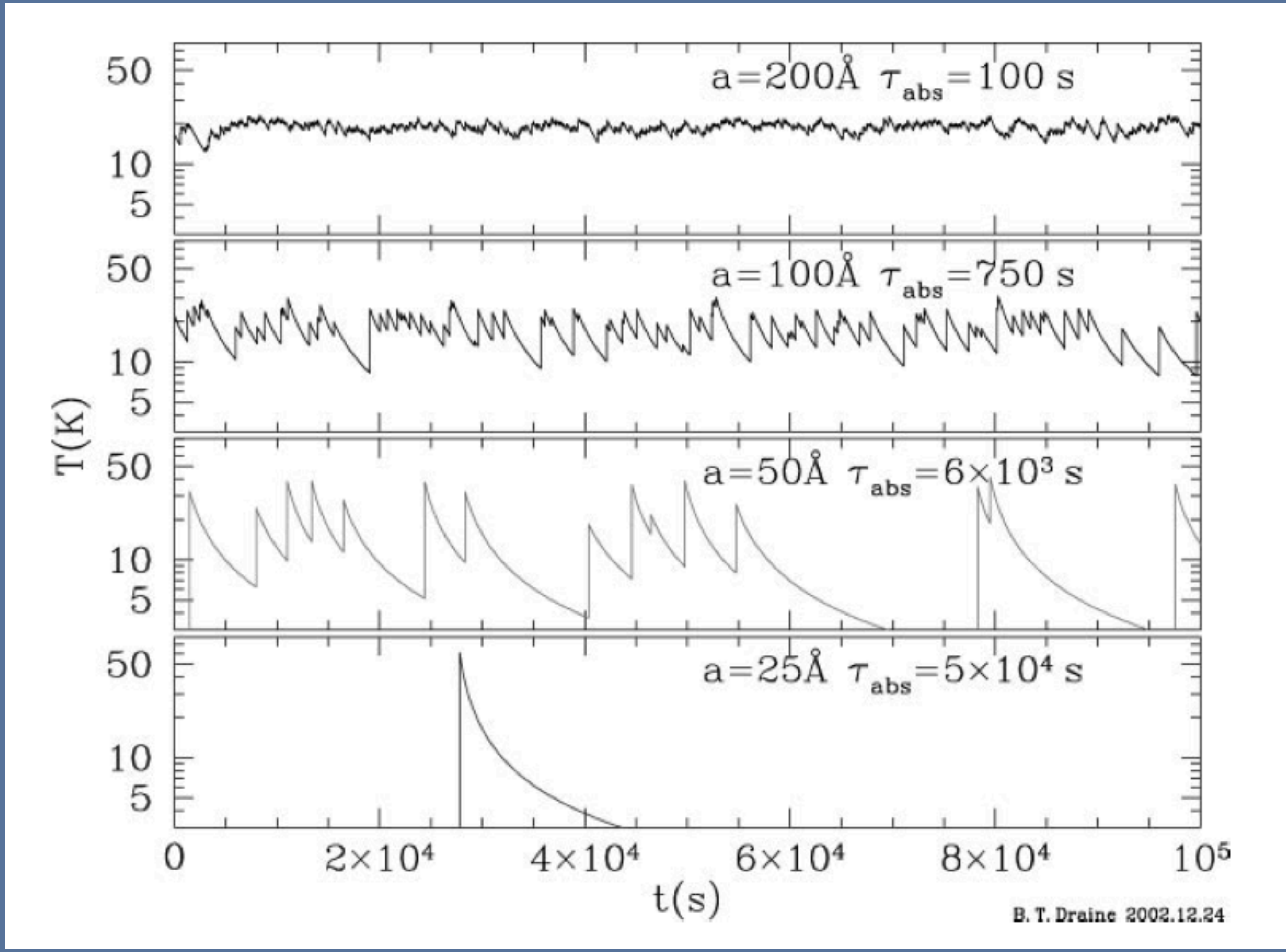
- The above equation highlights a difficulty of radiative transfer: the coupling in frequency.
- It is the total energy (i.e. integrated over frequencies) that is conserved, and the problem can no longer be considered monochromatic

2.1.1 For one type of dust

- The thermal equilibrium hypothesis for grains works well for “big” grains, but not in the case grains are small (nanograins) or for PAHs.
- Big grains can reach thermal equilibrium and emit like modified blackbodies at the temperature of equilibrium
- Small grains have a small heat capacity and the absorption of only one UV or visible photon can lead to a high temperature increase
- Small grains do not reach an equilibrium temperature but instead undergo temperature fluctuations that lead to emission at temperatures much higher than the equilibrium temperature
- The out of equilibrium emission of small grains is necessary to explain the MIR emission observed in many objects

2.1.1 For one type of dust

Temperature of stochastically heated grains



A day in the life of four carbonaceous grains, heated by the local interstellar radiation field. τ_{abs} is the mean time between photon absorptions (Draine 2003)

