
central object: a white dwarf, neutron star or black hole

$$\frac{\partial \rho}{\partial t} + \vec{\nabla} \cdot \left(\rho \vec{V}\right) = 0,$$

$$\frac{\partial \vec{V}}{\partial t} + \left(\vec{V} \cdot \vec{\nabla}\right) \vec{V} = -\frac{1}{\rho} \vec{\nabla} P - \vec{\nabla} \psi + \frac{1}{\rho} \vec{\nabla} \cdot \sigma$$

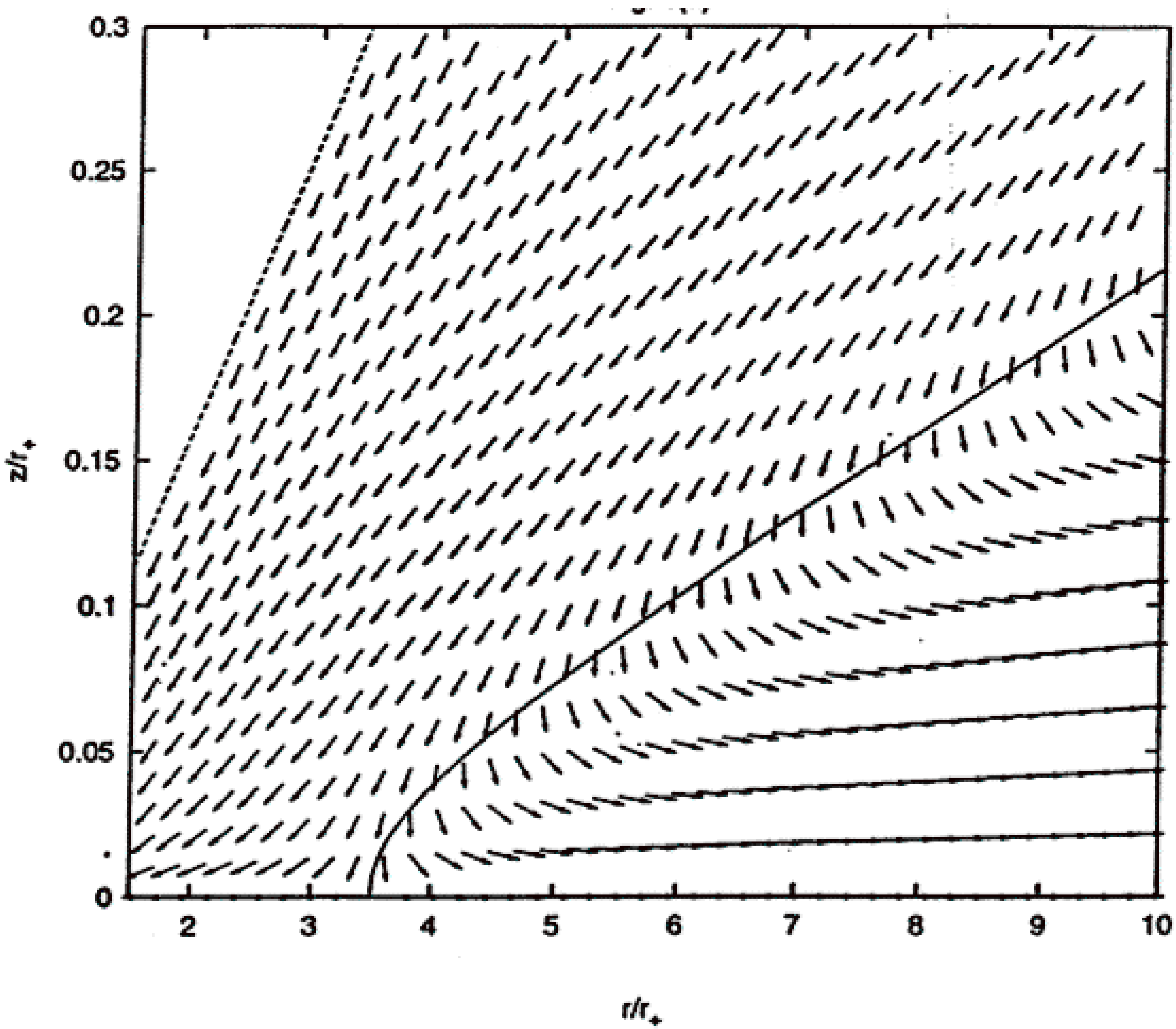
1/3 components:

$$v_r \frac{\partial v_r}{\partial r} + v_z \frac{\partial v_r}{\partial z} - \Omega^2 r = -\frac{\partial \psi}{\partial r} - \frac{1}{\rho} \frac{\partial P}{\partial r} + \frac{1}{\rho} F_r$$

$$F_r = \frac{2}{r} \frac{\partial}{\partial r} \left(\eta r \frac{\partial v_r}{\partial r}\right) - \frac{2 \eta v_r}{r^2} + \frac{\partial}{\partial z} \left[\eta \left(\frac{\partial v_r}{\partial z} + \frac{\partial v_z}{\partial r}\right)\right] + \frac{\partial}{\partial r} \left[\left(\xi - \frac{2}{3} \eta\right) \left(\vec{\nabla} \cdot \vec{V}\right)\right]$$

Usually these eqs are simplified (e.g., Shakura Sunyaev).

Exact solution of approximate eqs. NOT the same as approximate solution of exact equations.



Unit Velocity vectors (plot of analytic results)

see also Urpin 1984

Kluźniak Kita 1995, -ph/0006266

3

1987 models 21 century

1) 6 Hz QPO in Cyg X-1 & in Sco X-1 No model

NS oscillations for Sco X-1, no model for BHs!

2) 50 Hz QPO in Z sources No model

Beat frequency model, *NS spin - Kepler frequency of clump,*

250 Hz orbital (at inner edge) - 200 Hz spin ??

(Lamb et al. after Alpar & Shaham 1985,

after CV model of Patterson 1979).

---

3) 0.2 Hz QPO in an X-ray pulsar (Angelini et al. 1989)

B field ~ TG, beat freq. or Keplerian freq. HFQPO!

spin f >> 0.2 Hz ~ maximum Keplerian freq.

4

Resonance model for High-Frequency QPOs

in white dwarfs, neutron stars & black holes

Włodek Kluźniak

for Jean-Pierre Lasota's 65th birthday
ZAMEK na skale, 07.10.10. 11 AM

HFQPOs are a disk Oscillation!!
(DIFFICULTY in working out theory: we do not understand disks).

However, can make some general statements, e.g.:
1. Resonance can occur in any system
   (with sufficiently many d.o.f.).
2. If QPO a disk phenomenon, presence of surface
   at inner boundary could amplify effect
   (indeed observed: HFQPOs weaker in BHs).

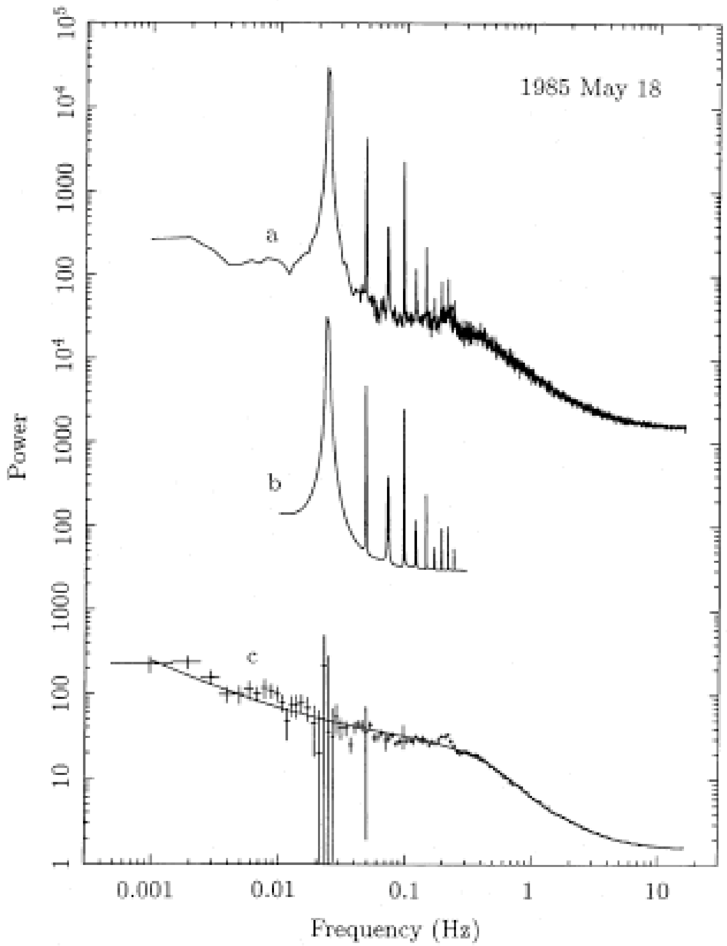
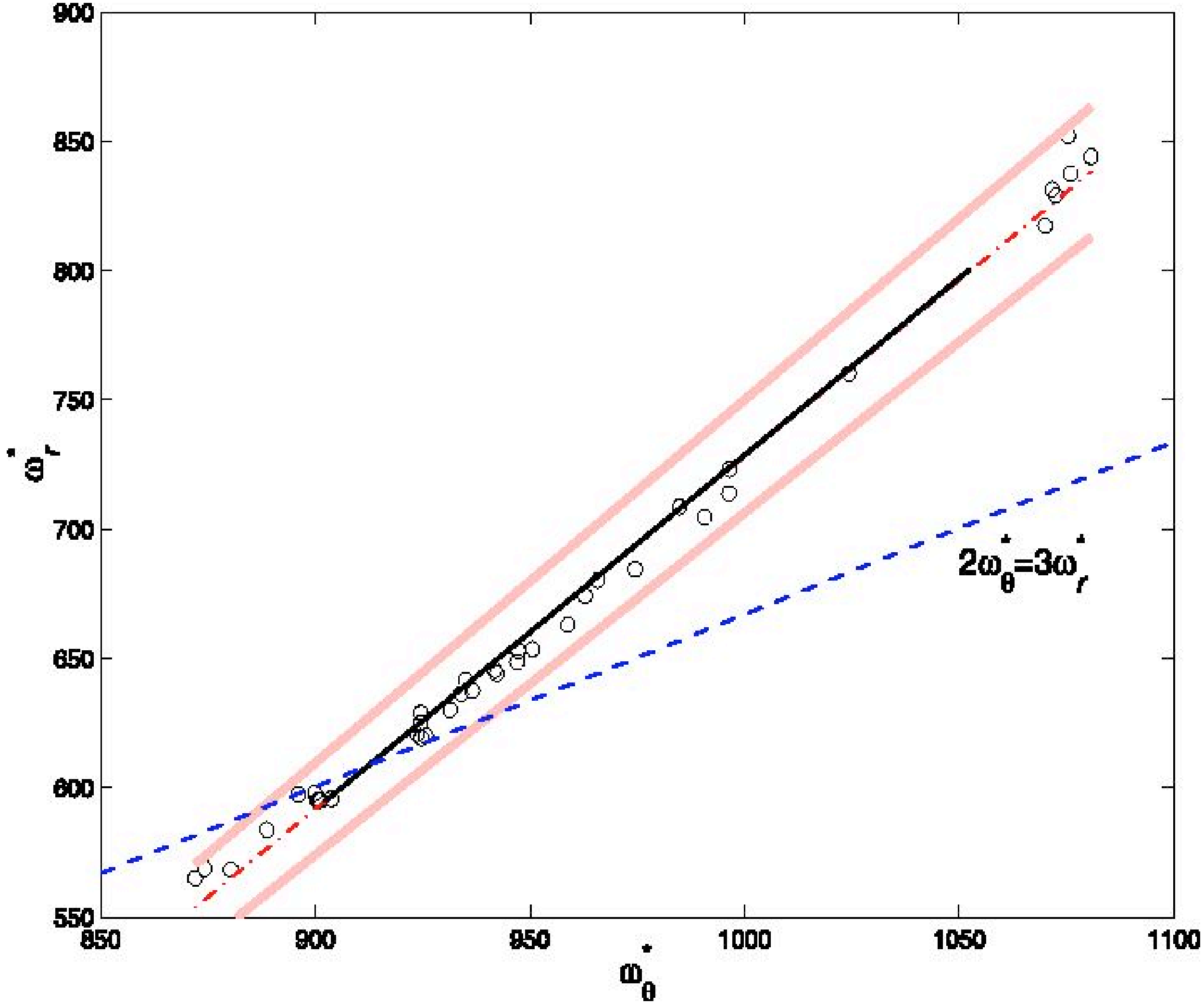


