

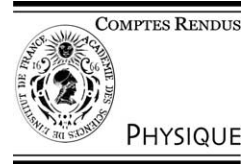


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The Cosmic Microwave Background/Le rayonnement fossile à 3K

Can the CMB reveal the topology of the universe?

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Abstract

This article summarizes recent progress in the development of tools to study the topology of the universe with the cosmic microwave background. The different signatures of the topology and observational constraints are described. The ability of future experiments to reveal the topological structure of our universe is then discussed. **To cite this article:** J.P. Uzan, A. Riazuelo, C. R. Physique 4 (2003).

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Résumé

Le fond diffus peut-il dévoiler la topologie de l'univers? Cet article résume les progrès récents concernant le développement des outils permettant d'étudier la signature topologique de l'univers dans le fond diffus cosmologique. Les différentes signatures de la topologie et les contraintes observationnelles sont décrites. Pour finir, la possibilité de dévoiler la structure topologique de notre univers est discutée. **Pour citer cet article :** J.P. Uzan, A. Riazuelo, C. R. Physique 4 (2003).

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1. Introduction

One major thrust of modern cosmology is that the properties of the large scale structures of the universe can be inferred from the knowledge of local physics. This started by the determination of the structure of spacetime from the local law of gravity by Einstein and its consequence regarding the expansion of the universe. It follows with the understanding by Gamow on how nuclear processes in the hot early phase of the universe imply primordial nucleosynthesis and the existence of a cosmic microwave background (CMB). It finally reaches a new dimension with the link between the quantum properties of matter and the large scale structures of the universe through the inflationary paradigm.

However, because we observe only one universe, we face a major problem in distinguishing boundary conditions from local physical laws. This has led to the need to investigate the global properties of our universe as deeply as possible. Among these properties is the question about the topology of the spatial sections of our universe. The CMB photons were emitted almost simultaneously at the time of last scattering and are the oldest electromagnetic signal we can ever observe in the universe. Because of this global property and its extension throughout the whole observable universe, this makes the CMB a perfect tool to constrain the topological structure of the universe.

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In standard relativistic cosmology, the universe is described by a Friedmann–Lemaître spacetime with locally isotropic and homogeneous spatial sections. In the case of a multiply connected universe, we visualize space as the quotient X/Γ of a simply connected space X (which is just a 3-sphere \mathbf{S}^3 , a Euclidean space \mathbf{E}^3 , or a hyperbolic space \mathbf{H}^3 , depending on the curvature) by a discrete and fixed point free group Γ of symmetries of X . This group Γ is called the holonomy group (see [1–3] for technical reviews and [4] for popular reviews) and it defines the boundary conditions on all the functions defined on the spatial sections, which subsequently need to be Γ -periodic. Hence, the topology leaves the local physics unchanged while modifying the boundary conditions on all fields living in the universe.

Here, we aim to overview the interplay between topology and the CMB and to explain the signatures that can be left by a change of these boundary conditions. In Section 2, we will describe the different observational effects of a non-trivial topology. We will then explain the main steps required to simulate CMB maps (Section 3). Such maps are necessary to test the ability of any algorithm to detect the topology on a given data set. We then summarize the actual constraints and their improvements from the WMAP data [5].

2. Observational imprints of the topology

The existence of a spatial topology has three distinct effects on the CMB.

2.1. Two-point correlation matrix

The two-point correlation matrix of the temperature field: if one decomposes the observed temperature field in term of spherical harmonics, then each coefficient $a_{\ell m}$ of this decomposition may be seen as a random variable for which one observes a single realization. In a simply connected space, the two-point correlation matrix, defined as the ensemble average $C_{\ell m}^{\ell' m'} \equiv \langle a_{\ell m} a_{\ell' m'}^* \rangle$ reduces to

$$C_{\ell m}^{\ell' m'} = C_{\ell} \delta_{\ell \ell'} \delta_{m m'}, \quad (1)$$

because the angular correlation function depends only on the relative angle between the two directions of observations (space is isotropic). In a multi-connected space, there exist preferred directions so that global isotropy, and possibly global homogeneity, is broken. The angular correlation function will then depend on the two directions of observations and possibly on the position of the observer. This will induce correlations between $a_{\ell m}$'s of different ℓ and m .