2.1.1 For one type of dust

- To take into account the emission of transiently heated grains, we can write the dust emissivity as follows:
- $j_\nu^{\rm dust}(\overrightarrow{x}) = \kappa_\nu^{\rm abs}\,\rho(\overrightarrow{x}) \int_0^\infty P(T,\overrightarrow{x})\,B_\nu(T)\,dT$
  where $P(T,\overrightarrow{x})$ is the grain temperature distribution at position $\overrightarrow{x}$
- The temperature distribution depends on the chemical composition and sizes of the grains, but also on the intensity and the spectrum of the radiation field.
- This term is a complex, non-linear function of the specific intensity, which adds up to the difficulty of radiative transfer
- Method to calculate the temperature distribution can be found in, e.g., Dwek (1986), Draine & Li (2001), Compiègne et al. (2011)

2.1.2 For a dust mixture

- In the inter/circumstellar medium different types of grains can be found, with different chemical compositions, sizes, shapes and densities

- Each type of grain i is characterised by its own absorption coefficient $\kappa_{\nu,i}^{\rm abs}$, its scattering coefficient $\kappa_{\nu,i}^{\rm sca}$ and its scattering phase function $\phi_i(\vec{n}, \vec{n}', \vec{x}, \lambda)$

- Let us denote $w_i(\vec{x})$ the relative contribution of each type of grain i at location $\vec{x}$ to the total density.

- The transfer equation is then:

$$\frac{dI_\nu(\vec{x}, \vec{n})}{ds} = -\sum_i w_i(\vec{x})\kappa_{\nu,i}^{\rm ext}(\vec{x})\,\rho(\vec{x})\,I_\nu(\vec{x}, \vec{n}) + j_\nu^*(\vec{x}, \vec{n}) + j_\nu^{\rm dust}(\vec{x}, \vec{n})$$
$$+\sum_i w_i(\vec{x})\,\kappa_{\nu,i}^{sca}(\vec{x})\,\rho(\vec{x})\int_{4\pi}\phi_i(\vec{n}, \vec{n}', \vec{x}, \lambda)\,I_\nu(\vec{x}, \vec{n}')\,d\Omega'$$

2.1.2 For a dust mixture

- This equation is identical to the previous one when the following quantities are defined

$$\kappa_\nu^{\mathrm{abs}}(\vec{x}) = \sum_i w_i(\vec{x})\,\kappa_{\nu,i}^{\mathrm{abs}}$$

$$\kappa_\nu^{\mathrm{sca}}(\vec{x}) = \sum_i w_i(\vec{x})\,\kappa_{\nu,i}^{\mathrm{sca}}$$

$$\kappa_\nu^{\mathrm{ext}}(\vec{x}) = \sum_i w_i(\vec{x})\,\kappa_{\nu,i}^{\mathrm{ext}}$$

And for the phase function

$$\phi(\vec{n}, \vec{n}', \vec{x}, \nu) = \frac{\sum_i w_i(\vec{x})\,\kappa_{\nu,i}^{\mathrm{sca}}\,\phi_i(\vec{n}, \vec{n}', \vec{x}, \nu)}{\sum_i w_i(\vec{x})\,\kappa_{\nu,i}^{\mathrm{sca}}}$$

- As far as primary emission, absorption and scattering are concerned, RT for dust mixtures is identical to transfer in a medium with one type of average particles

- No approximation is necessary

- What dimension for w_i? Do we have $\sum_i w_i = 1$?

2.1.2 For a dust mixture

- For a dust mixture the expression of dust emissivity is

$$j_\nu^{\text{dust}}(\overrightarrow{x}) = \sum_i w_i(\overrightarrow{x})\,\kappa_{\nu,i}^{\text{abs}}\,\rho(\overrightarrow{x})\,B_\nu(T_i(\overrightarrow{x}))$$

- The emissivity of a grain population i is a modified blackbody at temperature $T_i(\overrightarrow{x})$.

- The temperature $T_i(\overrightarrow{x})$ is determined as before with

$$\int_0^\infty \kappa_{\nu,i}^{\text{abs}}\,J_\nu(\overrightarrow{x})\,d\nu = \int_0^\infty \kappa_{\nu,i}^{\text{abs}}\,B_\nu(T_i(\overrightarrow{x}))\,d\nu$$

- At location $\overrightarrow{x}$, grains of different sizes or compositions will have different temperatures

2.1.2 For a dust mixture

- In what precedes, it is easy to combine absorption, scattering and extinction coefficients of the various types of grains in the RT equation without approximation
- This is no longer the case for the thermal reemission term
- Even though it is possible to calculate an average temperature for the different grains, this would result in a reduction of the complexity due to the grain mixture to only one average grain type
- This would be a physically incorrect simplification of the transfer problem (note that it could still be sufficient, useful or necessary depending on the application)