FIG. 1.—(*a*) The average power spectrum of the 1985 May 18 observation of EXO 2030+375. (*b*) The best fit in the 0.01–0.3 Hz frequency range for a model consisting of the first 10 harmonics of the coherent pulsar signal and a power law. (*c*) The residual power spectrum after subtraction of the model for the coherent pulsations. The solid line shows the best-fit for the continuum components (see text). The 20% broad QPOs that peak at a frequency of ~0.2 Hz can be clearly seen.

Angelini, Stella, Parmar 1989

X-ray pulsar (TeraGauss B-field)

Note harmonics of rotating polar spot



Data for Sco X-1 (circles), van der Klis et al.

Theoretical (black continuous) line: Abramowicz et al. 2003, see also Rebusco 2004, Horak 2005.

spots? (e.g. “relativistic precession model” Stella & Vietri)

Rotating spots have harmonics, but HFPOs do not,
so HFQPOs are NOT caused by orbiting clumps/vortices etc.

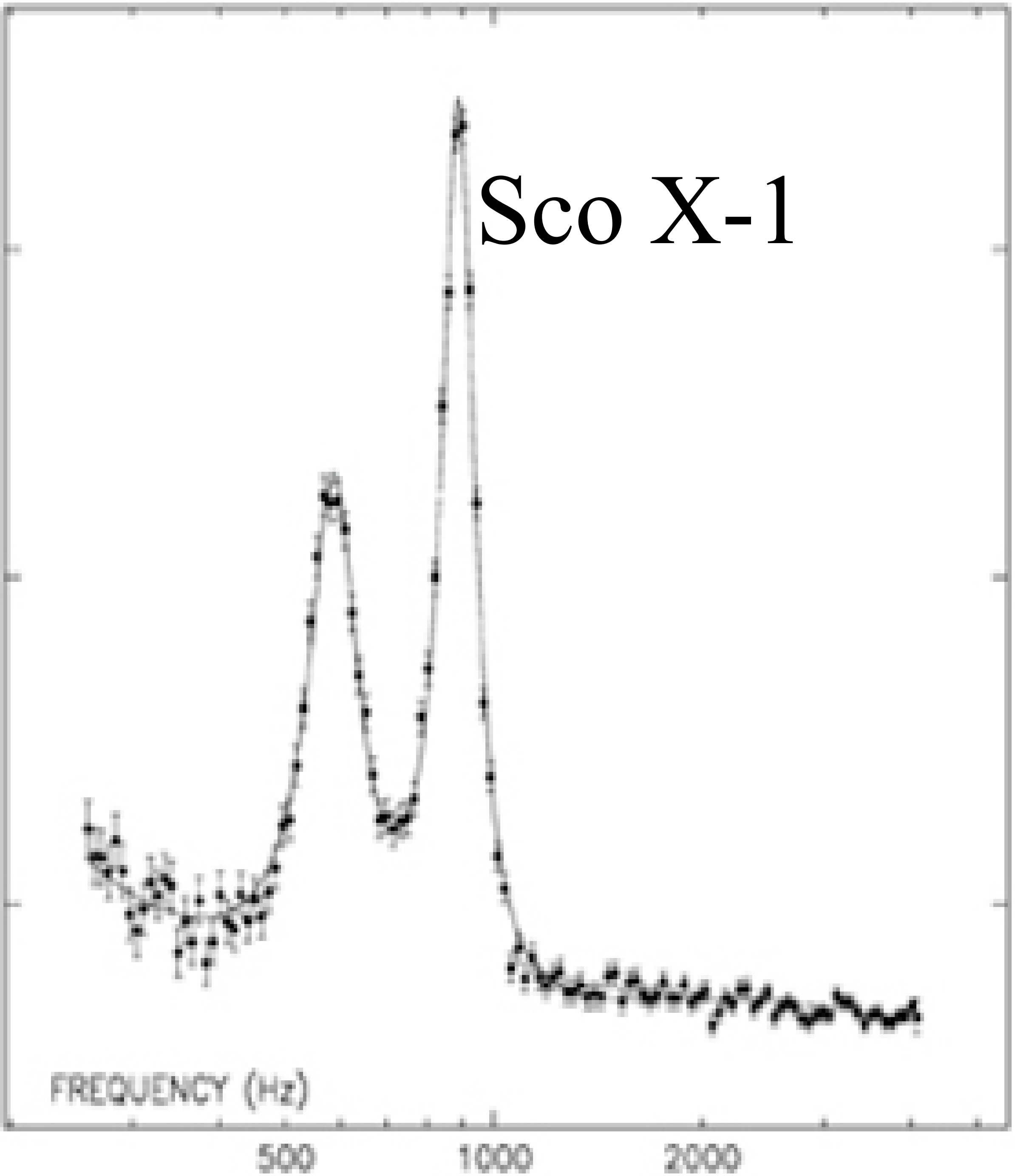
Also:

‘We show that ... *Q* can remain above 200 for thousands of seconds.
... we show that those involving clumps orbiting within or above the accretion disc are ruled out.’

Barret, D.; Kluźniak, W.; Olive, J. F.; Paltani, S.; Skinner, G. K.: MNRAS 2005

Ibid.: Different QPOs have different coherence times,
⇒‘relativistic precession’ model is ruled out for NS sources.

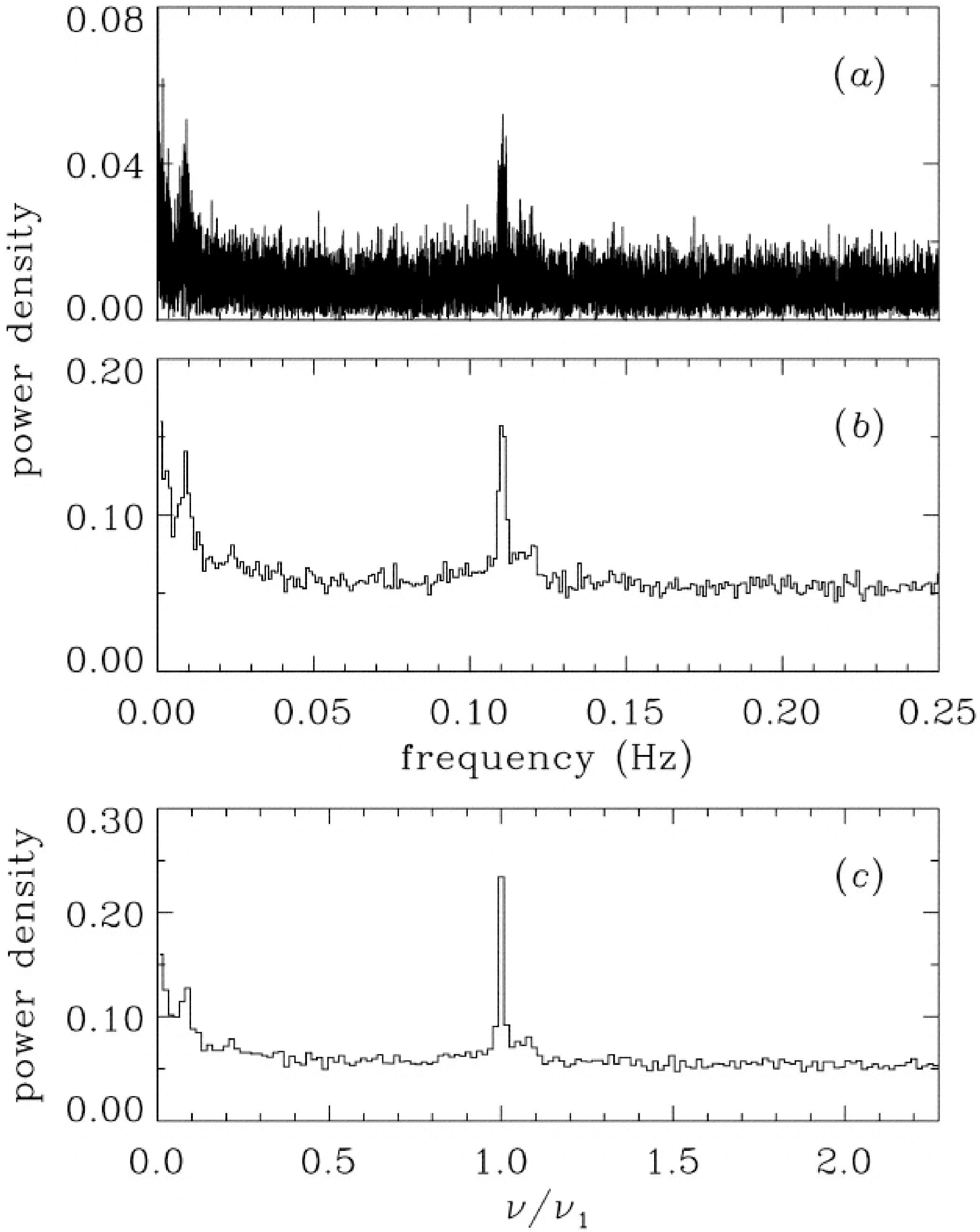
Power



High frequencies in the brightest X-ray source. Discovered by van der Klis et al. 1996 with RXTE:

