1. Introduction

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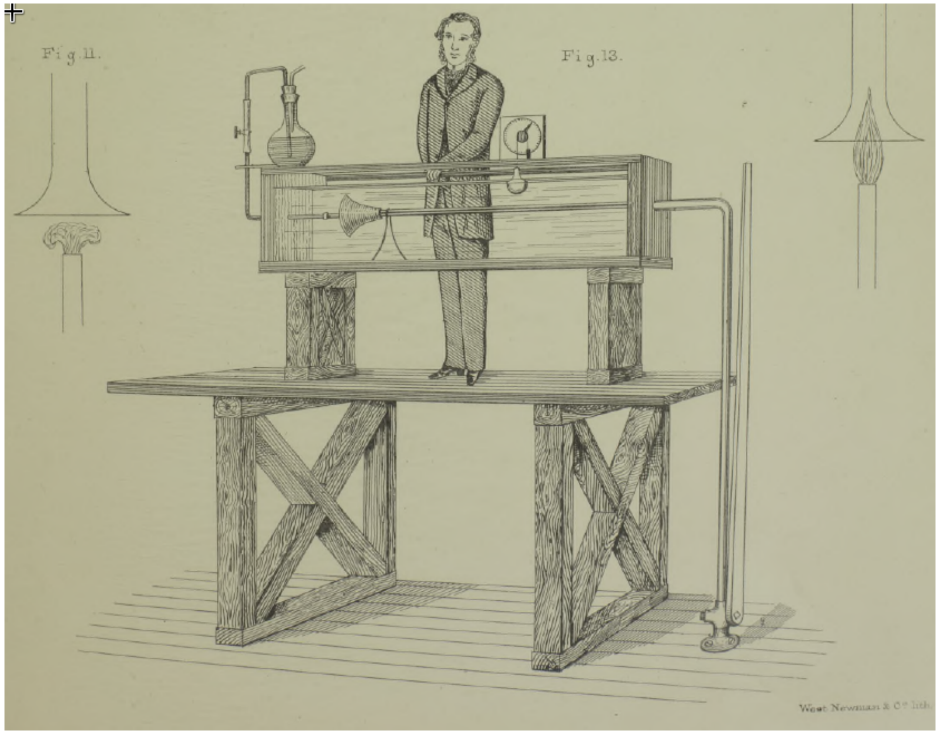
Two simple examples

- ▷ Pipe flow
- ▷ Channel flow
- ▷ The **velocity profile** can be derived from basic principles w/o invoquing involved equations

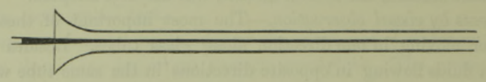
Transition to turbulence



The general results were as follows :—

(1.) When the velocities were sufficiently low, the streak of colour extended in a beautiful straight line through the tube, fig. 3.

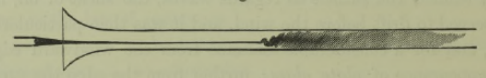
Fig. 3.



(2.) If the water in the tank had not quite settled to rest, at sufficiently low velocities, the streak would shift about the tube, but there was no appearance of sinuosity.

(3.) As the velocity was increased by small stages, at some point in the tube, always at a considerable distance from the trumpet or intake, the colour band would all at once mix up with the surrounding water, and fill the rest of the tube with a mass of coloured water, as in fig. 4.

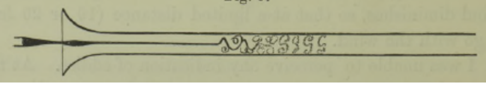
Fig. 4.



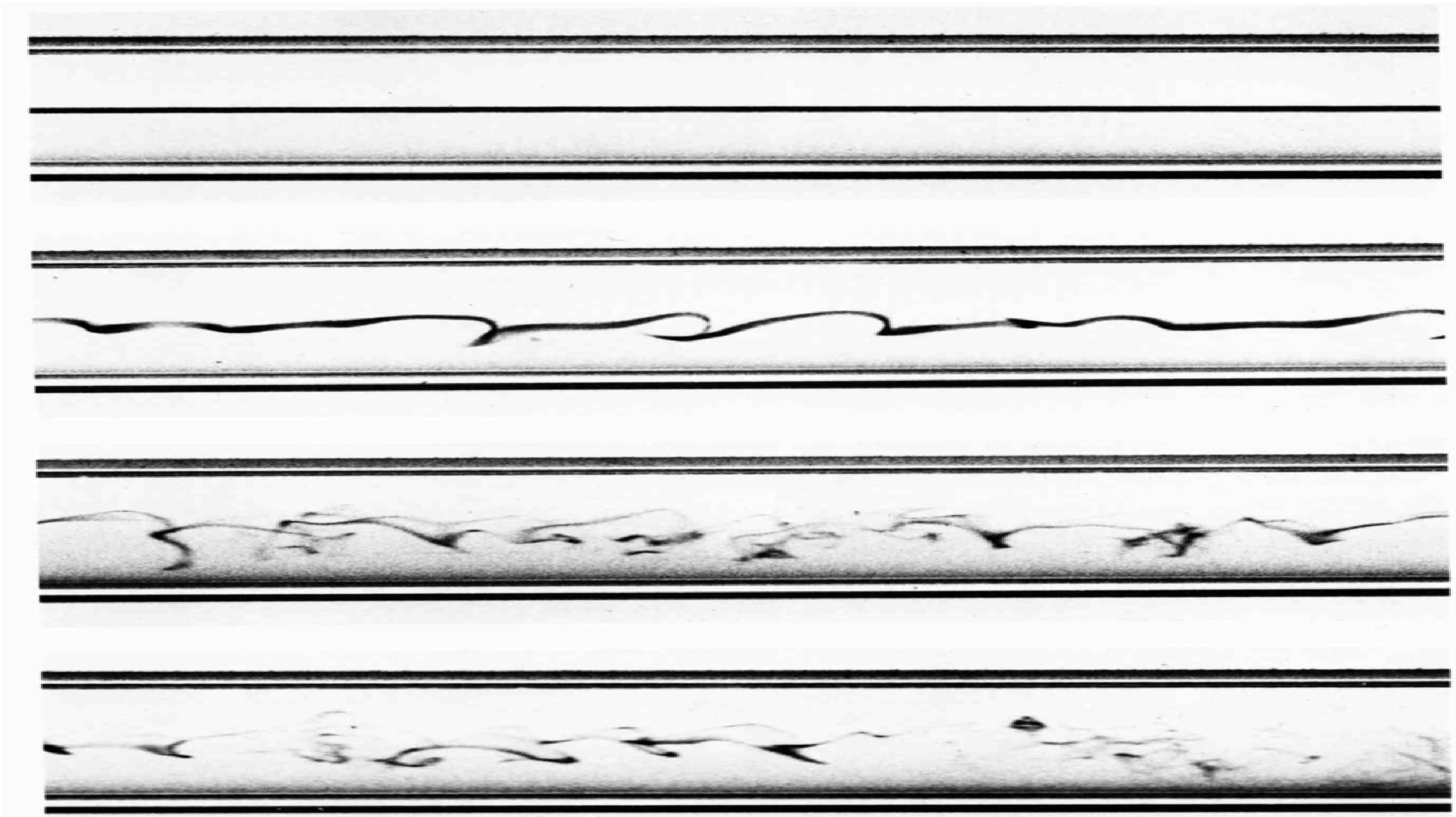
Any increase in the velocity caused the point of break down to approach the trumpet, but with no velocities that were tried did it reach this.