The exact value of the correlation matrix $C_{\ell m}^{\ell' m'}$ depends both on the local properties of the perturbations (initial power spectrum, statistics, etc.) as well as their global properties (topology) but the existence of ℓ - ℓ' and m - m' correlations is a direct signature of the breakdown of global isotropy and/or homogeneity. Topology, among other sources, leads to such a breakdown.

2.2. The angular power spectrum

The angular power spectrum, C_{ℓ} , obtained by averaging the correlation matrix diagonal coefficients, loses much of the topological information. However, most of the constraints on the topology that have been obtained up to now rely on the behavior of the angular power spectrum on large angular scales.

Unfortunately, there is a possible loophole that may reduce all the constraints derived from the angular power spectrum to ashes. These constraints rely on the assumption that the initial power spectrum is still almost scale invariant as predicted, e.g., from inflation. However, these inflationary models also predict that the spatial sections of the universe should be very large and almost flat so that we should not observe any deviation from a critical density universe, nor any topological structure on the size of the observable universe. On the one hand, observing a topology (or a slight curvature) would put these models in great difficulty (see, e.g., [6]) but on the other hand, there would then be no reason to expect an almost scale invariant power spectrum on large scales, one reason being that there will exist a new characteristic scale fixed by the topology. At present, there is no known model of the early universe leading to an observable topology and the entanglement between the effect of the topology and the local physics (initial power spectrum) has to be considered with care, particularly when trying to exclude a set of topologies.

The angular power spectrum is thus an interesting indicator but will not help in *proving* the existence (or absence) of any topological structure of the universe.

2.3. The existence of pattern correlation

The microwave background photons were all emitted at the time of last scattering, from the last scattering surface, which is a 2-sphere centered on us. Just as a large paper disk wrapped around a cylinder overlaps itself if it is big enough, the last scattering surface can wrap around the universe and self-intersect when the universe is multi-connected. The intersection of the

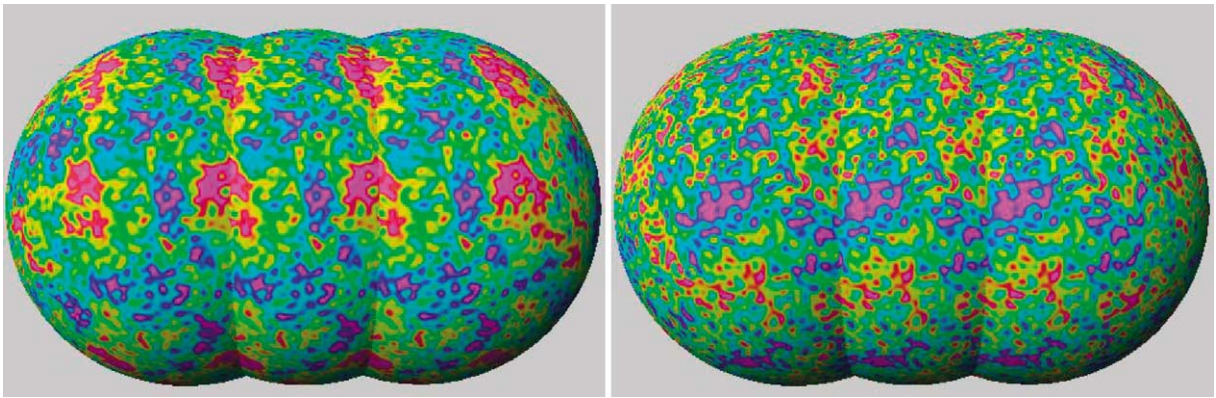


Fig. 1. The last scattering surface, with resolution $\ell = 120$, is seen from the outside along with two of its closest topological images after translation of L and $-L$ along one axis of the torus. The left picture shows the Sachs–Wolfe effect only. One can check by eye that the temperature fluctuations are very well correlated along matching circles. Note that, as expected, there are very few fluctuations on scales larger than the size of the torus (which is given here by the distance between the circles). In the right-hand picture, the matching between circles decreases when one includes the Doppler and integrated Sachs–Wolfe contributions. This map has comparatively more small scale power than the previous one as $\ell = 120$ is close to the first dip in the Sachs–Wolfe spectrum.

last scattering 2-sphere with itself is simply a circle that will appear twice in the cosmic microwave background. It follows that there might exist pairs of circles which share correlated patterns of temperature fluctuations (because they represent the same region of emission, see Fig. 1).