$\nu/\Delta\nu$ up to ~100

10

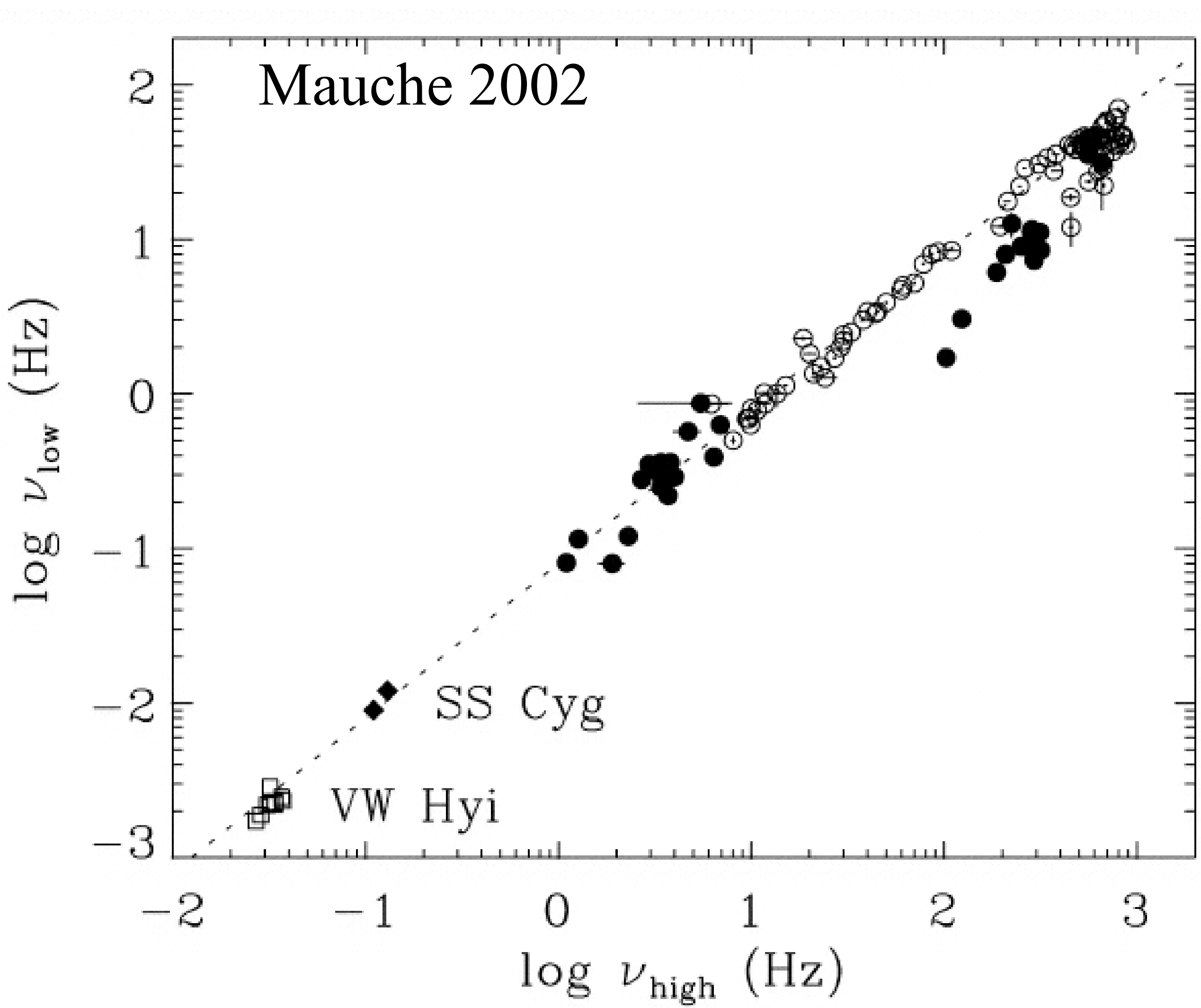


Mauche 2002
SSCyg

No twin peaks

No harmonics of HFQPO

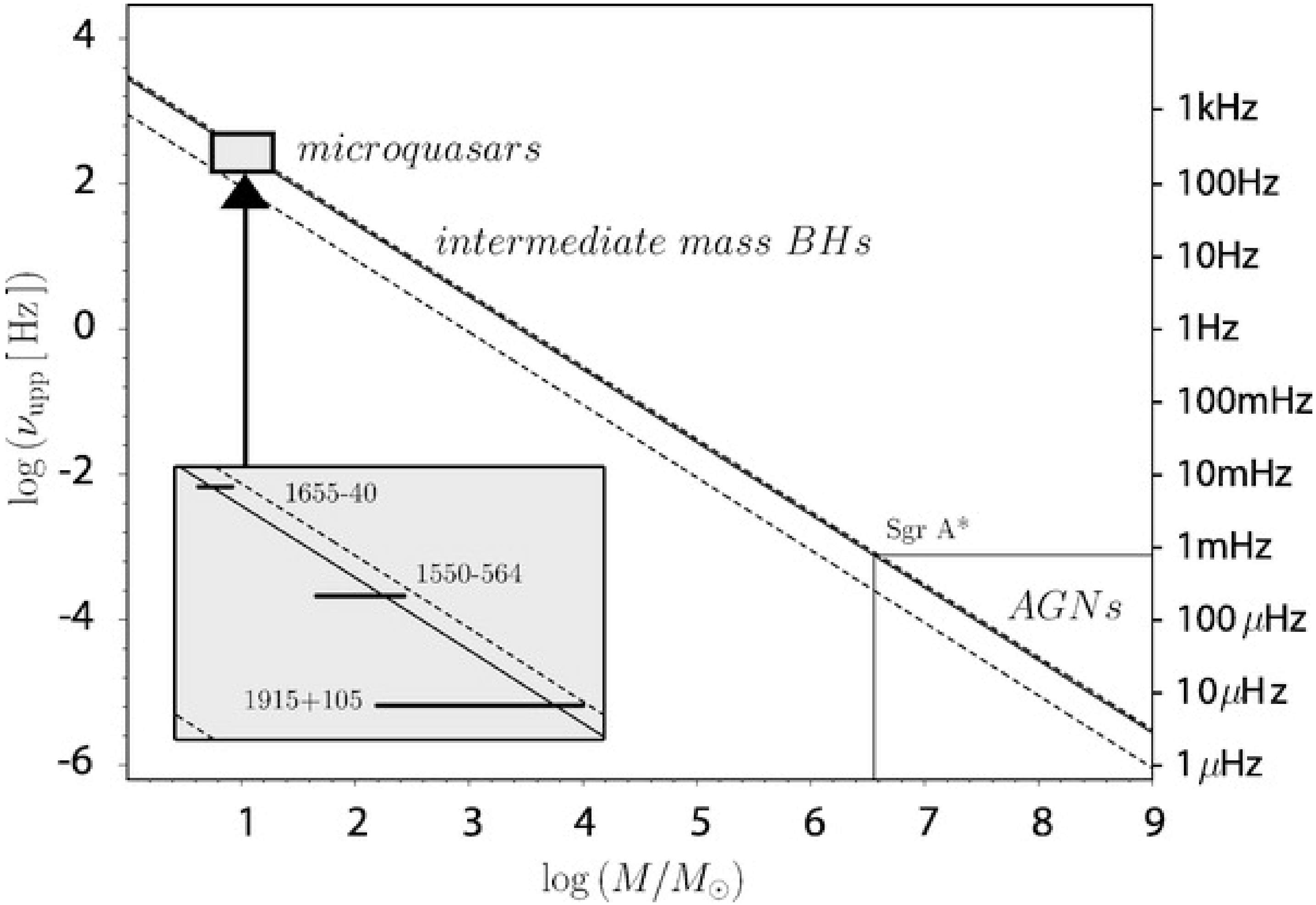
11



This correlation cannot have GR origin.
Possibly non-linear hydro (Benjamin-Feir instabil.)

In GR, $f \sim 1/M$. Case in point: HFQPOs
(McClintock & Remillard 2003)

Abramowicz, Kluźniak, McClintock, Remillard, 2004



←17 minute ?? IR flare
Genzel et al. 2003

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Theoretical work on CV disks:

Bath 1973: Characteristic frequency in disk~Keplerian orbital

1st scaling with radius (Kepler's II law !!)

(f depends on B-field, accretion rate, spin, and stellar radius).

---

we already used Bath, Evans & Papaloizou 1974:

Clumps will be sheared out in a few revolutions ⇒ low $Q = \nu/\Delta\nu$

Theoretical work on NS disks:

Kluźniak 1987; K, Michelson & Wagoner 1990

If B<100 MG then GR more important then magnetic field and

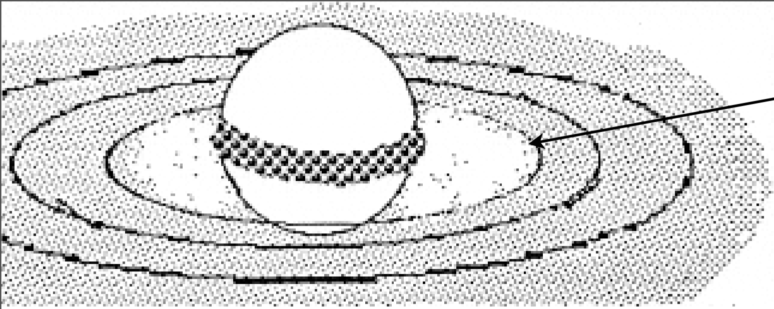
f (inner edge) = f (M,J)

i.e., 2nd scaling with mass

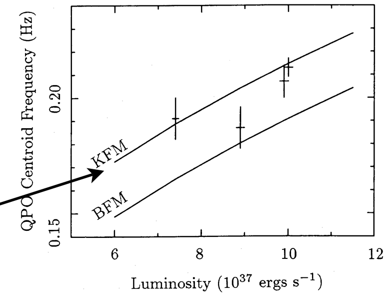
$f \sim 1/M$ (generally true in GR)

