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The Cosmic Microwave Background/Le rayonnement fossile à 3K

Cosmological implications from the observed properties of CMB

Alain Blanchard [a], James Bartlett [b], Marian Douspis [c]

[a] *Laboratoire d'astrophysique de l'observatoire Midi-Pyrénées, 14, avenue E. Belin, 31400 Toulouse, France*
[b] *APC, Université Paris 7, Paris, France*
[c] *Astrophysics, D. Wylikinson building, Keble Road, Oxford OX1 3RH, UK*

Presented by Guy Laval

Abstract

Observations of the cosmic microwave background represent a remarkable source of information for modern cosmology. Besides providing impressive support for the Big Bang model itself, they quantify the overall framework, or background, for the formation of large scale structure. Most exciting, however, is the potential access these observations give to the first moments of cosmic history and to the physics reigning at such exceptionally high energies, which will remain beyond the reach of the laboratory in any foreseeable future. Upcoming experiments, such as the Planck mission, thus offer a window onto the Physics of the Third Millennium. ***To cite this article: A. Blanchard et al., C. R. Physique 4 (2003).***

Résumé

Conséquences cosmologiques des caractéristiques observées du rayonnement fossile. L'ensemble des observations du fond cosmologique représentent une source d'informations remarquable pour la cosmologie moderne. Non seulement elle conforte le modèle du « Big Bang », mais précise notablement le cadre dans lequel les structures de l'univers se sont formées. Mais ce qui est sans doute le plus fascinant est que le fond cosmologique est la voie privilégiée d'accès à la physique des très hautes énergies qui régnaient dans les tous premiers instants de l'univers et qui pourraient demeurer à jamais inaccessibles de façon directe aux expériences de laboratoire. Les futures expériences, dont Planck en particulier, sont donc une porte ouverte sur la physique du troisième millénaire. ***Pour citer cet article : A. Blanchard et al., C. R. Physique 4 (2003).***

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Mots-clés : Anisotropies du fond de rayonnement cosmique ; Cosmologie ; Univers primordial

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1. Introduction

The program of modern cosmology was born with Lemaître's 1927 paper [1] in which he proposed a cosmological model primarily motivated by the desire of accounting for what he believed to be the two astronomical facts of major significance for the description of the Universe: its non-zero matter content and the apparent recession of galaxies that he interpreted as a direct evidence for the expansion of the Universe. A few years later, after the clear evidence for an expanding universe obtained by Hubble, Lemaître initiated a program [41] whose basic questions still represent fundamental lines of research in modern Cosmology: the very early history of the Universe, including the nature of the initial singularity and its connection to quantum mechanics, and the question of the history of structure formation. During the rest of the XXth century, cosmology

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E-mail addresses: Alain.Blanchard@ast.obs-mip.fr (A. Blanchard), bartlett@cdf.in2p3.fr (J. Bartlett), douspis@astro.ox.ac.uk (M. Douspis).

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underwent remarkable progress, by the continuation of this confrontation of some of the most recent, often regarded as the most exotic, theories in physics, with hard astronomical data. The determination of the values of the cosmological parameters, with a moderate error, has naturally always being one of the central goals of cosmology, although the strength of efforts in this direction has varied over time.

The concept of inflation (Guth, [2]), introduced more than twenty years ago, revolutionized the field, pointing out how Cosmology contained deep connections between high energy physics and some astronomical observations. Moreover, Inflation suggests that the actual value of several quantities could have a *physical origin*, rather than being just constants that have to be determined. In addition, the need for a *physical origin* of the fluctuations that seed structure formation reinforced the link between the question of large-scale structure and the physics of the Big Bang. It was recognised during the last twenty years that the properties of large-scale structure, as revealed by the galaxy distribution, was a potential source of key information for understanding the physics that occurred during the very first instants of the universe. For these reasons, the determination of cosmological parameters has become a scientific program which significance goes far beyond the question of establishing the numerical values of the few parameters describing the universe within the framework of general relativity.