On viewing the tube by the light of an electric spark, the mass of colour resolved itself into a mass of more or less distinct curls, showing eddies, as in fig. 5.

Fig. 5.



- ▷ Seminal paper: Osborne Reynolds 1883 (see here for some historical context)



Experiment redone w/ the original apparatus at Manchester University (but increased car traffic introduced additional perturbations)

1. *Objects and results of the investigation.*—The results of this investigation have both a practical and a philosophical aspect.

In their practical aspect they relate to the *law of resistance to the motion of water in pipes*, which appears in a new form, the law for all velocities and all diameters being represented by an equation of two terms.

In their philosophical aspect these results relate to the fundamental principles of fluid motion; inasmuch as they afford for the case of pipes a definite verification of two principles, which are—*that the general character of the motion of fluids in contact with solid surfaces depends on the relation between a physical constant of the fluid and the product of the linear dimensions of the space occupied by the fluid and the velocity.*

The results as viewed in their philosophical aspect were the primary object of the investigation.

As regards the practical aspect of the results it is not necessary to say anything by way of introduction; but in order to render the philosophical scope and purpose of the investigation intelligible it is necessary to describe shortly the line of reasoning which determined the order of investigation.

- In this paper, O. Reynolds introduces the 'Reynolds number':

$$\mathrm{Re} = \frac{U_l l}{\nu} = \frac{\rho U_l l}{\mu}$$

- ν: kinematic viscosity, μ: dynamic viscosity

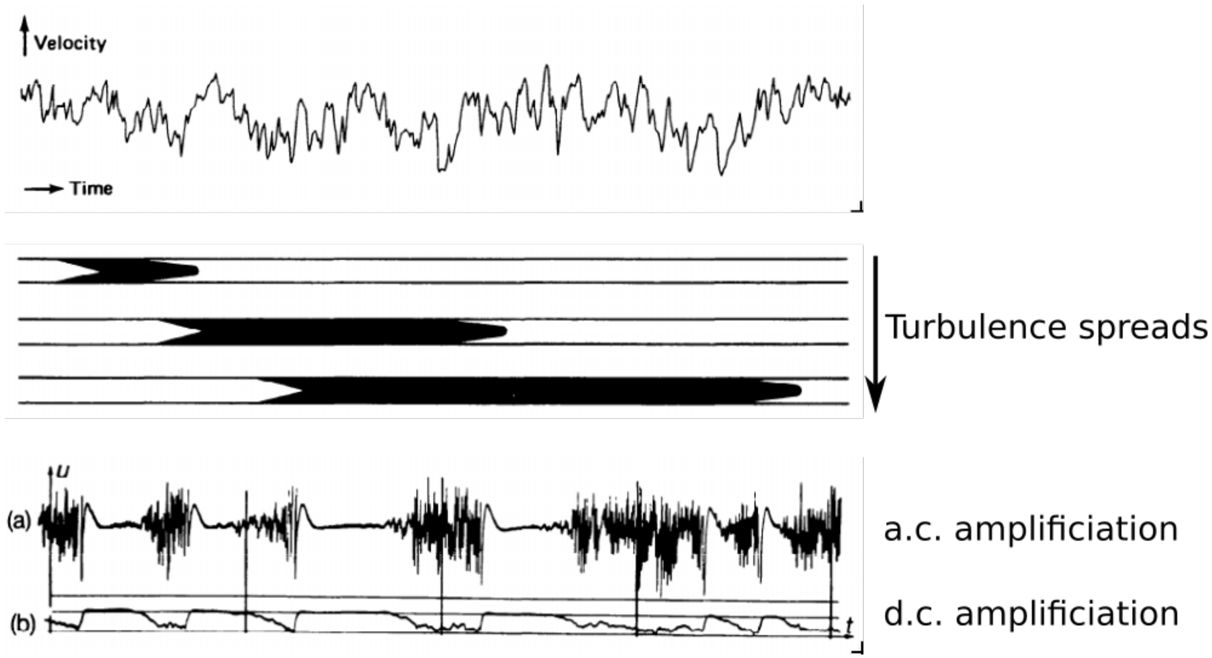
- ▷ The Navier-Stokes equation reads (neglecting e.g. gravity):

$$\frac{\partial \boldsymbol{u}}{\partial t} + (\boldsymbol{u} \cdot \boldsymbol{\nabla})\boldsymbol{u} = -\frac{1}{\rho}\boldsymbol{\nabla} p + \nu \nabla^2 \boldsymbol{u}$$