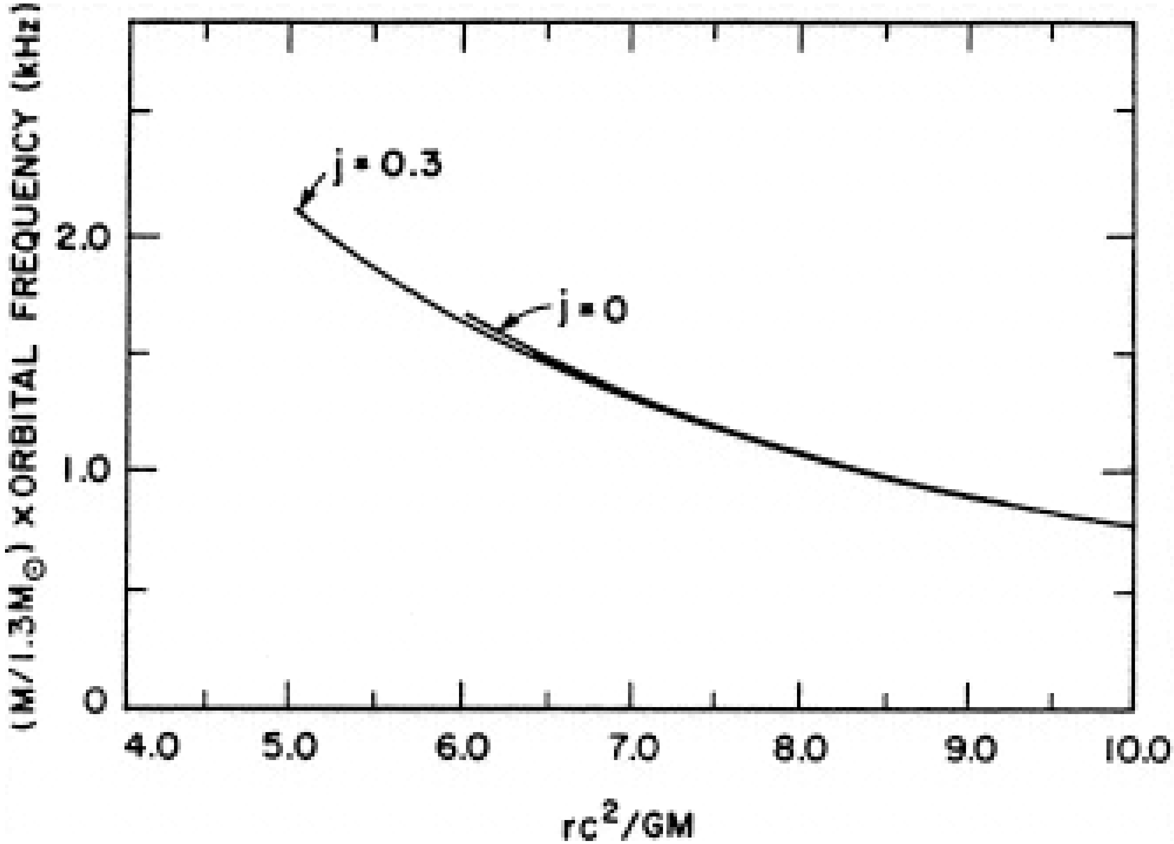
Theoretical work on BH disks:

Okazaki et al. 1987, trapped oscillations



characteristic frequency defined by inner edge, just as for QPO observed in X-ray pulsar by Angelini et al. 1989

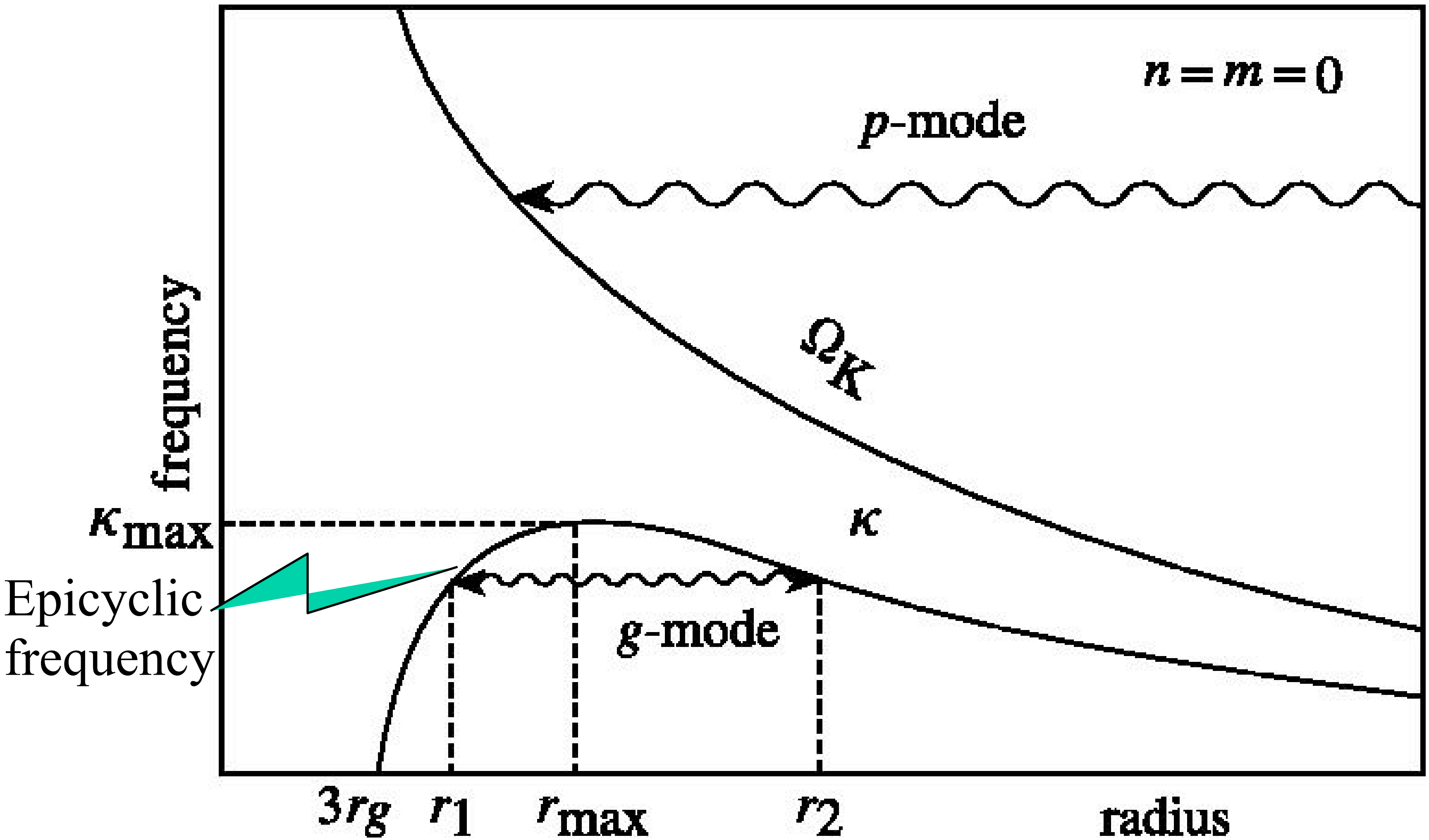




Schwarzschild metric (j=0) and Hartle metric for a neutron star with few ms spin (j=0.3). Each curve terminates at the smallest radius of a stable circular orbit allowed in Einstein's general relativity.

WK 1987; Kluźniak, Michelson, Wagoner 1990;

Okazaki, Atsuo T.; Kato, Shoji; Fukue, Jun: PASJ 39 (1987) 457
Global trapped oscillations of relativistic accretion disks



Kato 2001 $\Downarrow \Rightarrow$ Diskoseismology of Nowak & Wagoner 16

QPO phenomenon similar in WD, NS, BH.
The only structure common to these systems is an accretion disk.
Therefore QPOs are an accretion disk phenomenon.
But what kind of phenomenon?

Orbiting spot models ruled out by observations:
1. High Q in NS systems (Barret et al. 2005).
2. No harmonics (e.g. Remillard & McClintock stress "a single sinusoid is seen at a given time").

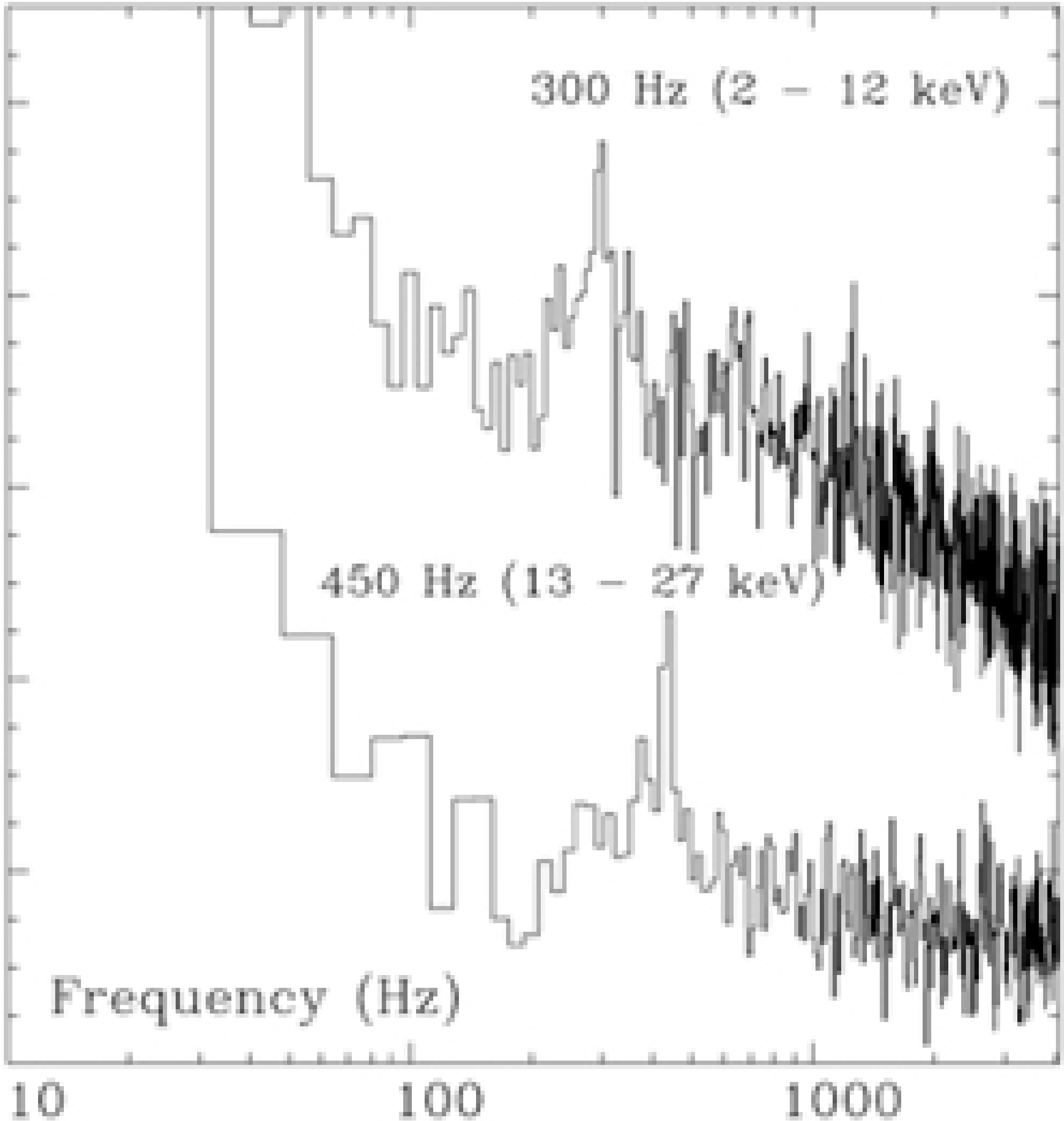
Only non-linear disk oscillation models are viable.