Establishing the precise abundance of light elements, which requires the modelling of the chemical evolution of galaxies and therefore the precise understanding of the physical process occurring in stellar interiors of stars, has been a fundamental test of the Big Bang during its first minutes, and is a good example of connections between modern cosmology, some fundamental physics (nuclear physics in this example) and classical astrophysics. There is now good convergence of data to a rather restricted range of possible values for the baryonic content (Charbonnel, [3]) of the Universe:

$$\Omega_b \sim 0.022h^{-2} \pm 10\%$$

This convergence makes Big Bang nucleosynthesis one of the piller of modern cosmology.

The discovery of the CMB provided the third fundamental piller on which the standard Big Bang is build. The verification of its remarkable black body spectrum by FIRAS/COBE represents the essential last achievement of the ‘classical cosmology’ program, allowing a reliable description of the major points of the history of the universe between the first billionth second and the present epoch. However the discovery by DMR/COBE of the fluctuations of the microwave sky has brought an essential observational fact that requires physical explanation beyond the physics well-established in laboratories. Whether inflation is the correct explanation of the origin of the fluctuations in the observed spectrum of the angular fluctuations in the microwave sky is still a matter of debate, although it is remarkable that this theory proposed more than twenty years ago has passed remarkably well several observational tests. However, the need for new physics is increasingly evident. It has also become clear that Cosmology will provide a test bed for this high energy physics that may well remain unattainable otherwise. In this respect, the observed properties of the CMB fluctuations appear as a remarkably clean tool for investigating early high energy physics. In fact, the possibility of constraining some parameters to the percent level with Planck, an extraordinary challenge, naturally leads to the idea of ‘high precision cosmology’ in a scientific domain where order of magnitudes were the only realistic perspective few years ago!

2. Why CMB does tell us something on cosmological parameters?

In the standard scenario of structure formation, the present distribution of matter results from the gravitational amplification of initially small perturbations of the matter density field. As the temperature of the universe goes down, the initial hot plasma will eventually recombine in neutral gas, suddenly leaving the universe essentially transparent. Therefore, observing the cosmic microwave background offers a direct image of the universe at this epoch, some 400 000 years after the Big Bang. (It is instructive to recall that this image is and will remain the most distant picture of the universe that light could ever reveal!) The physical conditions presiding at this epoch are well known and easy to describe (the density of matter at this epoch is still lower than the best vacuum one can obtain in laboratories!), the amplitude of the fluctuations being in the linear regime. Therefore, the calculation of the angular spectrum of temperature anisotropies, C_ℓ’s, resulting from a given initial matter fluctuation power spectrum $P(k)$, whose statistics is specified, is relatively straightforward even if it could be quite elaborate on the technical side. Qualitatively, fluctuations behave like waves in a viscous media – they oscillate and the amplitude decreases with time. This specific oscillating regime starts when the wavelength become smaller than the horizon. Therefore each wavelength starts oscillating with a fixed value of the initial phase but at an epoch varying with the wavelength. This oscillating regime stops rather brutally when the universe become transparent at recombination. The specific amplitude of the wave at that time depends on this phase (as well as on the detailed composition: baryonic and non-baryonic matter) and imply a specific pattern at some spatial wavelengths and its harmonics, which appear as successive peaks in the C_ℓ’s curve. These peaks are the angular equivalent of the specific spatial wavelengths. Therefore their numerical values depends on the angular distance to this surface corresponding to this epoch and involves various cosmological parameters. This allows one to understand why the C_ℓ curve depends on the characteristics of the spectrum of the initial fluctuations, on the matter content of the universe and on