- ▷ Reynolds number: definition and physical interpretation
- ▷ Low Reynolds number flows
  - Flows with low Reynolds numbers (Re) are laminar
  - When Re is very small (<1): 'creeping motion'

$$\nu \boldsymbol{\nabla}^2 \boldsymbol{u} = \rho \boldsymbol{\nabla} p$$

Turbulent flow in a pipe



- ▷ Velocity along the pipe in a turbulent regime
- ▷ A turbulent slug spreads with time
- ▷ Velocity fluctuations in a pipe at Re=2550
  - Alternance of laminar and turbulent regions separated by a sharp interface
- ▷ Transition to turbulence takes place for Re=2000 to 10^5

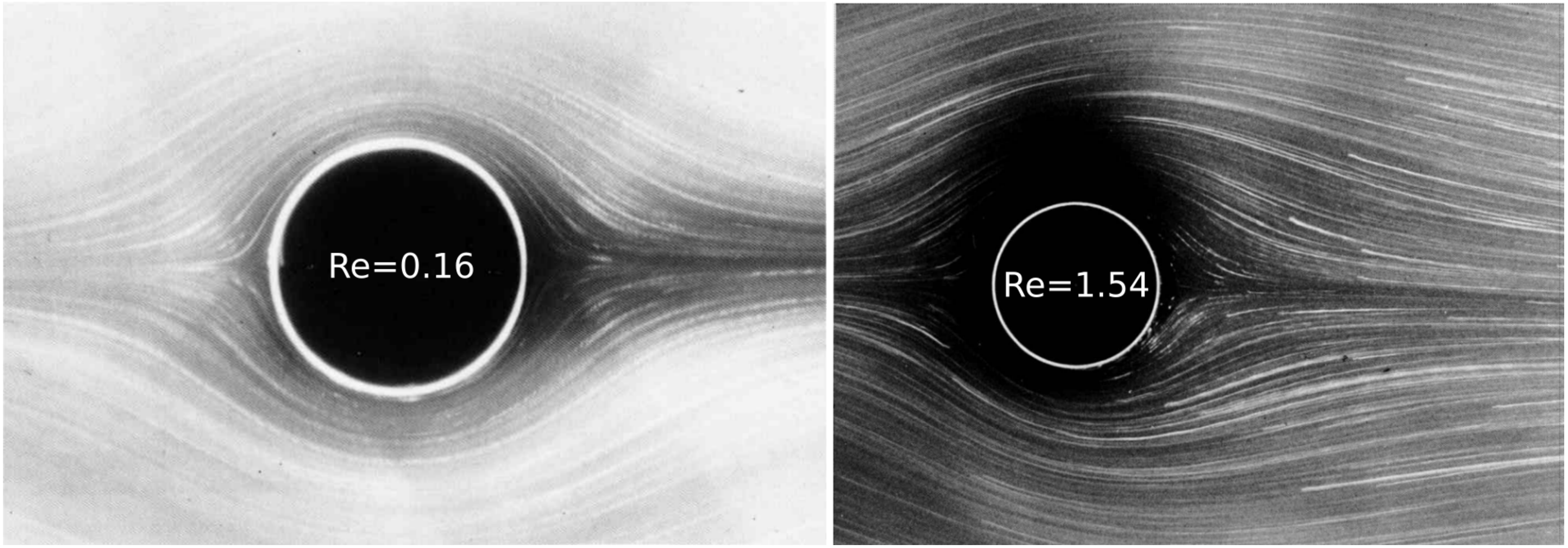
Observation of flows

- Observe controled flows in setups allowing to vizualize the **velocity field**
- Simple examples to catch basic properties of flows and see what types of phenomena fluids commonly produce
- Most of the following images are taken from the classical textbook *An album of fluid motion* (Milton van Dyke, 1988), a collection of photographs realized by various researchers worldwide.