Kluźniak & Abramowicz (2000 submitted to PRL):

kHz QPO phenomenon similar in neutron stars reflects a non-linear (2:1 or 3:2) resonance in the general-relativistic accretion disk.

Prediction of a second high-frequency QPO in BH.

Rejected by the referees:
"No evidence for quantization of frequencies."
Resubmitted after evidence was in. Rejected again:
"There is nothing new here, it's all in the discovery paper."



Abramowicz & Kluźniak 2001, *A precise measurement of BH spin* (A&A):

'We note that the recently discovered 450 Hz frequency in the X-ray flux of the black hole candidate GRO J1655-40 is in a 3:2 ratio to the previously known 300 Hz frequency...'

GRO J 1655-40 X-ray power spectra : discovery of second HFQPO in a BH (Strohmayer 2001).

Abramowicz, Bulik, Bursa, WK 2003, A&A

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Sco X-1

al. (1997)

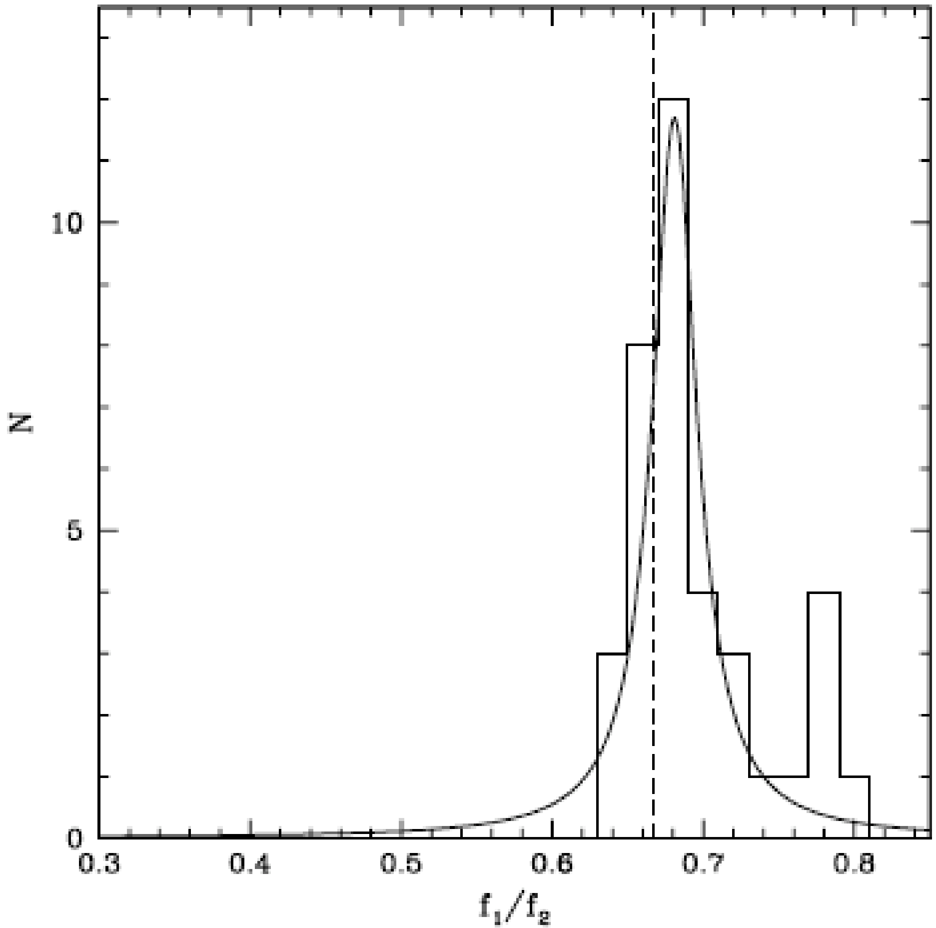
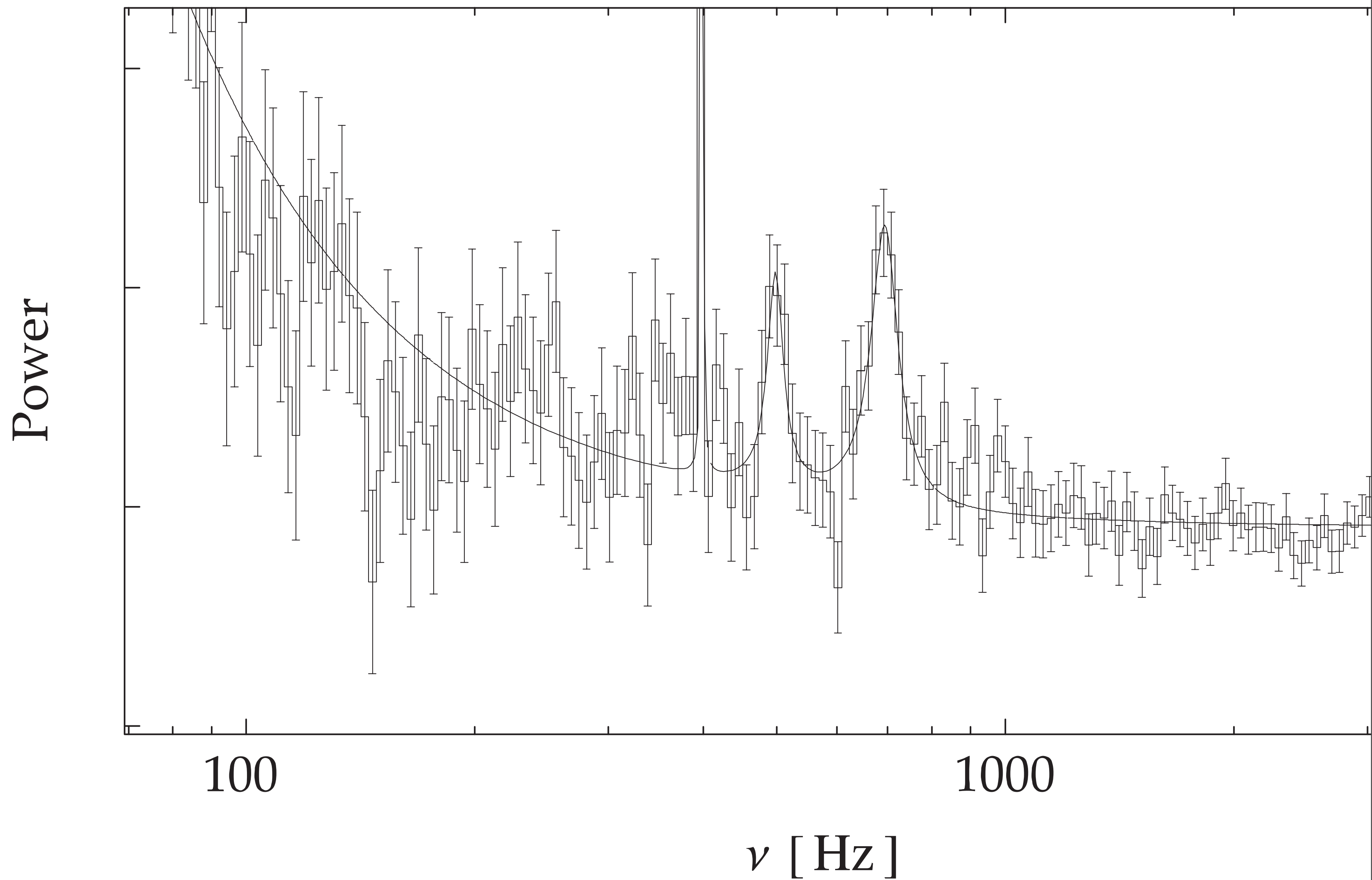
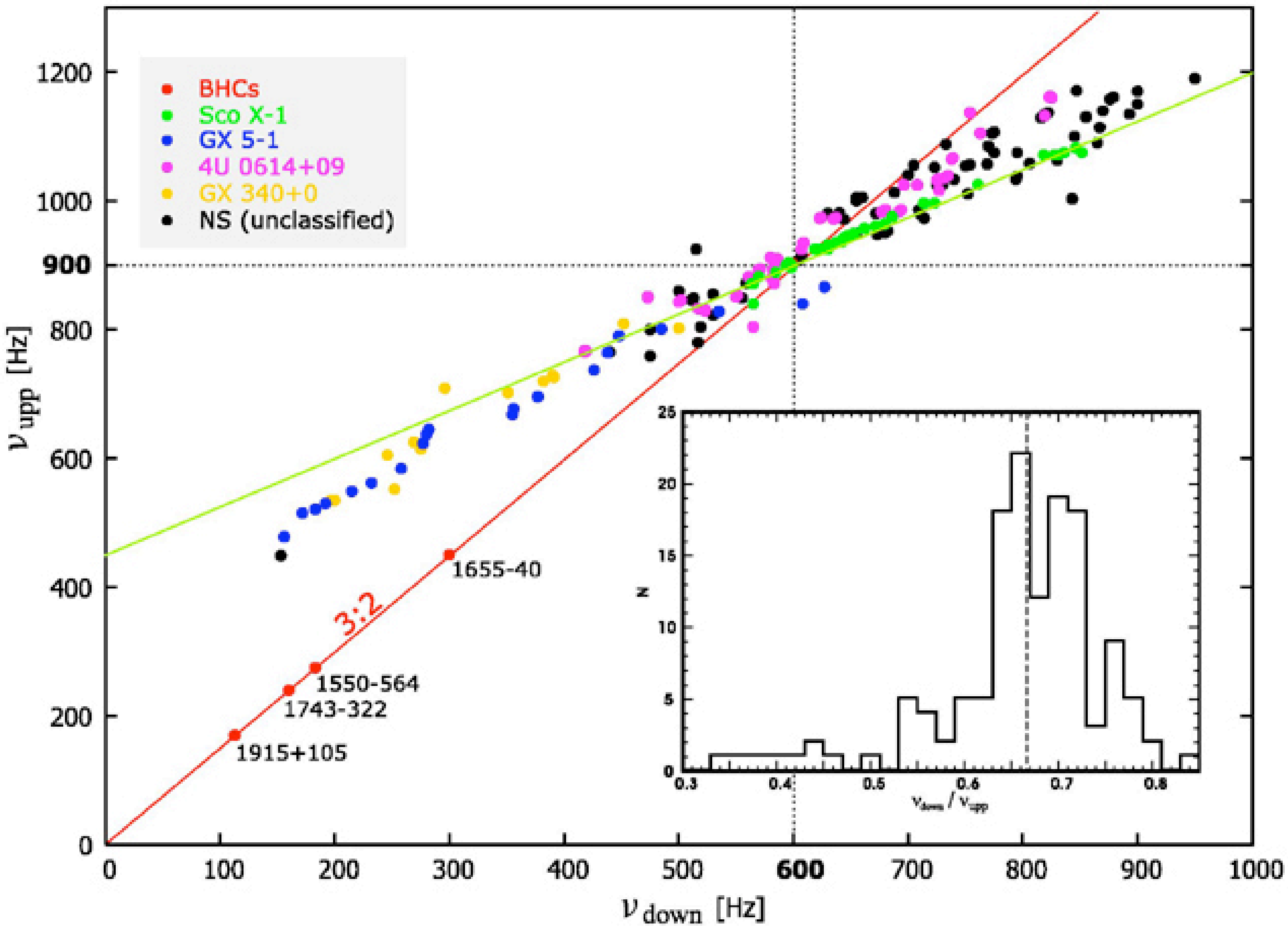