2.1.2 For a dust mixture

- For stochastically heated grains, the emissivity of the dust becomes

$$j_\nu^{\text{dust}}(\overrightarrow{x}) = \sum_i w_i(\overrightarrow{x})\, \kappa_{\nu,i}^{\text{abs}}\, \rho(\overrightarrow{x}) \int_0^\infty P_i(T, \overrightarrow{x})\, B_\nu(T)\, dT$$

- $P_i(T, \overrightarrow{x})$ is the temperature distribution of the grains i at location $\overrightarrow{x}$

2.2 Radiative transfer for dust of known temperature

- We are first going to focus on a very simple case, which is that when the dust temperature is known
- This is a very useful application when we need to determine physical quantities (column density, mass) of an object from its dust emission
- Indeed, given a gas-to-dust ratio, dust emission can be used as a proxy for the amount of gas, in particular when H_2 does not emit.
- In fact, the dust temperature is rarely known, but it can be estimated if we have an idea of the object's environment (protostellar envelope, protoplanetary disk, prestellar core, etc.)
- We will also assume that we can neglect scattering
- Under which circumstances can we neglect scattering?

2.2 Radiative transfer for dust of known temperature

- To simplify the problem further, we will assume a homogeneous medium with no background radiation at the wavelength of interest.
- In reality there has to be a radiation field, which is responsible for the dust temperature. We will ignore it in this application.
- The transfer equation is $\frac{dI_\nu}{ds} = -\kappa_\nu^{\text{abs}}\,\rho\, I_\nu + j_\nu^{\text{dust}}$ with $j_\nu^{\text{dust}} = \kappa_\nu^{\text{abs}}\,\rho\, B_\nu(T_{\text{dust}})$

$$\Rightarrow \quad \frac{dI_\nu}{ds} = -\kappa_\nu^{\text{abs}}\rho\ [I_\nu - B_\nu(T_{\text{dust}})]$$

- We have already solved this equation: $\frac{dI_\nu}{d\tau_\nu} = I_\nu - B_\nu(T_{\text{dust}})$, with $d\tau_\nu = -\kappa_\nu^{\text{abs}}\,\rho\, ds$

2.2 Radiative transfer for dust of known temperature

- The solution is: $I_\nu(\tau_\nu) = B_\nu(T_{\rm dust})(1 - {\rm e}^{-\tau_\nu})$
- Having measured I_ν, we can derive τ_ν in the optically thin case
- The optically thick case is generally not interesting because it underestimates the column density)
- In the (sub)millimetre regime (emission of cold dust), the dust emission is rarely optically thick, except maybe at high resolution or towards high-mass star forming regions
- Therefore, $I_\nu^{\rm obs} = I_\nu(\tau_\nu) \simeq \tau_\nu\, B_\nu(T_{\rm dust})$
- From the optical depth, we can derive the column density: $\tau_\nu = \kappa_\nu^{\rm abs}\, \rho\, D$

  where D is the medium's thickness and ρ the dust density (in ${\rm g\,cm^{-3}}$)

2.2 Radiative transfer for dust of known temperature

- We want to determine the H_2 column density N_{H_2}
- $N_{H_2} = \int n_{H_2} \, ds$, where n_{H_2} is the number density of hydrogen molecules
- We define μ the mean molecular weight per hydrogen molecule
  - $\mu \, m_H \, \mathcal{N}(H_2) = \mathcal{M}$, where $\mathcal{M}$ is the total mass of a volume containing $\mathcal{N}(H_2)$ molecular hydrogen molecules.
  - m_H is the mass of a hydrogen atom
  - μ takes into account the fact that the gas contains H_2, He and other heavy atoms
  - $\mu \sim 2.8$ (Kauffmann et al. 2008)

2.2 Radiative transfer for dust of known temperature

- We can write again the H_2 column density
- $N_{H_2} = \int \frac{\rho_{\rm gas}}{\mu\, m_H}\, ds = \frac{1}{\mu\, m_H\, \kappa_\nu^{\rm abs}}\, \mathscr{R} \int \rho\, \kappa_\nu^{\rm abs}\, ds$
    - $\rho_{\rm gas}$ is the gas (mass) density in $\rm g\, cm^{-3}$ (ie taking H_2, He , etc. into account)
    - ρ is the dust (mass) density in $\rm g\, cm^{-3}$
    - $\mathscr{R}$ is the mass gas-to-dust ratio, ie $\mathscr{R} = \frac{\rho_{\rm gas}}{\rho}$. Its value is around 100 in the ISM.
    - Note that sometimes, $\mathscr{R}$ is included in the definition of $\kappa_\nu^{\rm abs}$, and in this case, ρ in the RT equation is $\rho_{\rm gas}$