Flow past a cylinder

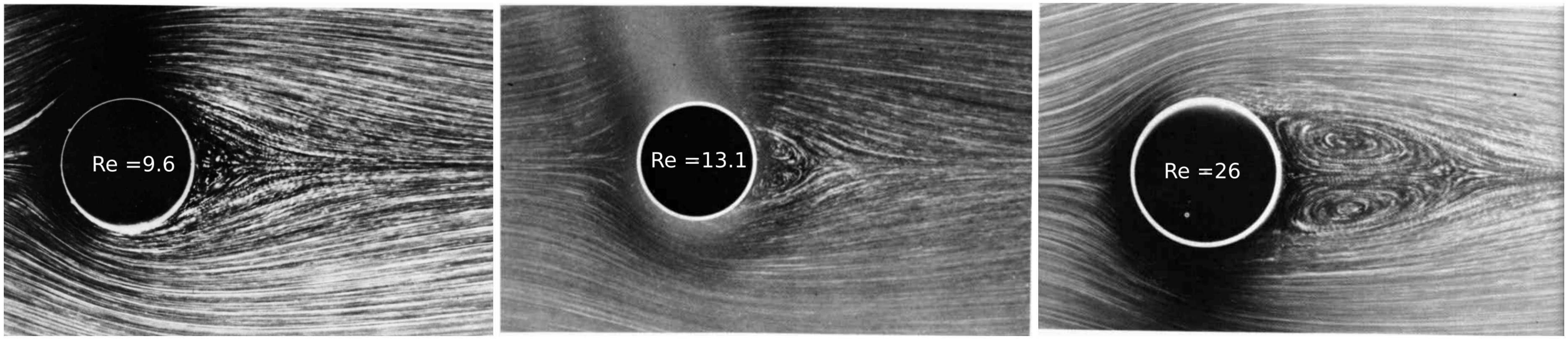
- ▷ This is a simple setup to study two-dimensional flow
- ▷ The cylinder has a length $\gg$ diameter
- ▷ It is either at rest and the flow moves from left to right
- ▷ Or equivalently, moves from right to left, in a fluid at rest

Low Re



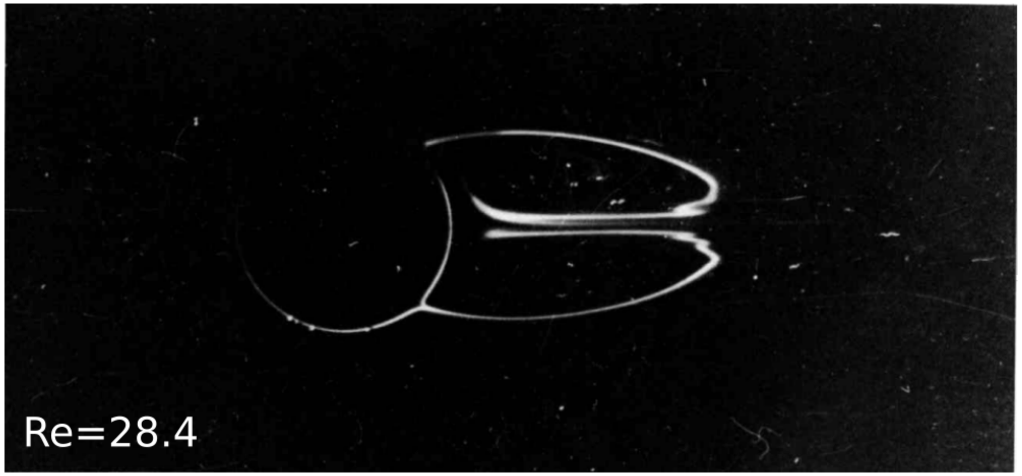
- ▷ Left: ’creeping flow’; a creeping motion is one at very low Reynols number such that only the viscous and pressure gradient terms remain;
- ▷ Right: ’laminar flow’
- ▷ The left/right (fore-and-aft) symmetry is lost when Re is increased

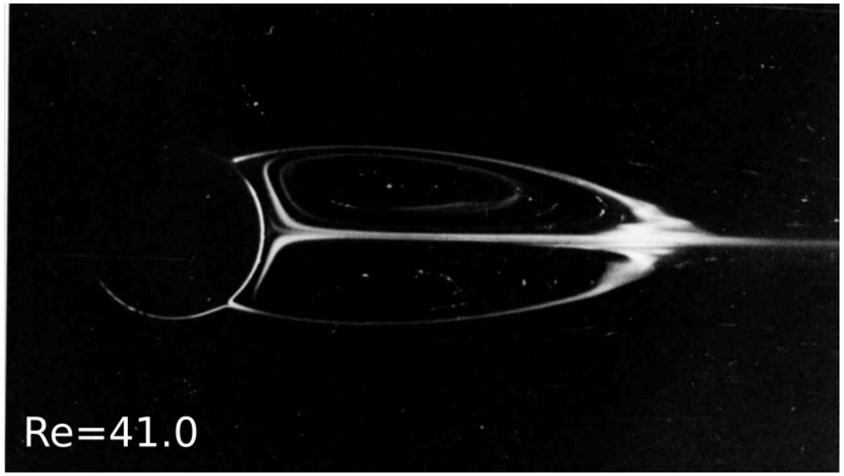
Moderate Re



- ▷ Left: as Re (hence the flow velocity) is increased, eddies form behind the cylinder; the process is called 'separation';
- ▷ Middle: eddies are larger as they are stretched by the flow; the size of the eddies increase with Re, until they become unstable (Re$\sim$40).
- ▷ Right: the eddies are $\sim$ twice more extened in the flow direction, and $\approx$ $\sqrt{2}$ in the perpendicular direction, and their centers are farther from the cylinder;