Fig. 3. The observed distribution of frequency ratio of the two kHz QPOs in Sco X-1. The data is from van der Klis et al. (1997). Also shown is the best fitting Lorentzian.

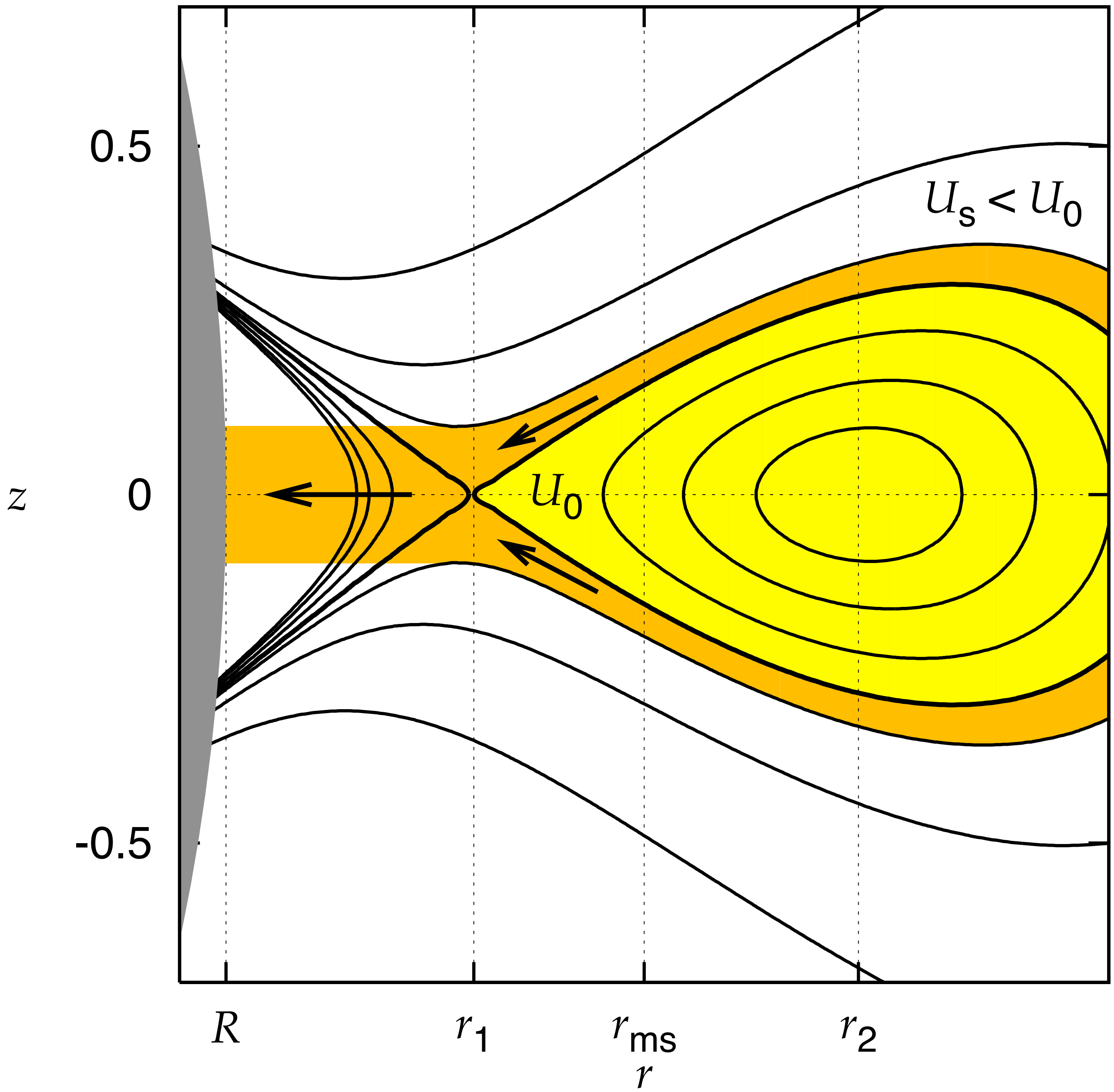
Initially rejected: “this way of plotting data is misleading.”



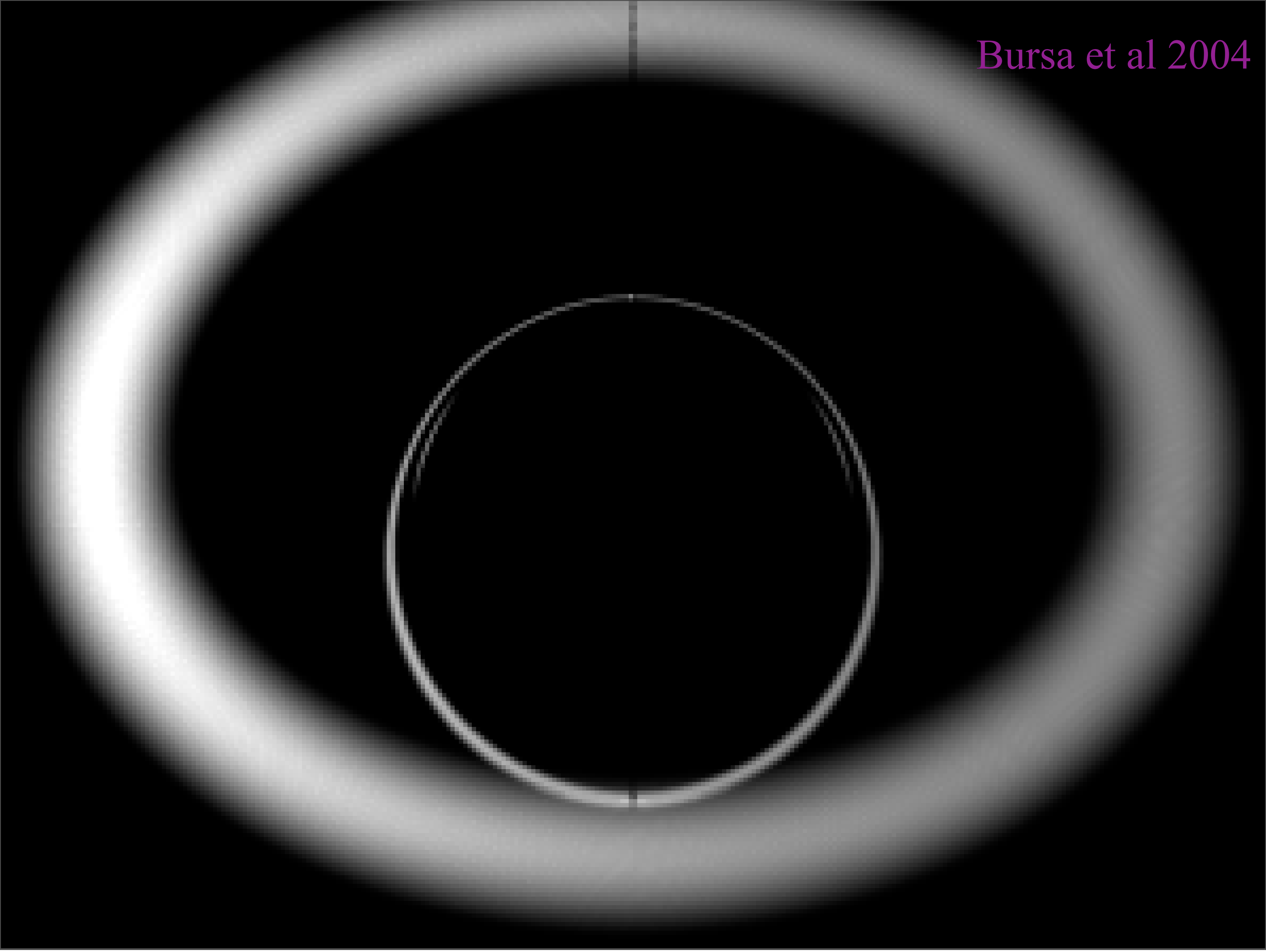
401 Hz accreting pulsar (Wijnands et al. 2003) 1/2 spin difference.
Smoking gun of non-linear resonance in NS disk (WK et al. 2004).



Bursa unpublished, Kluźniak 2005, Abramowicz et al.. 2003
McClintock & Remillard 2005



Abramowicz,
Horak, WK
Acta A. 2007


Bursa et al 2004

Parametric resonance: Mathieu eq.