2.2 Radiative transfer for dust of known temperature

- In this case, $N_{H_2} = \dfrac{1}{\mu\, m_H\, \kappa_\nu^{\rm abs}} \displaystyle\int \rho_{\rm gas}\, \kappa_\nu^{\rm abs}\, ds$
    - $N_{H_2} = \dfrac{\tau_\nu}{\mu\, m_H\, \kappa_\nu^{\rm abs}} = \dfrac{I_\nu^{\rm obs}}{\mu\, m_H\, \kappa_\nu^{\rm abs}\; B_\nu(T_{\rm dust})}$
    - Units: sometimes $I_\nu^{\rm obs}$ can be given in units like mJy/beam. The size of the beam (in sr) has then to be taken into account
    - $\mu\, m_H$ is not the mass of a hydrogen molecule but the gas mass per hydrogen molecule
    - This equation can be used in practical cases: an intensity of 13 mJy/beam has been measured at $\lambda = 1.3\,\mathrm{mm}$. The telescope beam is $13''$. The mass absorption coefficient $\kappa_\lambda^{\rm abs}$ at this wavelength is $0.005\,\mathrm{cm}^2\,\mathrm{g}^{-1}$. What is the H_2 column density, if we assume a temperature of 10 K?

2.2 Radiative transfer for dust of known temperature

Estimating the gas mass

- The mass is obtained by integrating the column density over the source

$$M = \mu m_{\rm H} \int N_{\rm H_2} dA$$

dA: surface element

$$= \frac{1}{\kappa_\nu^{\rm abs} B_\nu(T_{\rm dust})} \int I_\nu^{\rm obs} \, dA$$

- If d is the distance to the source, we have $dA = d^2 \, d\Omega$, with $d\Omega$ the solid angle element

$$M = \frac{d^2}{\kappa_\nu^{\rm abs} B_\nu(T_{\rm dust})} \int I_\nu^{\rm obs} \, d\Omega = \frac{d^2 \, F_\nu}{\kappa_\nu^{\rm abs} B_\nu(T_{\rm dust})}$$

- F_ν is the flux in the solid angle subtended by the source

2.3 Determining the temperature with radiative transfer

- In the previous section, we assumed that we already knew the temperature. In fact, this is rarely the case
- If we make a small error in the determination of the temperature, we risk obtaining a spectrum that violates energy conservation
- For example if we overestimate the temperature by a factor of 2, we make an error of a factor of $2^4 = 16$ in the energy
- In many cases, dust emission comes from the absorption by the dust of radiation emitted by neighbouring stars. Such an overestimate would mean that the dust radiates 16 times more energy than it receives! Spectral energy distributions would be completely wrong
- Even an error of 20% on the dust temperature leads to a factor of 2 on the energy emitted by dust
- Obeying the energy conservation law is fundamental. It is important to calculate dust temperature self consistently with radiative transfer

2.3.1 Dust grain at radiative equilibrium

- Already seen in section §2.1. We consider big grains at equilibrium with the radiation field
- The heating and cooling rates per dust mass have to be equal
  - Heating: $Q_+ = \int_0^\infty \kappa_\nu^{\rm abs} \, J_\nu \, d\nu$
  - Cooling: $Q_- = \int_0^\infty \kappa_\nu^{\rm abs} \, B_\nu(T_d) \, d\nu$
- In the general case, J_ν contains I_ν that can depend on the dust temperature
- We will start with a simple case in which the dust ist optically thin and illuminated by a star

2.3.1 Dust grain at radiative equilibrium

- Assume F_ν^* is the stellar flux at frequency ν.
- The heating rate is: $Q_+ = \int_0^\infty \kappa_\nu^{\mathrm{abs}} F_\nu^* \, d\nu$

  Watch out, it is not the same dimension as before

- The cooling rate is: $Q_- = 4\pi \int_0^\infty \kappa_\nu^{\mathrm{abs}} B_\nu(T_d) \, d\nu$

  The factor 4π comes from the fact that the energy is emitted in all directions (integration over the solid angle to have the same dimension as the stellar flux)

- At radiative equilibrium: $Q_+ = Q_-$

$$4\pi \int_0^\infty \kappa_\nu^{\mathrm{abs}} B_\nu(T_d) \, d\nu = \int_0^\infty \kappa_\nu^{\mathrm{abs}} F_\nu^* \, d\nu$$

2.3.1 Dust grain at radiative equilibrium

- This equation can be solved numerically in an iterative fashion: for each iteration on T_d a complete integral over ν has to be calculated, which makes solving the radiative transfer very cumbersome

- It is also possible to tabulate $Q\_(T_d)$ and then while solving the RT calculate Q_+, and look for the zero of the expression $Q\_(T_d) - Q_+$ using the table (possibly interpolating the value for more precision)