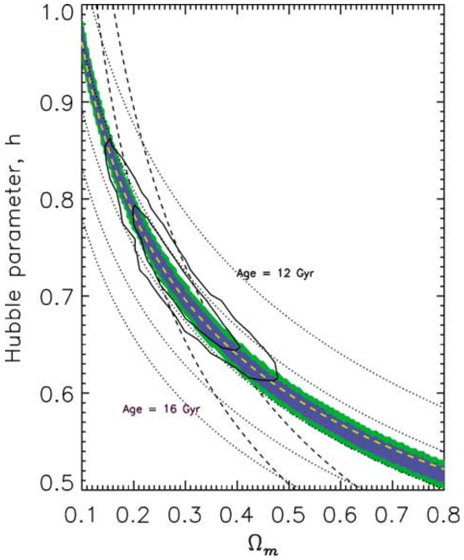

Fig. 1. Constraints set by the properties of the first peak as seen by WMAP in the $\Omega_m - h$ plane for a flat universe (filled region, blue, 1σ, green 2σ). The constraint region follows almost exactly constant age lines (dotted). Additional constraints can be obtained on the dark matter content for powerlaw CDM models (dashed lines), leading to tight contours when combined. From Page et al. [33].

cosmological parameters. In the standard inflationary scenario, additional contributions can come from primordial gravitational waves. Further complications could occur: the simplest models on the origin of the primordial fluctuations assume adiabaticity, but other possibilities do exist (like isocurvature modes, see Langlois, this issue [4]). In addition, although active perturbations, corresponding to topological defects, are ruled out as the primary seeds of structure formation and thereby of CMB fluctuations, the possibility remains that a non-zero contribution does exist (Bouchet et al., [5]) which might affect parameter estimations from the C_ℓ's. The increase in precision measurements is therefore vital in order to ensures that our vision is not blurred by such exotic contributions. Hereafter, we will comment essentially on the interpretation of the C_ℓ's curve within inflationary scenarios (see Parentani, this issue [6]), i.e. on passive initial Gaussian fluctuations (although non-gaussianity is possible in inflationary scenarios).

The formalism to compute expected fluctuations in the CMB has been developed quite early (Sachs and Wolfe, [7]; Peebles and Yu, [8]) and useful constraints from upper limits on CMB fluctuations have been used quite widely in the 1980s (Wilson and Silk, [9]; Vittorio and Silk, [10]; Bond and Efstathiou, [11]). However the detection of the first fluctuations by COBE on large scale (Smoot et al., [12]) represents what can be considered as the most important observational fact in Cosmology during the last twenty years of the XXth century (although some tantalising evidence existed before DMR, there is no doubt that the DMR instrument obtained the first reliable detection of anisotropies beyond the dipole component). Indeed, this discovery lead to a deep change in modern cosmology: the DMR observations reveals that predictions of early universe physics theories, like inflation, were actually testable by astronomical observations. At the same time, the DMR observations called for further effort on the observational side: because COBE could not reveal the fluctuations on angular scales smaller than 7 degrees, the actual information that one can get from the DMR measurement was very limited. It has therefore become clearer and clearer that small scale fluctuations would be critical in bringing more stringent constraints on cosmological scenarios. These ideas have strongly motivated the two space missions WMAP and Planck Surveyor, as well as many balloon and ground based experiments.

2.1. First fundamental result: the universe is nearly flat

However, in order to fully exploit the result of space missions, or even to fully explore their actual capabilities in constraining cosmological parameters, the need for accurate and fast codes to compute the C_ℓ's for large sets of parameters has become obvious. Such extremely fast codes have become available (CMBFAST: Seljak and Zaldarriaga, [13]; CAMB: Lewis et al., [14]; DASH: Kaplinghat et al., [15]) allowing the computation the C_ℓ for a given model in few seconds, while hours were necessary a few years ago. Detailed investigations then become possible on a large number of parameters (see Douspis, this issue [16]). During the same period, tantalising observational evidence for the presence of the first peak was reported for the first time by the Saskatoon experiment (Netterfield et al., [17]). Soon after, several experiments provided measurements on similar scales. These early detections were consistent with the presence of the so-called first Doppler peak, a maximum in