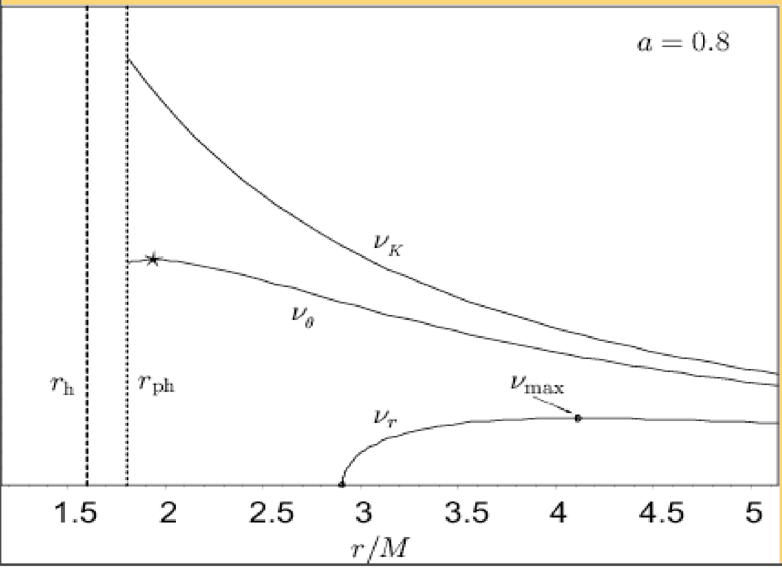
$$\ddot{U} + \omega_0^2[1 + h\cos(\omega_1 t)]\, U = 0$$

Resonance condition: $\omega_0 = (n/2)\, \omega_1$, $n = 1,2,3\ldots$

Suppose $\omega_1 < \omega_0$ (as true for epicyclic frequencies),

then first possibility $n=3 \Rightarrow$ 3:2 ratio.

WK & Abramowicz 2002



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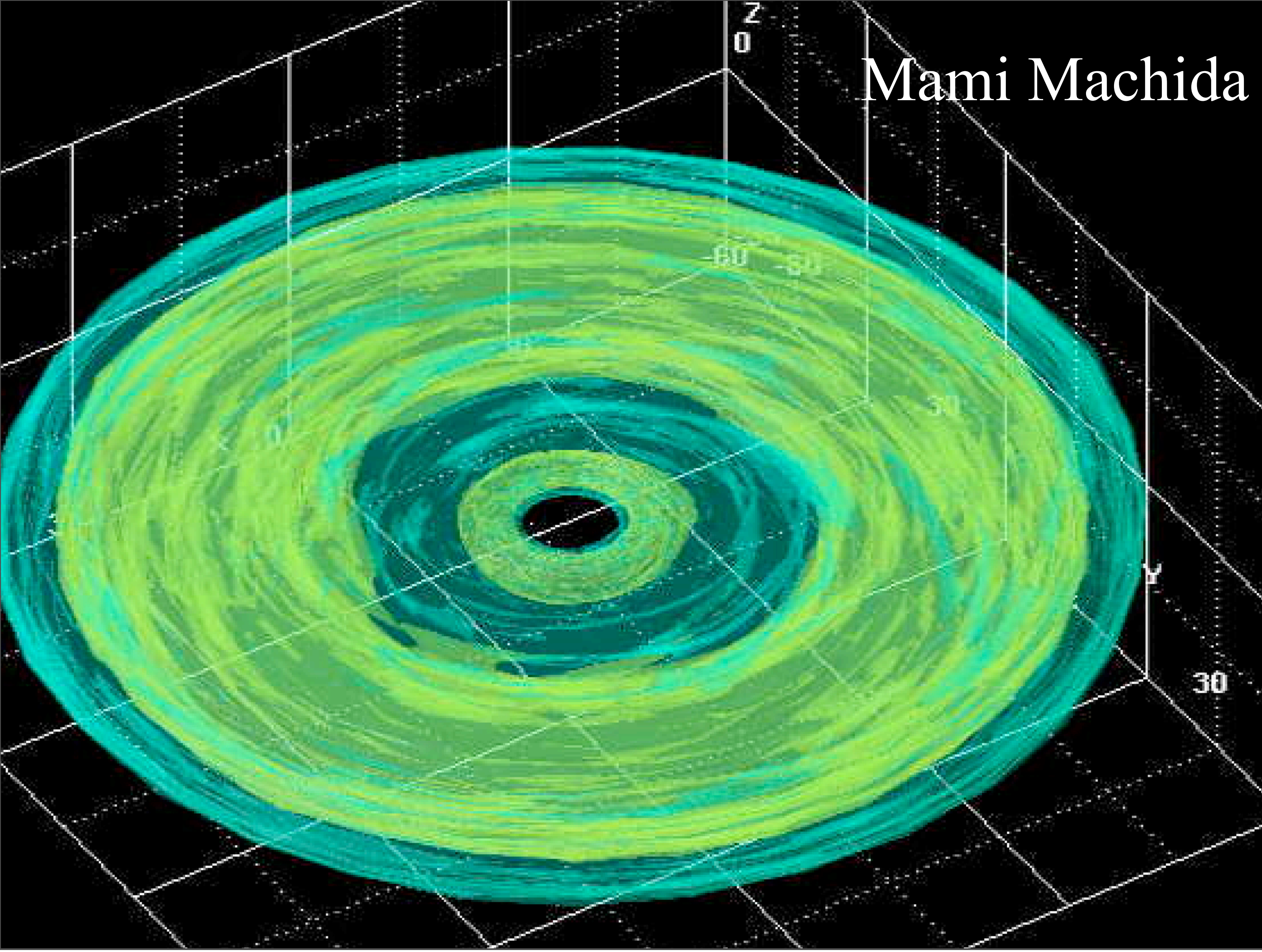
Non-linear resonance in disk oscillations.
Progress report for the past 5 years:

We **{M. Abramowicz, D. Barret, O. Blaes, T. Bulik, M. Bursa, J. Horak, S. Kato, V. Karas, W. Kluźniak, J.-P. Lasota, J. McClintock, W. Lee, J.F. Olive, P. Rebusco, R. Remillard, E. Šramkova, N. Stergioulas, Z. Stuchlík, G. Török, ...}**

have

1. identified possible origin of 3:2 resonance
2. identified possible X-ray modulation mechanisms
3. - - cause of frequency-frequency correlation in NS
4. discovered & classified new modes of thin tori
5. identified two modes which are weakly coupled &
6. the same two modes have fixed frequency to ~10%
7. discussed implications for BH spin


Mami Machida
Z
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Regardless of the model, identification of HFQPOs as a non-linear resonance in disk oscillations is rock solid.

In black hole systems this must be a strong-gravity phenomenon.

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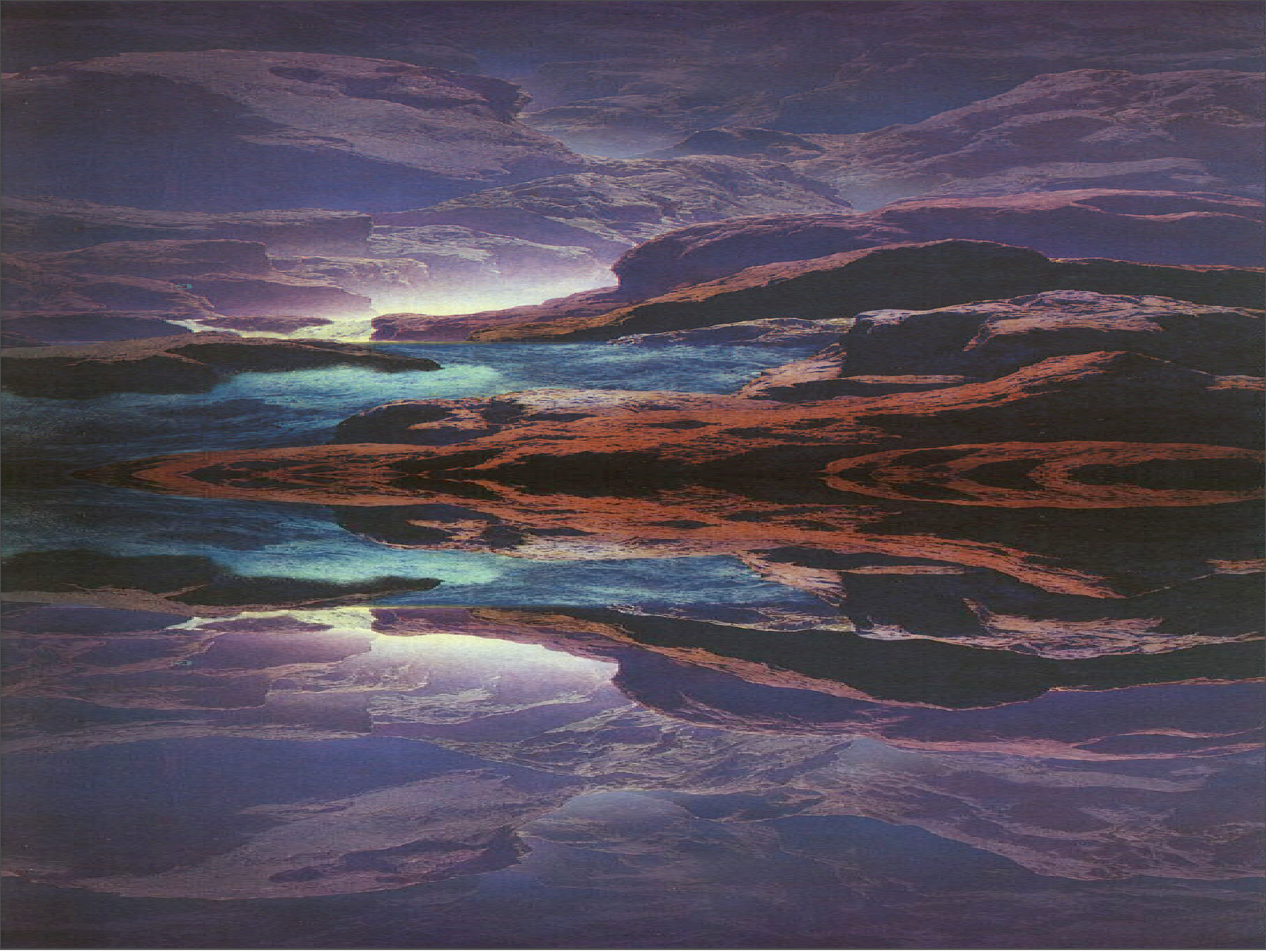
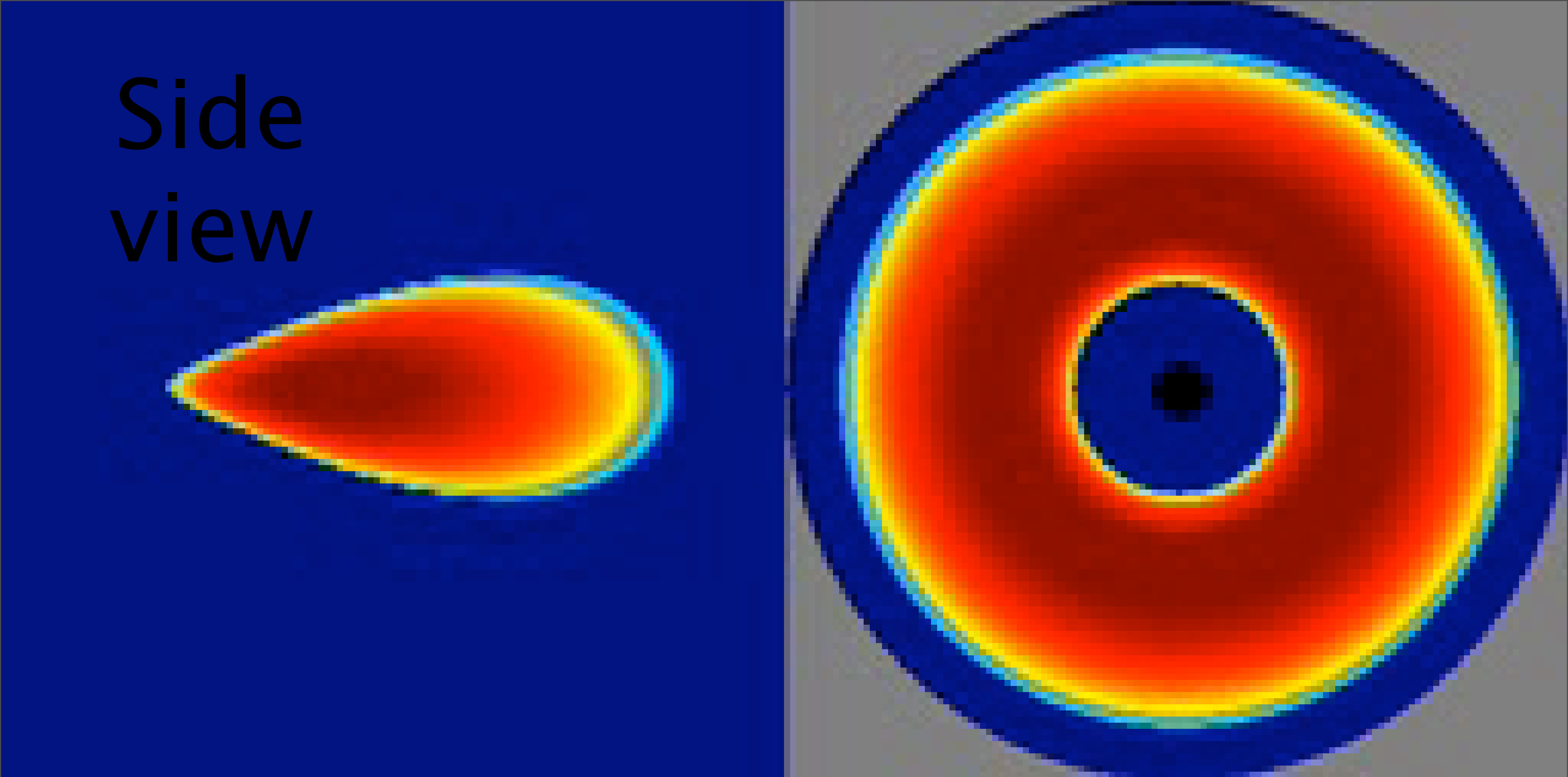
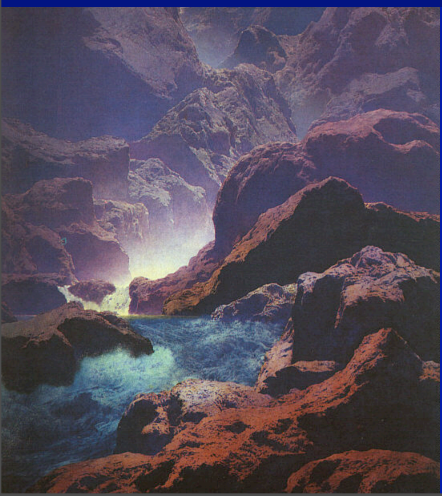