- Another method uses the mean Planck opacity:
  - $\kappa_P^{\text{abs}}(T) = \dfrac{\int_0^\infty \kappa_\nu^{abs} B_\nu(T)\, d\nu}{\int_0^\infty B_\nu(T)\, d\nu} = \left(\dfrac{\sigma}{\pi} T^4\right)^{-1} \displaystyle\int_0^\infty \kappa_\nu^{abs} B_\nu(T)\, d\nu$
  - It is the mean opacity weighted by the Planck function at temperature T_d

2.3.1 Dust grain at radiative equilibrium

- We can now rewrite the thermal equilibrium equation:

$$4\kappa_P(T_d)\,\sigma\,T_d^4 = \int_0^\infty \kappa_\nu^{abs}\,F_\nu^*\,d\nu$$

- This quantity enables us to calculate the transfer equation rapidly: first, the $\kappa_P(T_d)$ values are tabulated in advance. Then after a first estimate of T_d, $\kappa_P(T_d)$ is calculated and the following equation is solved for, yielding a new estimate of T_d:

$$T_d = \left(\frac{1}{4\kappa_P(T_d)\,\sigma}\int_0^\infty \kappa_\nu^{abs}\,F_\nu^*\,d\nu\right)^{\frac{1}{4}}$$

- With the new T_d value, we calculate a new estimate of $\kappa_P(T_d)$, and solve for T_d using the equation above, etc., until convergence.

- Convergence is usually obtained within a few iterations

2.3.1 Dust grain at radiative equilibrium

- We now suppose that the star has a radius $R_* \ll r$ and emits like a perfect blackbody at T_*.

  The flux received at a distance r is: $F_\nu^* = \dfrac{4\pi R_*^2 \pi B_\nu(T)}{4\pi r^2}$

- The equilibrium equation yields

$$4\kappa_P(T_d)\,\sigma T_d^4 = \frac{\pi R_*^2}{r^2}\int_0^\infty \kappa_\nu^{abs} B_\nu(T_*)\,d\nu = \frac{\pi R_*^2}{r^2}\kappa_P(T_*)\frac{\sigma}{\pi}T_*^4$$

$$\Rightarrow T_d = \sqrt{\frac{R_*}{2r}}\left(\frac{\kappa_P(T_d)}{\kappa_P(T_*)}\right)^{\frac{1}{4}} T_*$$

- This is also iteratively solved, with a fast convergence

2.3.1 Dust grain at radiative equilibrium

- We define the thermal cooling efficiency factor: $\epsilon = \dfrac{\kappa_P(T_d)}{\kappa_P(T_*)}$

$$\Rightarrow T_d = \sqrt{\frac{R_*}{2r}} \frac{1}{\epsilon^{1/4}} T_*$$

- If $\epsilon < 1$ the cooling is less efficient than stellar radiation absorption
- Typically for small grains $\epsilon < 1$ and for big grains ($> 100\mu\mathrm{m}$) $\epsilon \simeq 1$, so large grains are cooler than small grains
- Small silicates in a radiation field are usually cooler than small carbonaceous grains, because carbonaceous grains have a higher opacity in the visible and NIR (and therefore absorb stellar radiation better)
- We can also imagine that carbon monomers coagulating on silicate monomers can help heating the silicates
- $\epsilon = 1$ is the “grey” case, which is like having $\kappa_\nu^{\mathrm{abs}}$ independent of ν. In this case κ_P is independent of T

2.3.3 Thermal radiative transfer and optical depth effects

- If the dusty medium is very optically thin, the temperature of the dust is given by the equations in the previous sections
- Optical depth can nevertheless play a role in many cases, with two main consequences
  - If the optical depth at the wavelength of the stellar radiation (typically in the visible, NIR, or even UV) is not negligible, the stellar radiation will be attenuated. As a consequence, the dust that is shielded by the direct stellar radiation will be cooler than given by the previous equations in the optically thin case
  - If the optical depth at the wavelength of the dust thermal emission is not negligible, the radiation emitted by a grain can be reabsorbed by another grain elsewhere in the medium.
    - The radiative energy cannot immediately escape and can be absorbed and reemitted several times before leaving the medium.
    - The cooling of one regions leads to the heating of another one, and vice-versa.
    - Thermal radiative transfer has therefore a non-local character. Because we do not know in advance the temperature of the other regions of the medium, we do not know which heating to expect

2.3.3 Thermal radiative transfer and optical depth effects

- The extinction of the stellar flux by dust is easy to take into account: $F_\nu^* = \dfrac{L_\nu^*}{4\pi\, r^2}\,\mathrm{e}^{-\tau_\nu^*}$

  τ_ν^*: optical depth towards the star

  L_ν^*: mnochromatic luminosity of the star

  We have assumed that the star is a point source. The above term corresponds to $I_\nu(0)\,\mathrm{e}^{-\tau_\nu}$ when integrating the transfer equation $\dfrac{dI_\nu}{ds} = -\,\kappa_\nu^{\mathrm{abs}}\rho\; I_\nu + j_\nu^*$