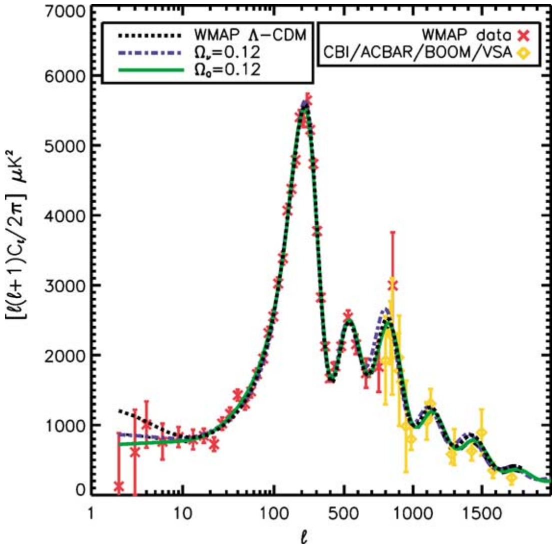

Fig. 2. The temperature power spectrum for the best-fit power-law ΛCDM model (dotted black line) from Spergel et al. [29], and for two broken-power-law models (both having $\Omega_\Lambda = 0$) with $\Omega_\nu = 0.12$ (dot-dashed blue line) and $\Omega_{\mathrm{Q}} = 0.12$ (solid green line), compared to data from WMAP and other experiments [34–40]. Such models have a low Hubble constant ($H_0 \sim 46$ km/s/Mpc) and of course are rejected by the interpretation of the SNIa Hubble diagram but are consistent with most major cosmological data (large scale structure, abundance of local clusters, dark matter distribution on large scales as observed from weak lensing, primordial nucleosynthesis). From Blanchard et al. [30].

the amplitude in the C_ℓ, found to lie close $l \sim 250$, although their consistency was far from obvious. The first analyses of cosmological implications revealed that this data were consistent with flat cosmological models and inconsistent with open cosmological models (Lineweaver et al., [18]; Handcock et al., [19]; Lineweaver and Barbosa, [20,21]). During the period 1997–2000 several small scale experiments brought further observational evidence for the detection of the first peak as the number of measurements increased at smaller scales (larger ℓ). These data consistently pointed toward a nearly flat model, but were also pointing toward an index for the power spectrum of initial fluctuations close to 1 as expected in inflationary models. Constraints obtained from the CMB received increasing attention, and the observational results from Boomerang and Maxima (see Stompor et al., this issue [22]) who provided maps with unprecedented S/N brought undeniable evidence for the presence of the Doppler peak at $\ell \sim 220$, thereby providing the definitive evidence for a nearly flat Universe (when data are interpreted within the framework of General Relativity; actually the evidence for flatness – or nearly so – is not direct). This is certainly one of the most important observational facts in modern cosmology: the DMR result demonstrated the need for new physics, but the present observations demonstrated without any ambiguities that theorists had provided models whose predictions were very close to the actual data. It is now clear that investigations of early universe physics can be constrained – actually quite severely – by astronomical observational data.

The detailed existing observations of the fluctuations of the CMB also implied that the case for the simplest general framework, the gravitational growth of passive Gaussian fluctuations, is very strong. Indeed this idea is now completely accepted. It was also realised that much tighter constraints on cosmological parameters could be obtained by *combinations*: large scale structure, SNIa Hubble diagram, Hubble constant measurements could be used in order to almost entirely specify the value of the cosmological parameters. This technique has been extremely fruitful with the increased accuracy of second generation experiments (Boomerang, Maxima, CBI, Archeops, ACBAR) (see Benoît et al., [23,24]), although early investigations did already provide crucial evidence which lead to a standard model, the so called concordance model, now recognised as a model able to reproduce most of existing observations.