This idea, called the *circle in the sky method* and introduced by Cornish, Spergel and Starkmann [7] is the most direct probe of the global structure of our universe and is unaffected by the uncertainty in the local cosmological model (value of the cosmological parameters, properties of the initial perturbations, etc.).

The correlation between two matched circles would be perfect if the temperature fluctuation were a scalar function (i.e., if it did not depend on the direction of observation) and if the patterns were not distorted. However, in addition to the intrinsic temperature fluctuations of the emission region, known as the Sachs–Wolfe effect, the observed temperature fluctuations have two direction dependent contributions: (i) the Doppler effect, which depends on the relative motion of the emitting region with respect to us; and (ii) the integrated Sachs–Wolfe effect which depends on the cumulative gravitational effects experienced by the photons while travelling from the last scattering to us. From an observational point of view, the galaxy cut can also suppress part of the circles and the foreground removal can thus affect the matching.

2.4. Summary

To summarize, the angular power spectrum is an unreliable indicator of topology but can be used to tentatively constrain the topology, keeping in mind there is a possibly unjustified assumption on the initial power spectrum; the existence of ℓ – ℓ' and m – m' correlations is a direct proof of the breakdown of global isotropy and/or homogeneity; while the existence of correlated patterns (such as circles) is a direct proof of the existence of the topology, which is independent of the assumptions of the local physics and the model of structure formation. However, the applicability of the latter method requires (i) evaluating the magnitude of the effects that may blur the topological signal; and (ii) validating search algorithms.

In order to detect the topology of the universe, one therefore needs to be able to simulate high resolution CMB maps for various topologies.

3. CMB computation with a non-trivial topology

In standard relativistic cosmology, the equations of evolution of the cosmological perturbations reduce to a set of coupled differential equations involving a Laplacian. The system is therefore conveniently solved in Fourier space. Studying the observational signatures of the topology thus requires the following steps:

1. classify the relevant topologies;
2. compute the eigenmodes of the Laplacian in each topology;
3. implement the topology in a CMB code.

3.1. Classification of the topologies

The classification of the topologies of three-dimensional spaces of constant curvature depends on the geometry of the universal covering space.

For locally Euclidean spaces, there exist 18 different topologies [8]: 10 compact spaces (6 orientable and 4 non-orientable), 5 chimney spaces having only two compact directions (2 orientable and 3 non-orientable), 2 slab spaces having one compact direction (1 orientable and 1 non-orientable) and the Euclidean space \mathbf{E}^3 . Each holonomy group is a finite subgroup of the isometry group $\text{Isom}(\mathbf{E}^3)$ of Euclidean space.

For hyperbolic spaces, the classification is not yet known. The holonomy group is a finite subgroup of the isometry group of hyperbolic 3-space, $\text{Isom}(\mathbf{H}^3) = \text{PSL}(2, \mathbf{C}) \equiv \text{SL}(2, \mathbf{C})/Z_2$ (see [9] for the status of this classification).

Spherical spaces were originally classified by Threlfall and Seifert [10] in 1930. The classification was recently revisited in terms of single, double and linked action manifolds by Gausmann et al. [11]. Borrowing from Thurston’s approach [9], these authors used the fact that any finite group of unit quaternions determines a fixed point free group Γ of isometries of \mathbf{S}^3 , which then serves as the holonomy group of a multiply connected spherical space. The spaces arising in this way are called *single action* spaces and are in one-to-one correspondence with the finite subgroups of \mathbf{S}^3 , thought of as the group of all unit length quaternions. In a *double action* space, two groups of relatively prime order (excluding perhaps a common factor of two) act simultaneously so that Γ is the product of a cyclic group by either a cyclic or a binary polyhedral group. *Linked action* spaces are similar to double action spaces except that the orders of the factors are not relatively prime and only certain elements of one factor are allowed to act simultaneously with a given element of the other factor.