- The second effect is more difficult. Another term has to be added to the radiative equilibrium equation

$$4\pi \int_0^\infty \kappa_\nu^{\mathrm{abs}}\; B_\nu(T_d) d\nu = \int_0^\infty \kappa_\nu^{\mathrm{abs}}(F_\nu^* + 4\pi\, J_\nu^d)\; d\nu$$

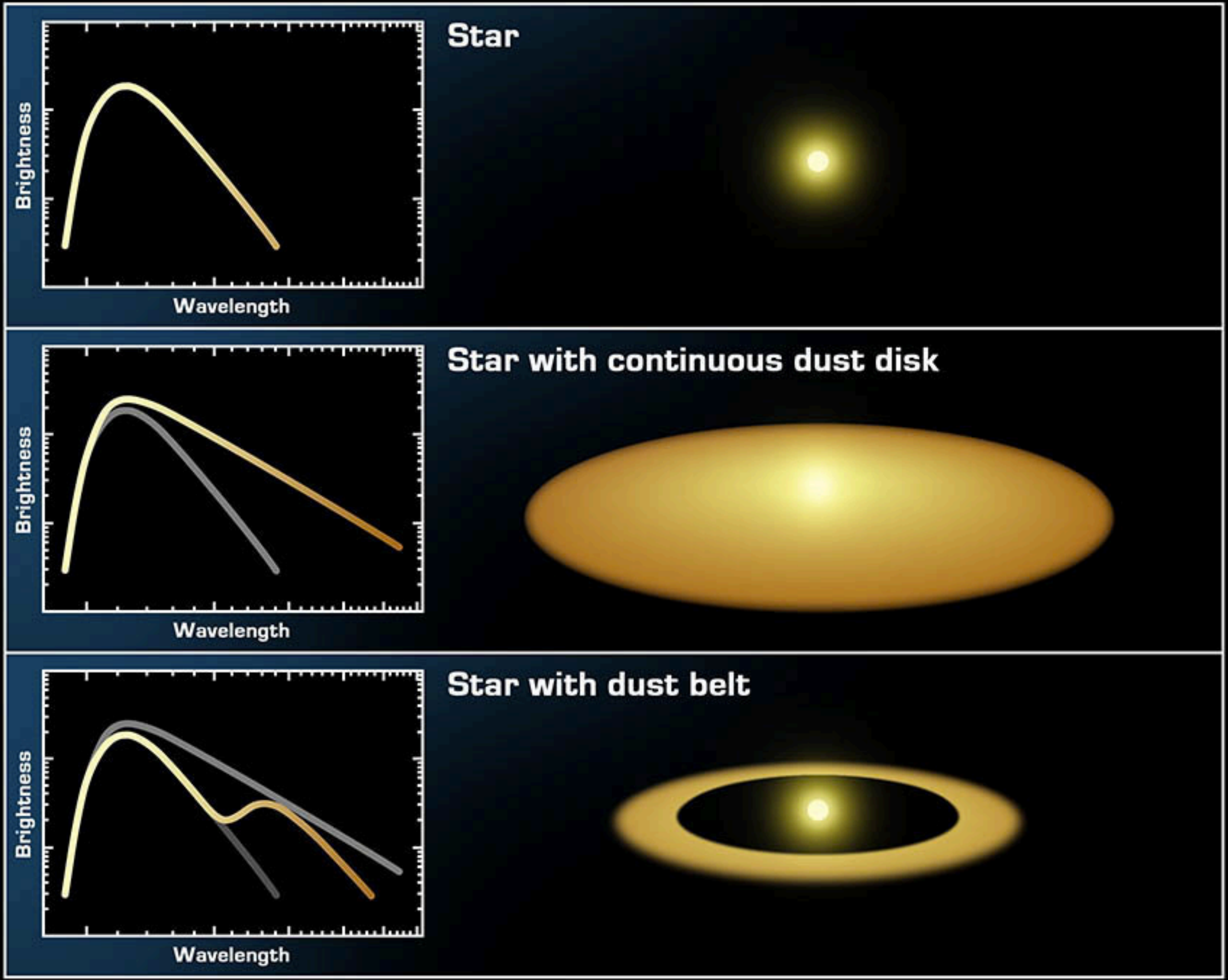
with $J_\nu^d = \dfrac{1}{4\pi}\oint I_\nu^d(\vec{x}, \vec{n})\; d\Omega$, which is the mean intensity of the thermal radiation I_ν^d emitted by the other grains