2.2. BBN: CMB and light element abundance

The quality of constraints that can be obtained from the C_ℓ is truly remarkable. This is well illustrated by the constraints that can be put on the baryonic content of the Universe Ω_b. Somewhat surprisingly the value of Ω_b can be constrained from available data on the C_ℓ, in a way which is relatively independent of the other parameters. Few years ago, obtaining information on this quantity was possible only through the comparison of predictions of primordial nucleosynthesis and the observed abundance of light elements. Deuterium is the light element which is the most sensitive to primordial baryon abundance. Furthermore it has now been observed in Lyman α clouds which are likely not to have suffered significant chemical evolution. A few years ago, there was a controversy on the actual abundance of Deuterium, even before the Boomerang data the CMB clearly favoured the lower value, indicative of a high baryonic content. The controversy has since disappeared, and the agreement between the baryonic content from CMB and from Deuterium in Lymanα clouds is excellent (Kirkman et al., [25]), although it is not clear whether the abundance of Helium 4 (Gruenwald, Steigman and Viegas, [26]) is fully consistent with Deuterium in standard BBN. It is remarkable that within a few years the CMB has been able to provide constraints in this domain that are of the same quality as primordial nucleosynthesis, whose reign lasted for decades.

There is no doubt that the satellites WMAP and Planck are going to provide measurements whose accuracy will be close to the fundamental limit implied by the so-called cosmic variance (i.e., the limited possible knowledge on C_ℓ due to the finite size of the celestial sphere). However, it is still a somewhat open question to infer what accuracy can be achieved on Cosmological parameters. Actually, such a question could be answered only within a specified model, and there is some arbitrariness in deciding whether the models investigated are of enough generality to make firm statements. For instance, CMB data can be fit by models with $\Omega_M > 1.2$; therefore the conclusion that CMB prefer nearly flat models relies on some a priori. This remains a fundamental result of modern cosmology, because the important result is that models with low Ω_M content without cosmological constant are strongly ruled out (to my knowledge there is no such model which could accommodate CMB data). However in the area of *precision cosmology* this question deserves special attention. Indeed, if one writes an accurate constraint on a cosmological parameter that CMB implies, it is wise to specify the model in which this has been obtained. For instance, the accurate age constraint obtained by WMAP is only meaningful within the specified scenario (the flat power law pure CDM with adiabatic fluctuations).

2.3. Can we be fooled?

This question is connected to the problem of degeneracy among cosmological parameters in estimation from the C_ℓ. Indeed it is well known that very different combinations of parameters could lead to indistinguishable C_ℓ (Zaldarriaga et al., [27]), differences being smaller than the cosmic variance. In addition, current investigations are performed assuming pure power law power spectrum for the initial fluctuations. If there are some complexities in the shape of the initial power spectrum may well render cosmological constraints erroneous if this complexity is not dealt with in the analysis (Kinney, [28]). A good example of this is the status of the cosmological constant from the WMAP data: the detection of such term from the CMB data is strong in pure power law λCDM models (Spergel et al., [29]). Allowing some type of variations in the shape of the power spectrum leaves this conclusion essentially unchanged. However, an Einstein de Sitter ($\Omega_M = 1$ and $\Omega_\Lambda = 0$) in which the power spectrum presented two different spectral indexes over different scales has been shown to be able to reproduce the WMAP results as well as the concordance model (see Blanchard et al., [30]).

This comes from the fact that the primary cosmological quantity which determines the C_l curve is the angular distance to the last scattering surface. Therefore, although the position of the first peak clearly points toward a nearly flat model, there is some degeneracy left in the Ω_M–H_0 plane. This degeneracy might be easily broken in a specific model. Indeed, the six independent parameters of flat pure power law ΛCDM models, can be accurately determined from the WMAP data alone. Within this framework, the emerging picture is fully consistent with the concordance model: the index of the primordial spectrum is very close to 1: $n = 0.99 \pm 0.04$, $H_0 = 72 \pm 5$ km/s/Mpc. Such cosmological model is also in agreement with others measurements of cosmological relevance: the Hubble diagram of distant SNIa, the measurement of the Hubble constant by the HST, estimations of the matter content of the universe by various methods. In contrast, an Einstein de Sitter model could be made consistent with WMAP data only at the price of a low Hubble constant ($H_0 \sim 46$ km/s/Mpc), which is however a value that some data would favour (Kochanek and Schechter, [31]).