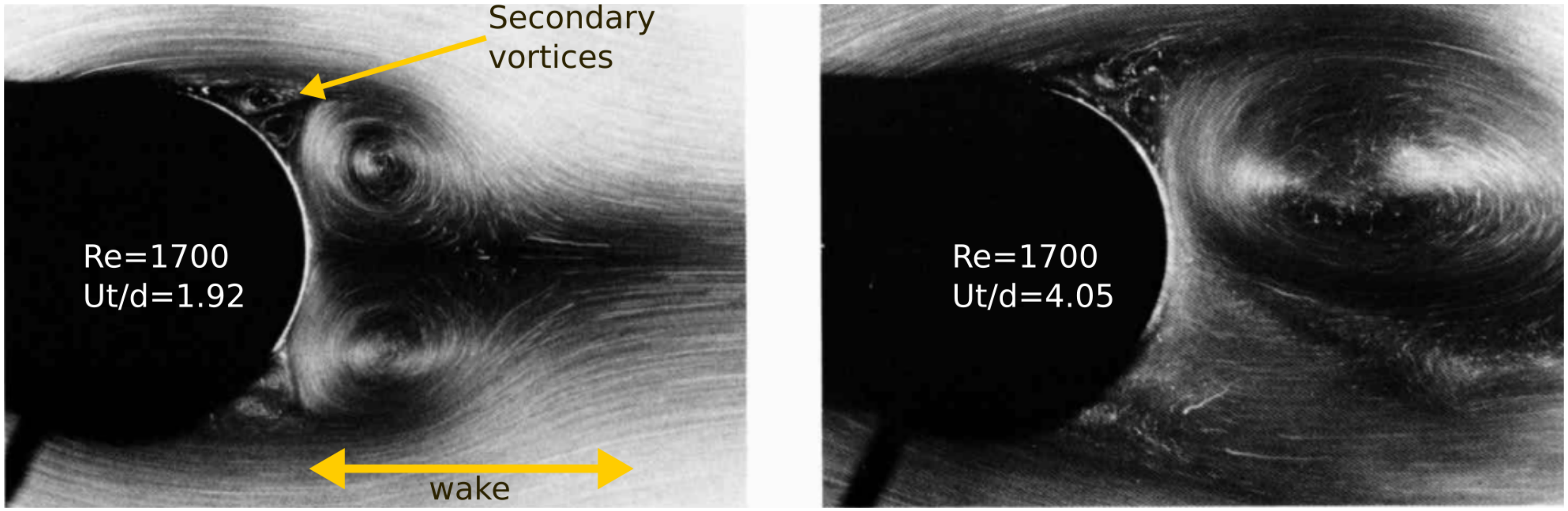
Circulation flow past the cylinder





- We look at condensed milk coating the cylinder
- We thus see the path of fluid particles trapped in eddies, or moving away;

High Re



- ▷ The cylinder is moved (from R to L)
- ▷ Fractional displacement relative to dimater = Ut/d
- ▷ Formation of secondary vortices upstream of the two main ones
- ▷ When Ut/d=4
  - up/down symetry is lost;
  - Vortex shedding ('émission de tourbillons')