2.3.3 Thermal radiative transfer and optical depth effects

- The intensity I_ν^d obeys the following equation
- $\dfrac{dI_\nu^d(\vec{x}, \vec{n})}{ds} = \kappa_\nu \, \rho \, [\epsilon_\nu \, B_\nu(T_d) + (1 - \epsilon_\nu) \, S_\nu^{\rm sca}(\vec{x}) - I_\nu^d(\vec{x}, \vec{n})]$
- It is possible to separate the stellar and the dust terms, because of the linearity of the equation. We then have $I_\nu = I_\nu^* + I_\nu^d$
- To solve this problem, we have to use an iterative scheme (Λ-iteration) because we do not know a priori J_ν^d.
  - The above RT equation is integrated along many rays
  - The mean intensity J_ν^d is then derived at every location
  - The thermal equilibrium equation is solved to determine T_d at each location.
  - Then the RT equation is again solved with the new temperature, and so on.
- This method works well with moderately optically thick media. If the optical depth is very high, convergence will be very slow.

4. Spectral energy distributions

- It is the (wide band) spectrum of an astrophysical object
- For a star, it is (close to) a blackbody
- Spectral energy distributions (SED) are a powerful way of studying astrophysical objects
- Dust emission covers a wide bandpass, because of the large band dust opacities and the wide range of dust temperatures: dust grains close to a star are hotter than those that are far away.
- This can be seen on the SED
- Often, it is the presence of an IR excess emission with respect to a stellar blackbody that reveals the presence of dust


Star
Brightness
Wavelength
Star with continuous dust disk
Brightness
Wavelength
Star with dust belt
Brightness
Wavelength

4. Spectral energy distributions

- The emission close to the peak of the Planck function (or the peak of $\nu\, B_\nu(T)$, or $\kappa_\nu\, \nu\, B_\nu(T)$) contains most of the energy, whereas the Rayleigh-Jeans part often contains only a small amount of energy
- The typical SED of an object containing dust can be considered as a discrete or continuous sum of contributions of the type $\kappa_\nu\, \nu\, B_\nu(T)$ at different temperatures T
- With a look at one SED, we can try to decompose it into several components.
  - The peak wavelength of each component gives their temperature
  - The intensity of each component indicates how much dust is present

4. Spectral energy distributions

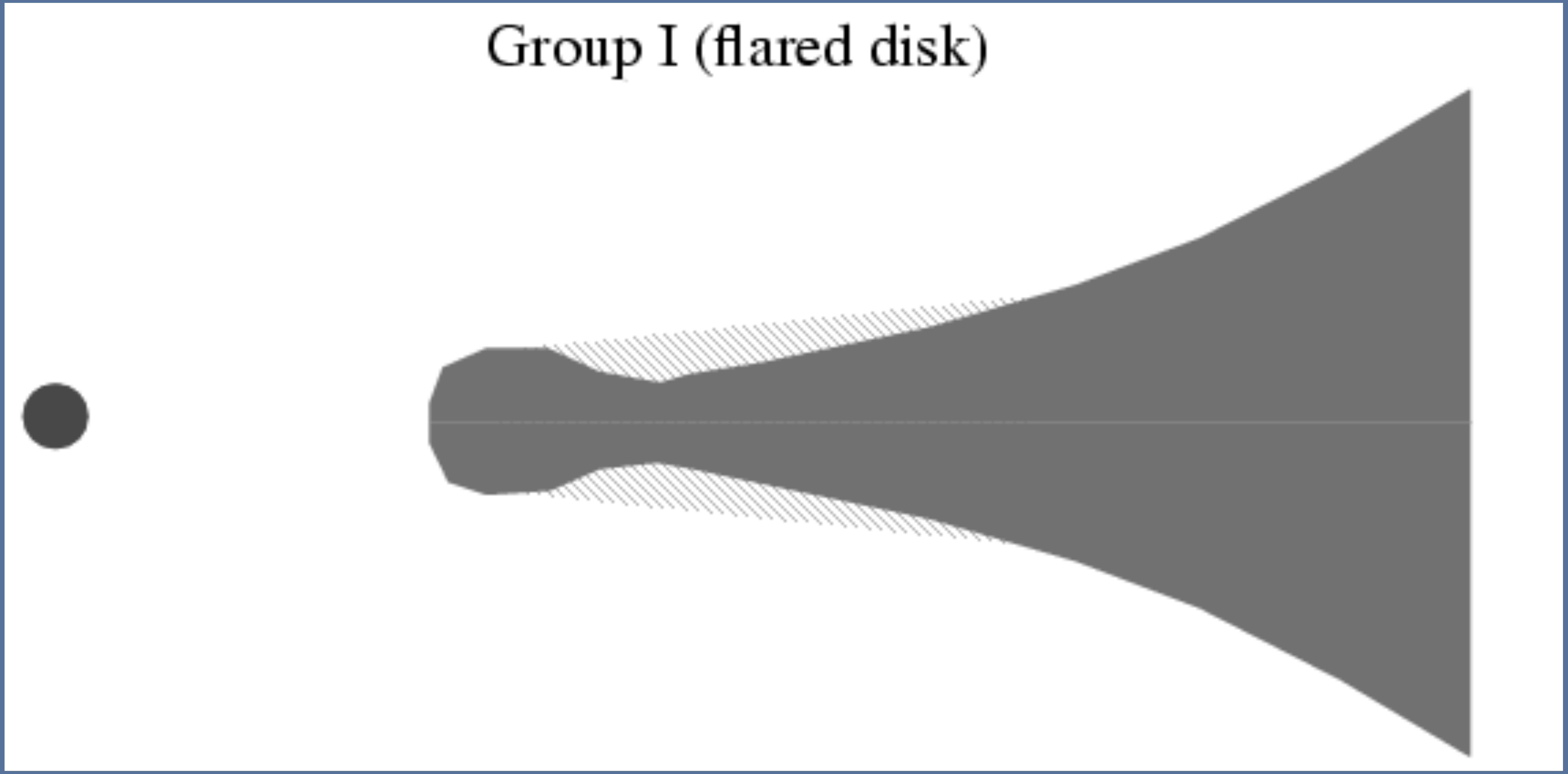
- Another important notion is the "covering fraction". Let us consider several cases
  - A star entirely surrounded by a geometrically thin dust shell, but with an optical depth at the peak wavelength of stellar emission (typically in the visible) is $\gg 1$. In this case, nearly all the radiation emitted by the star will be absorbed by the dust and reemitted in the IR. If the dust shell is optically thin in the IR, the reemitted radiation will escape immediately. Because we assume radiative equilibrium, all stellar luminosity will be reemitted in IR radiation, and we have $L_{\rm IR} \simeq L_*$
  - Same configuration as before but the shell has holes and covers only 50% of the sky as seen by the star. We then have $L_{\rm IR} \simeq 0.5 L_*$
  - If the shell entirely covers the star but is optically thin at stellar wavelengths ($\tau_* \ll 1$), the shell absorbs only a small part of the stellar radiation
- This conversion of stellar luminosity in IR luminosity is called reprocessing of stellar radiation