The fundamental conclusion at this level is that the concordance model is clearly the simplest cosmological model in order to reproduce the WMAP data. Although one should keep in mind that formally the WMAP data *rejected* the best model at more than 95%, such a model reach a good agreement with several data of cosmological relevance.

Given the importance of the hypothesis on the primordial spectrum, it is certainly critical to have independent measurement of the power spectrum of matter fluctuations on all scales. Surveys of galaxies as well as the power spectrum of Lymanα clouds the provide such estimation. Although they certainly provide a reasonable estimation of the amplitude of matter fluctuations over a wide range of scales, typically from 1 h^{-1} Mpc to 100 h^{-1} Mpc, it is much more difficult to properly evaluate by which amount of ‘bias’ they could be affected.

More direct measurements of the level of fluctuations in the matter content of the Universe, commonly expressed as σ_8, the root mean squared amplitude over a sphere of $8h^{-1}$ Mpc, are possible through two techniques: the abundance of clusters and the measurement of the weak lensing signal over large scales. Both methods allow rather direct measurement of a combination of Ω_M and of the amplitude of matter fluctuations σ_8. These methods can be extended to break the degeneracy. Both methods suffer from different systematics which limit the present day accuracy to something like 20% but rapid progress from large scale weak lensing surveys are likely to allow a significant reduction of this uncertainty, allowing a measurement of the power spectrum over a wider range of scales than from X-ray clusters. Again a concordance model normalised to WMAP is able to reproduce quite well the observed amplitude.

This illustrates the remarkable success of the concordance model: without any significant further adjustment, it is in good agreement with what we know about large scale structure. In contrast, in an Einstein de Sitter universe, the amplitude of matter fluctuations derived within pure CDM models produced amplitude of matter fluctuations on small scales which are unacceptably large. Such a disagreement can be alleviated by the introduction of a modest component of matter like neutrinos or quintessence with $w \sim 0$. In addition the power spectrum of matter fluctuations is then in agreement with the observations on large scale structures.

3. Conclusion

The so-called concordance model provides a remarkable simple cosmological model which reproduces well the WMAP results and which is in agreement with a number of astrophysical observations of cosmological relevance. Despite of this success, it should be realised that the WMAP data by themselves do not require the introduction of a cosmological constant. Actual direct evidence for the existence of a non-zero cosmological constant that dominates the density of the universe are rather limited: the Hubble diagram of distant SNIa and the possible detection of the correlation between deep galaxy surveys and CMB. Therefore, it is essential to confirm the actual non-zero value of the cosmological constant (or one of its generalisation like quintessence) by other data. The next generation of large projects dedicated to cosmology will undoubtedly allow the reliable establishment of a non-zero cosmological constant (if this is actually the case...). This will allow cosmologists to work within the robust framework of a standard model. The high precision that should be obtained from satellite CMB experiments, typically 1% in the Planck experiment (see Bouchet et al., this issue [32]), will open the possibility of determining the cosmological parameters with a precision of the same order. This will be possible by combining different data that will provide accurate, complementary information, including those on the power spectrum of matter fluctuations. In order to take full advantage of the accuracy of CMB data, the precision of data with which they are combined should be similar and therefore systematic uncertainties should be controlled with a similar precision. This is the great challenge for precision cosmology but the rewards will be the establishment of a standard model of cosmology to a high precision and probably unique access to physics at energies much beyond what would be attained directly from laboratory experiments.

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