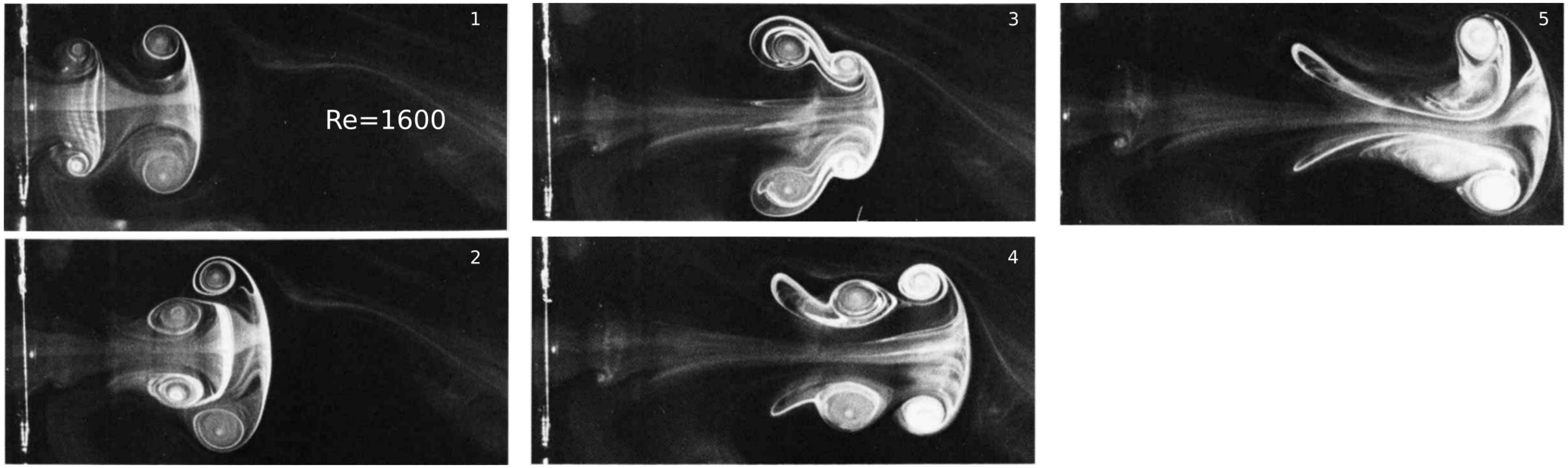
Vortices

- Vorticity

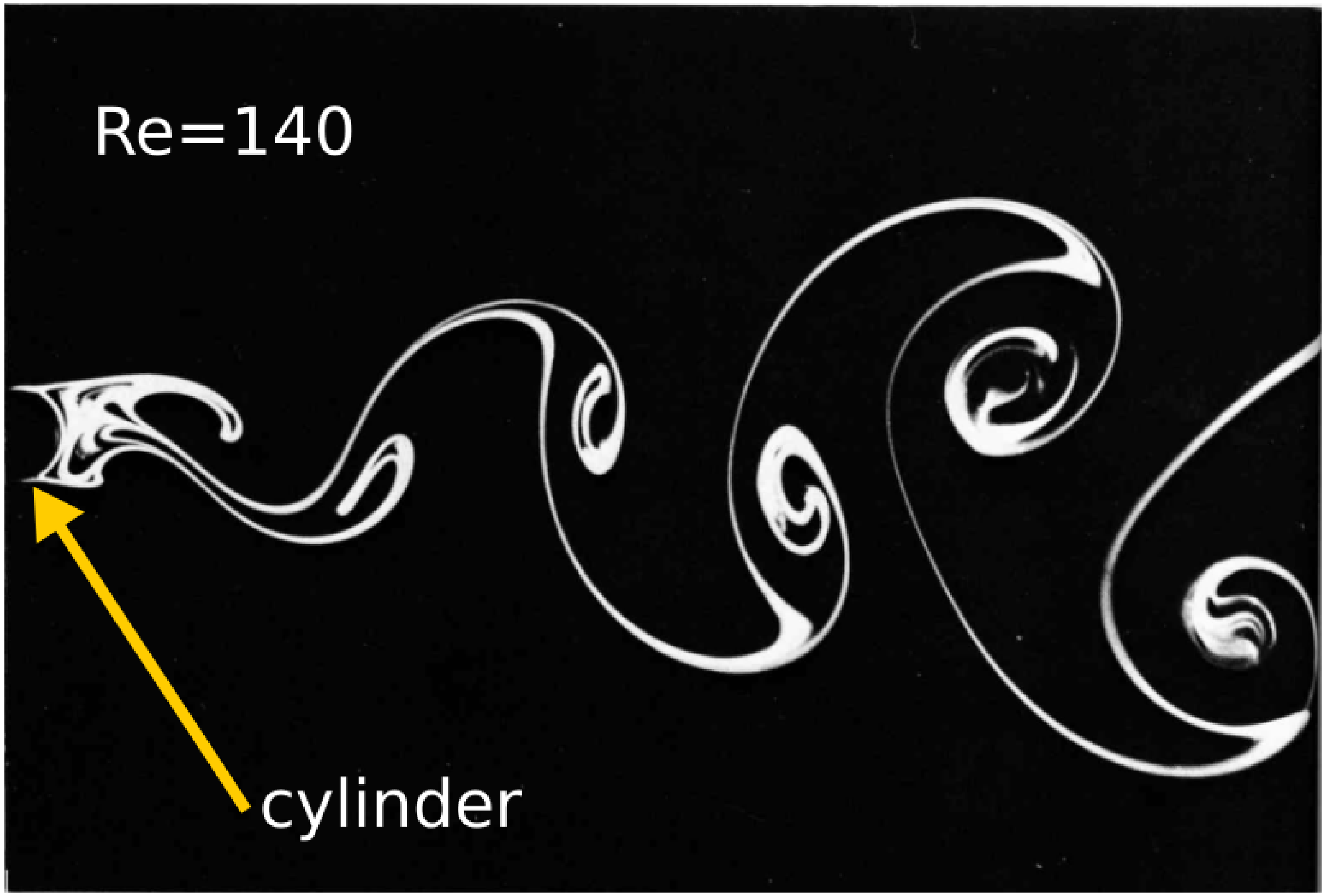
$$\boldsymbol{\omega} = \nabla \times \boldsymbol{u}$$

- Vorticity is a fundamental quantity in fluid dynamics; it is related to the deformation of a fluid particle.
- Vorticity obeys an equation similar to the Biot-Savart law; vorticity shares some properties with magnetic fields ($\boldsymbol{B}$).
- Vortices play an essential role in the transition from laminar to turbulence