3.2. Eigenmodes of the Laplacian

Once the topology is fixed, we must determine the eigenmodes $\gamma_k^{[\Gamma]}(\mathbf{x})$ and eigenvalues $k^2 - K$ of the Laplacian on X/Γ through the generalized Helmholtz equation

$$\Delta \gamma_k^{[\Gamma]}(\mathbf{x}) = -(k^2 - K) \gamma_k^{[\Gamma]}(\mathbf{x}), \tag{2}$$

where k indexes the set of eigenmodes, and where the eigenmodes satisfy the periodicity conditions

$$\gamma_k^{[\Gamma]} \circ g(\mathbf{x}) = \gamma_k^{[\Gamma]}(\mathbf{x}) \quad \forall \mathbf{x} \in X, \quad \forall g \in \Gamma. \tag{3}$$

These modes, on which any function on X/Γ can be expanded, respect the boundary conditions imposed by the topology: they correspond precisely to the modes of X that are invariant under the action of the holonomy group Γ so that any linear combination of such modes will satisfy, by construction, the required boundary conditions.

It is fruitful to expand the modes of X/Γ on the basis $\mathcal{Y}_{k\ell m}^{[X]}$ of the eigenmodes of the universal covering space as

$$\gamma_{ks}^{[\Gamma]} = \sum_{\ell=0}^{\infty} \sum_{m=-\ell}^{\ell} \xi_{k\ell m}^{[\Gamma]s} \mathcal{Y}_{k\ell m}^{[X]}, \tag{4}$$

so that all the topological information is now encoded in the coefficients $\xi_{k\ell m}^{[\Gamma]s}$, where s labels the various eigenmodes sharing the same eigenvalue. The sum over ℓ runs from 0 to infinity if the universal covering space is non-compact (i.e., hyperbolic or Euclidean).

The computational challenge is to find this Γ -invariant subspace and construct an orthonormal basis for it. In the case of flat manifolds the eigenmodes can be found analytically [12]. In the case of hyperbolic manifolds, many numerical investigations have been performed (see Ref. [2] for a reviews). In the case of spherical manifolds, the eigenmodes have been found analytically for lens and prism spaces [13], and otherwise can be found numerically [14].

3.3. CMB computation

The temperature fluctuation in a given direction in the sky can be related to the eigenmodes of the Laplacian by a linear convolution operator $O_k^{[X]}(\mathcal{Y}_{k\ell m}^{[X]})$ depending only on the modulus k of the wavenumber, and a 3-dimensional random variable $\hat{\epsilon}_{\mathbf{k}}$ related to the initial conditions. Specifically,

$$\frac{\delta T}{T}(\theta, \varphi) = \frac{(2\pi)^3}{V} \sum_{k,s} O_k^{[X]}(\mathcal{Y}_{k\ell m}^{[X]}) \sqrt{\mathcal{P}(k)} \hat{\epsilon}_{\mathbf{k}}, \tag{5}$$

where \mathcal{P} is the initial power spectrum of the fluctuations. Since the topology does not affect the local physics, the only change arises from the eigenmodes on which the functions are expanded. It can be shown [15] that the correlation matrix defined in Eq. (1) takes the general form

$$C_{\ell m}^{\ell' m'} = \frac{(2\pi)^3}{V} \sum_k \mathcal{P}(k) O_k^{[X]}(R_{k\ell}^{[X]}) O_k^{[X]}(R_{k\ell'}^{[X]}) \sum_s \xi_{k\ell m}^{[\Gamma]s} \xi_{k\ell' m'}^{[\Gamma]s*} \tag{6}$$

This approach, originally developed in [15], turns out to be very efficient for simulating high resolution CMB maps both in the Euclidean case [12,15] and in the spherical case [16] (that is, in all the spaces where the $\xi_{k\ell m}^{[\Gamma]s}$ coefficients are known).

In conclusion, once the eigenmodes of the Laplacian are known (i.e., the coefficients $\xi_{k\ell m}^{[\Gamma]s}$), one can compute the correlation matrix which contains all the information about the temperature field and from which one can extract the C_ℓ , simulate maps, etc.