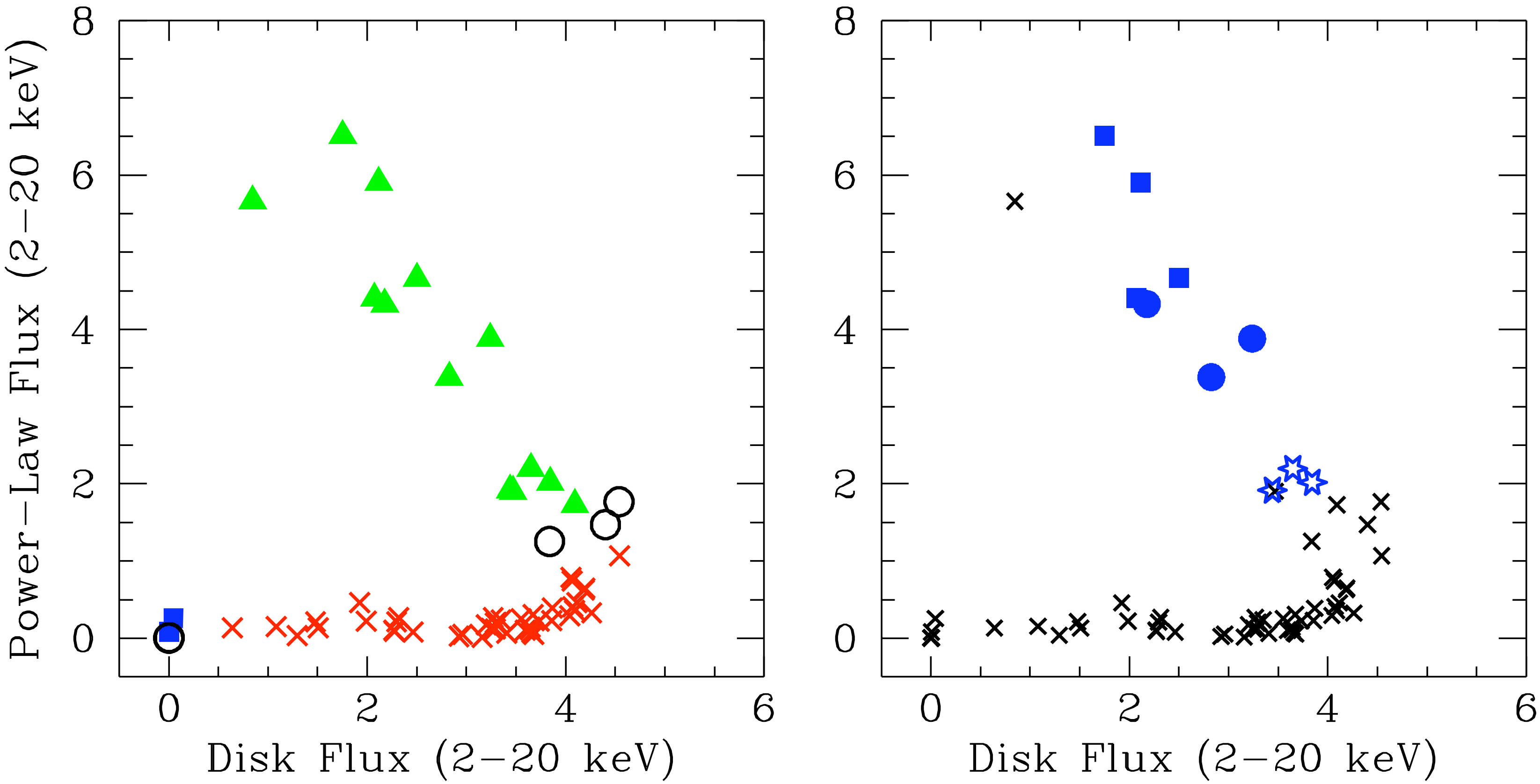


Simulation of flow of matter
around a black hole
(Balbus & Hawley, MRI)

Remillard & Mc Clintock

300 Hz & 450 Hz

f=f(M, J)



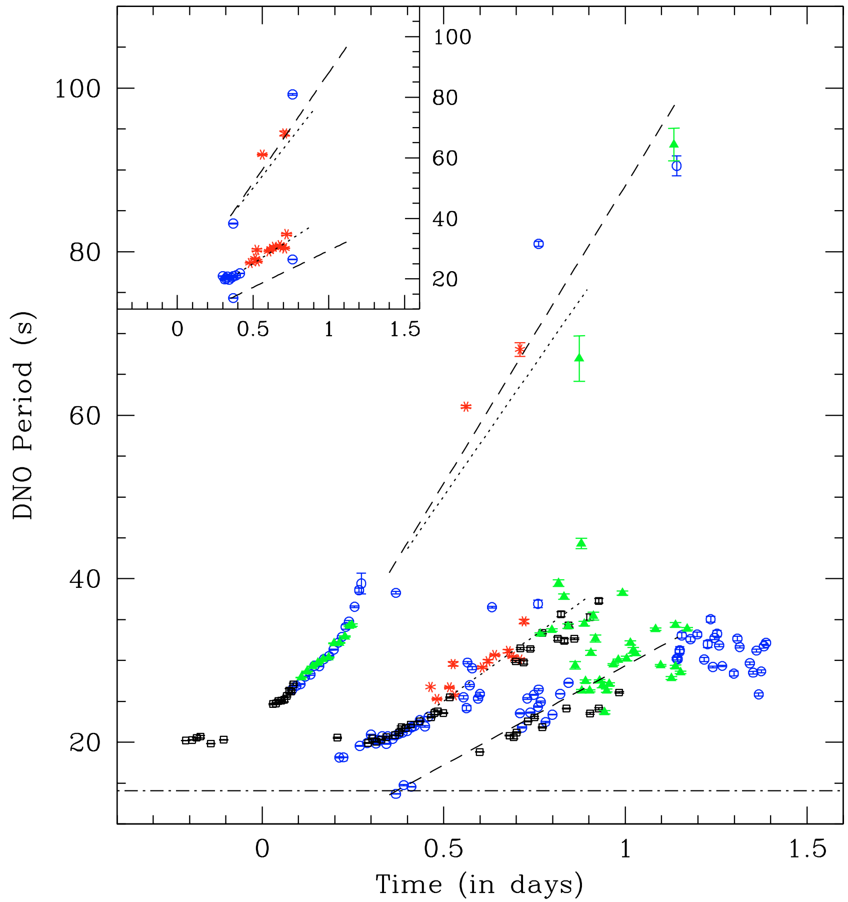
Source (BH) came back from quiescence after 9 years, and the two frequencies are the same as before! Hence frequencies depend on Mass and Spin alone.


Phoebus on Halzaphron

Warner & Woudt 2005

VW Hyi

WD system



This is a textbook example of non-linear response of forced oscillator (WK, Lasota, Abramowicz, Warner 2005), e.g. fixed eigen-frequency, decreasing forcing frequency (or vice versa).