Interaction of pairs of vortices

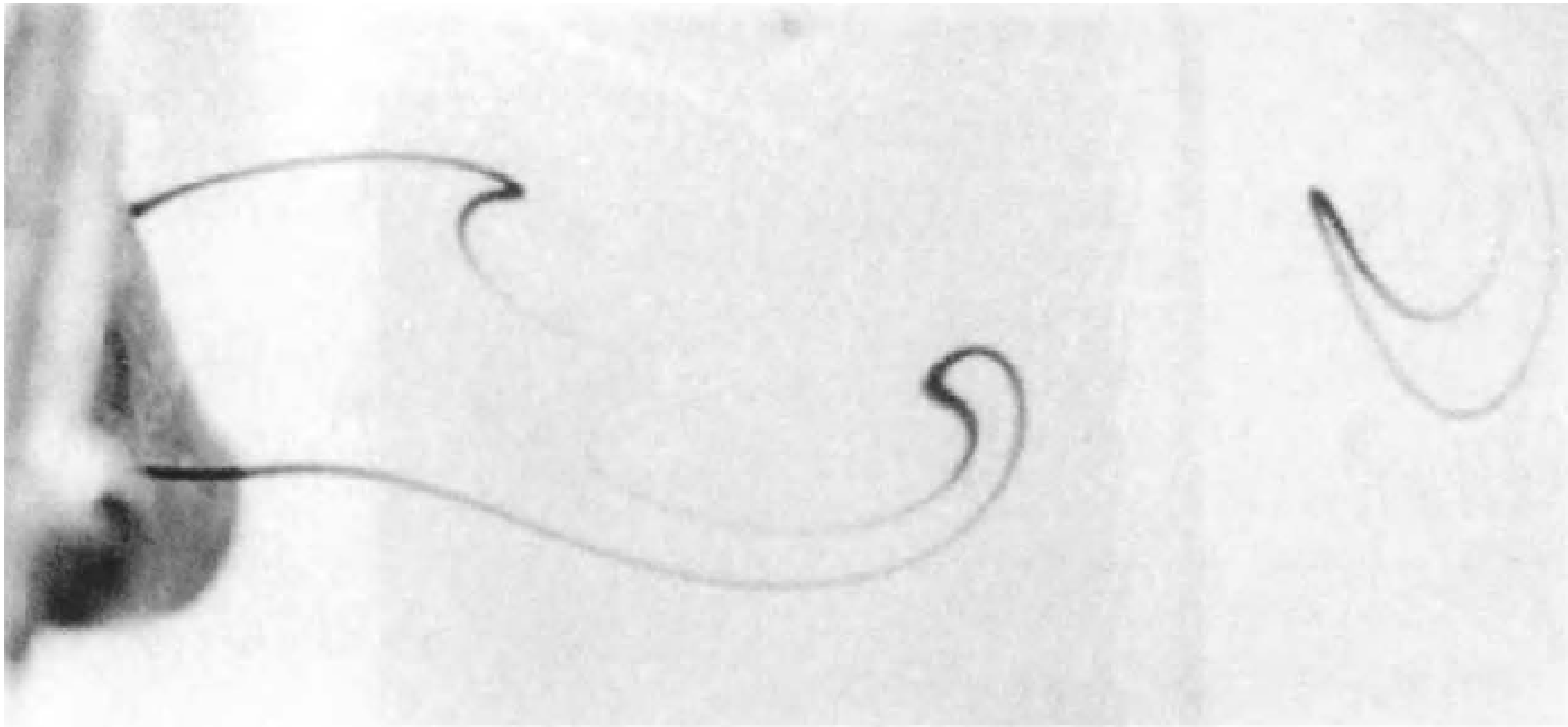


- ▷ Two puffs of air injected
- ▷ The second moves faster in the velocity field created by the first
- ▷ The 2nd goes through the 1st
- ▷ The same process repeats (1st going through the 2nd)



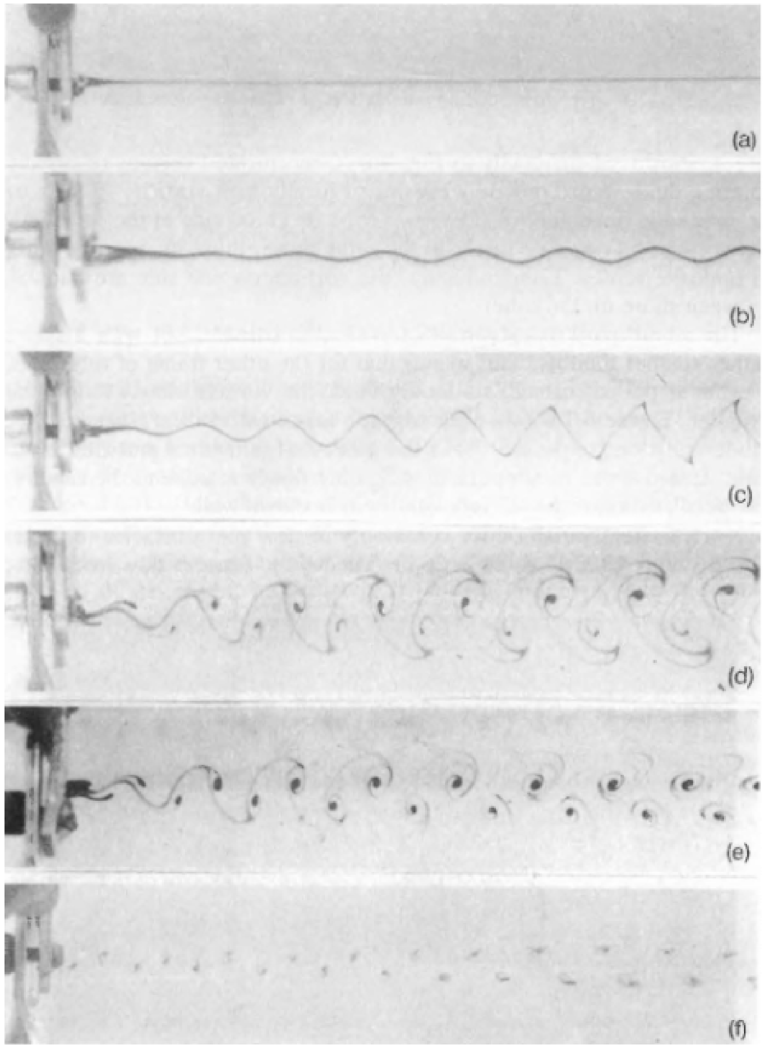
- Illustrates the Kelvin-Helmholtz instability
- Structure of the Kármán vortex street ('allée de tourbillons de Kármán) no longer depends on Re whence Re>100 (K-H instability does not depend on viscosity)

Eddy shedding



- At Reabove $\sim$100
- Eddies are 'emitted' periodically: whilst an eddy on one side is formed and being emitted (shed), another eddy, on the other side (with respect to symetry axis) is forming;