4. Observational prospects

The recent WMAP results interestingly indicate that a closed universe seems to be marginally preferred [5]. In particular, with a prior on the Hubble constant, one gets a density parameter $\Omega_0 = 1.03 \pm 0.05$, while further constraints including type Ia supernovae data leads to $\Omega_0 = 1.02 \pm 0.02$. Moreover, the WMAP angular correlation function seems to lack signal on scales larger than 60 degrees [5]. This may indicate a possible discreteness and a cutoff in the initial power spectrum, as would be expected from a multi-connected topology [6]. Nevertheless $|\Omega_0 - 1|$ is small, which has important implications concerning the observability of the topological structure of our universe [17].

The possibility of detecting the topology of a nearly flat universe was discussed in [17]. It was noted that the chances of detecting a multiply connected topology are worst in a large hyperbolic universe. The reason is that the typical translation distance between a cosmic source and its nearest topological image seems to be on the order of the curvature radius $|1 - \Omega_0|^{-1/2}$, and that when $\Omega \simeq 1$ the distance to the last scattering surface is less than the half of that distance (see [18]). In a flat universe, the topology scale is completely independent of the horizon radius, because Euclidean geometry has no preferred scale and admits similarities. In a spherical universe the topology scale depends on the curvature radius, but, in contrast to the hyperbolic case, as the topology of a spherical manifold gets more complicated, the typical distance between two images of a single cosmic source decreases. No matter how close Ω is to 1, only a finite number of spherical topologies are excluded from detection.

At present, the constraints on the topology of the universe are still very preliminary. Regarding locally Euclidean spaces, it was shown on the basis of the COBE data that in the case of a vanishing cosmological constant the size of the fundamental domain of a 3-torus has to be larger than $L \geq 4800 h^{-1}$ Mpc [19], where the length L is related to the smallest wavenumber $2\pi/L$ of the fundamental domain, which induces a suppression of fluctuations on scales beyond the size L of the fundamental domain. This constraint does not exclude a toroidal universe since there can be up to eight copies of the fundamental cell within our horizon. This result was generalized to four other compact Euclidean manifolds in [20]. A non-vanishing cosmological constant induces more power on large scales, via the integrated Sachs–Wolfe effect. For instance, if $\Omega_A = 0.9$ and $\Omega_{\text{mat}} = 0.1$, the constraint is relaxed to allow for 49 copies of the fundamental cell within our horizon [21]. Recently we computed the CMB signatures of the 17 Euclidean spaces with non trivial topology [15]. Among the various shapes for a topological 3-torus we found that a right torus of size $L_x = L_y = 4$ and $L_z = 2$ in units of the Hubble radius seems to be favored [22].

Recently, we constrained a family of spherical spaces on the basis of the WMAP data. A lens space’s fundamental domain is constructed by identifying the two faces of a lens shaped solid with a $2\pi q/p$ rotation, for relatively prime integers p and q such that $0 < q < p$. The result is the lens space $L(p, q)$. Exactly p copies of the fundamental domain tile the 3-sphere, their faces lying on great 2-spheres filling a hemisphere of each, just as the 2-dimensional surface of an orange may be tiled with p sections of orange peel, meeting along meridians spaced $2\pi/p$ apart. For lens spaces of the form $L(p, 1)$ we were able to show that, if $\Omega = 1.02$, p needs to be larger than 7 for the topology to be detectable and smaller than 15 for being compatible with the WMAP angular power spectrum.

The reason for this latter constraint comes from the fact that these spaces induce an increase of power on large angular scales, contrary to what one would expect from the simple example of a torus. Such an increase lies in the properties of the spectrum of lens spaces. The smallest nonzero eigenvalue of $L(p, 1)$ is always $k = 2\sqrt{K}$ (K being the comoving space curvature), and has constant multiplicity 3 for all $p > 2$, contrasting sharply to the behavior of the cubic 3-torus of size L , for which the smallest eigenvalue scales as L^{-1} . This behavior can be understood by realizing that as p increases the space is becoming smaller in only one direction and remains large in perpendicular directions: on large scales we see a 2-dimensional repartition of modes that are perpendicular to the axis of the lens, and the relative weight of large scale modes to small scale modes is greater in two dimensions than in three.