4. Spectral energy distributions

- The covering fraction Ω can be defined as the probability for each stellar photon to be absorbed and reemitted by the dust

- If the dust is optically thick at stellar wavelengths, $\Omega = \Omega_{\text{geom}}$, where Ω_{geom} is the geometric covering fraction, ie the fraction of the sky as seen from the star which is covered by the dust
  - $L_{\text{IR}} \simeq \Omega\, L_*$
  - This is only an estimation because the result is modified by the presence of scattering, geometric effects, etc.
  - "Shadowing" has to be taken into account: if a cloud already covers part of the sky as seen from the star, then another cloud located at a greater distance will only receive part of the radiation which has been reprocessed

What is the SED of such an object (flared disk)?



4. Spectral energy distributions

- SEDs are useful but they retain information only on the energy and not on its spatial distribution
- If we consider, for example in the case of a disk, that the overall SED is the sum of the contributions of dust at different distances (and therefore at different temperatures), we make an approximation because the dust close to the star will contribute (in addition to the stellar radiation itself) to the heating of more distant dust. The problem itself is rather complex
- Moreover the geometry of circumstellar disks (with, e.g., flaring) is often complex and modifies the temperature distribution
- The interpretation of an SED without radiative transfer remains to the first order
- Of course, it is always possible to use a full radiative transfer calculation to model an SED.

5. Perspectives - massive star forming regions

- Generally, for radiative transfer in dusty media, it is possible to separate a stellar component (in the UV/visible) and a thermal component (in the MIR)
  - The UV/visible component does not usually contain a thermal component (the dust temperature is not high enough for this), but scattering on dust has to be taken into account, because it is not negligible at these wavelengths
  - The second component is the thermal reemission by the grains after reprocessing, which usually takes place in the MIR or even longer wavelengths. At these wavelengths, scattering can be neglected, but thermal radiative transfer has to be treated
- To sum up

UV/visible → scattering

MIR/submm/mm → thermal transfer

5. Perspectives - massive star forming regions

- The situation is different in massive star forming regions.
- The flux emitted by these stars is very high, in particular in UV/visible, and dust temperatures can reach 1500 K (beyond this temperature, silicates sublimate)
  - Dust grains at 1500 K have their peak emission around 1 µm (NIR)
  - At these temperatures, scattering can take place
- Massive star forming regions combine both difficulties: scattering and thermal transfer in the same wavelength domain, which have to be treated simultaneously in the transfer equation
- In addition, we should take radiation pressure on grains into account, the complex geometries (multiple systems), the drastic opacity change where dust sublimates, and high optical depths (even in the FIR, the emission is not always optically thin)
- All the ingredients are there to make it a particularly difficult problem to solve