This leads to the conclusion that such an increase of power on large angular scale will also appear for slab and chimney space and to any space that is strongly anisotropic. This leads to the idea that a ‘well proportioned’ universe may be favored [23].

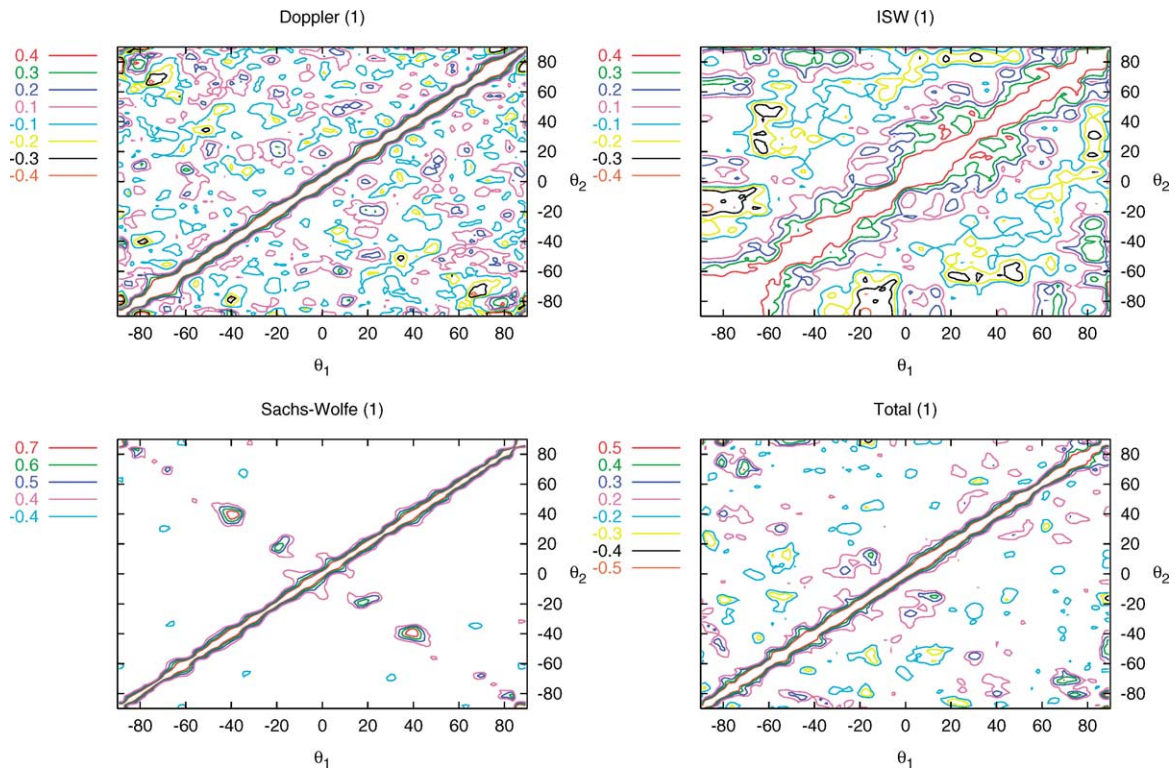


Fig. 2. Contour plots of the correlation estimator function between pairs of parallel circles of latitudes θ_1, θ_2 for a cubic toroidal universe. In this model, the circles are expected to lie at latitude $\theta = \pm 18^\circ, \pm 30^\circ, \pm 71^\circ$. In the upper row we show the Doppler and integrated Sachs–Wolfe contributions, which can be considered as noise. In the lower row, we show on the left the Sachs–Wolfe contribution, which exhibits significant correlations at the expected latitudes. Summing the three contributions, one can check that the expected correlation remains, although at a lower level (lower right picture).

Concerning the circle method, we recently studied the influence of the Doppler and integrated Sachs–Wolfe effects [15] and found that the topological signal was not excessively blurred (see Fig. 2). This is a first step toward finding an optimal method to detect pairs of matching circles in the CMB temperature fluctuation map.

In conclusion, tools to study the topology have been developed and maps can be confidently simulated. This enables one to discuss in detail the detectability of the topology (effects of noise, foreground, etc.) with the WMAP or the Planck Surveyor data.

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