Kármán vortex street behind a cylinder



Re=30

Re=40

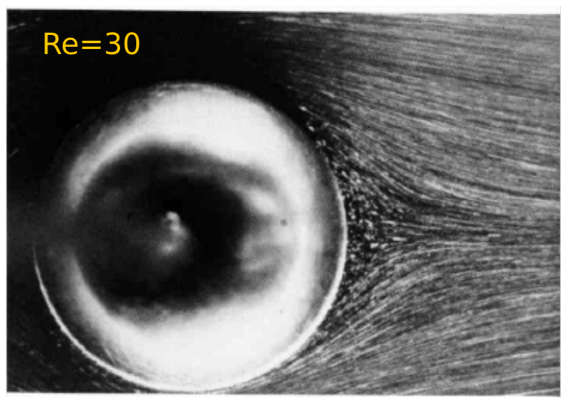
Re=47

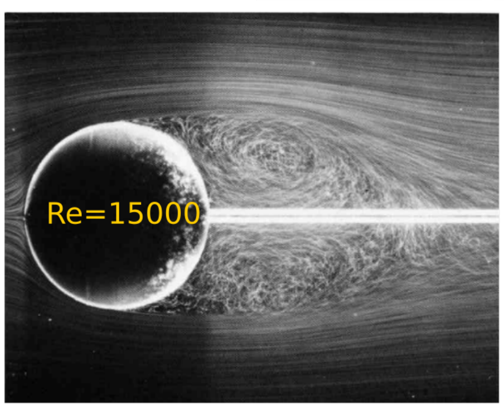
Re=55

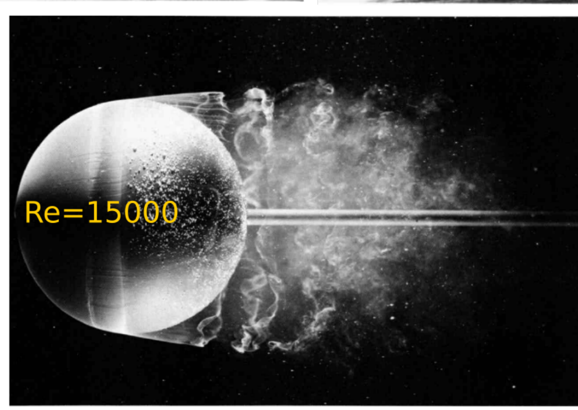
Re=67

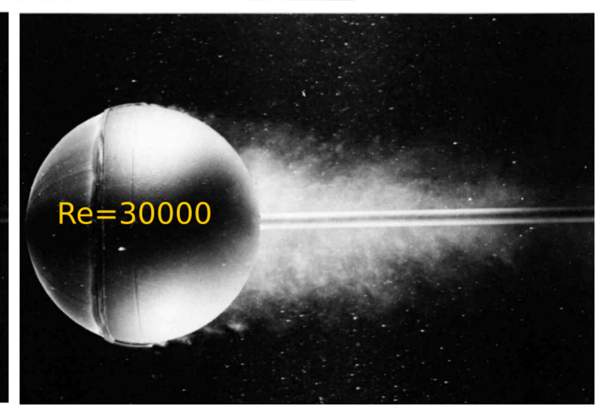
Re=100

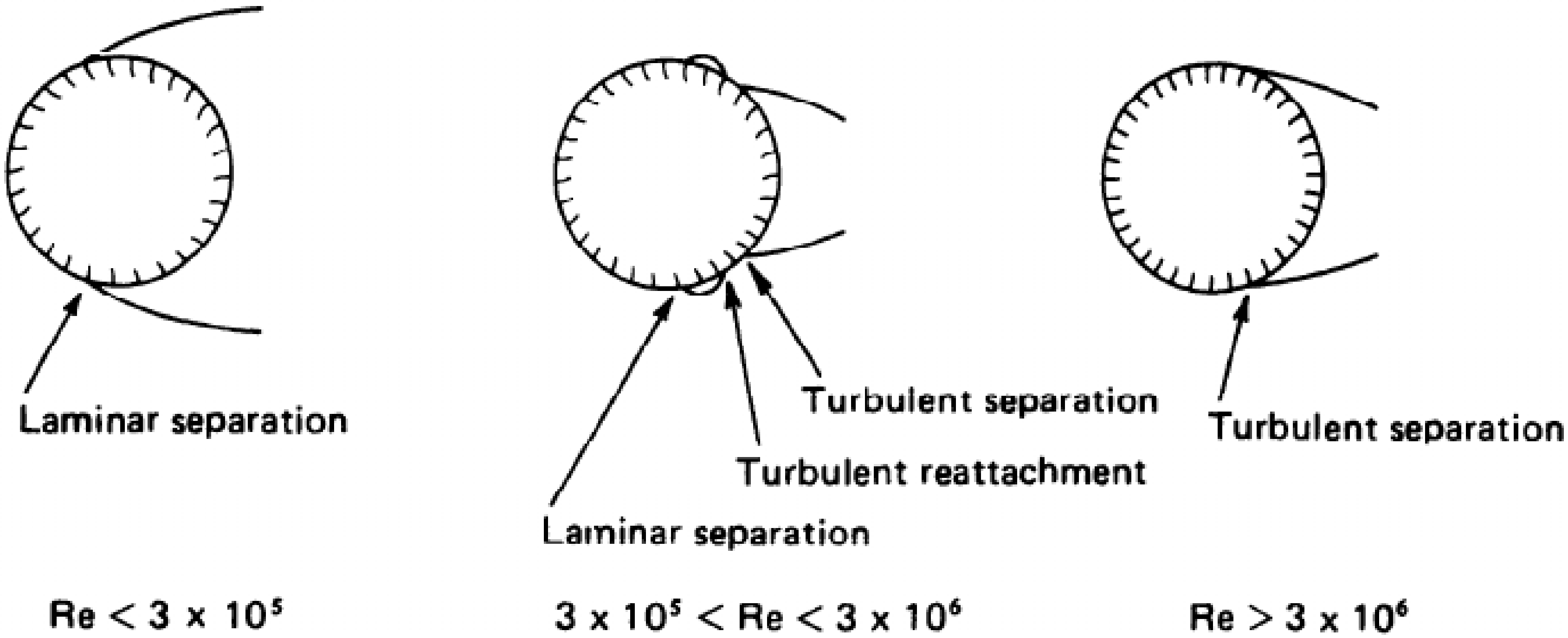
Boundary layers







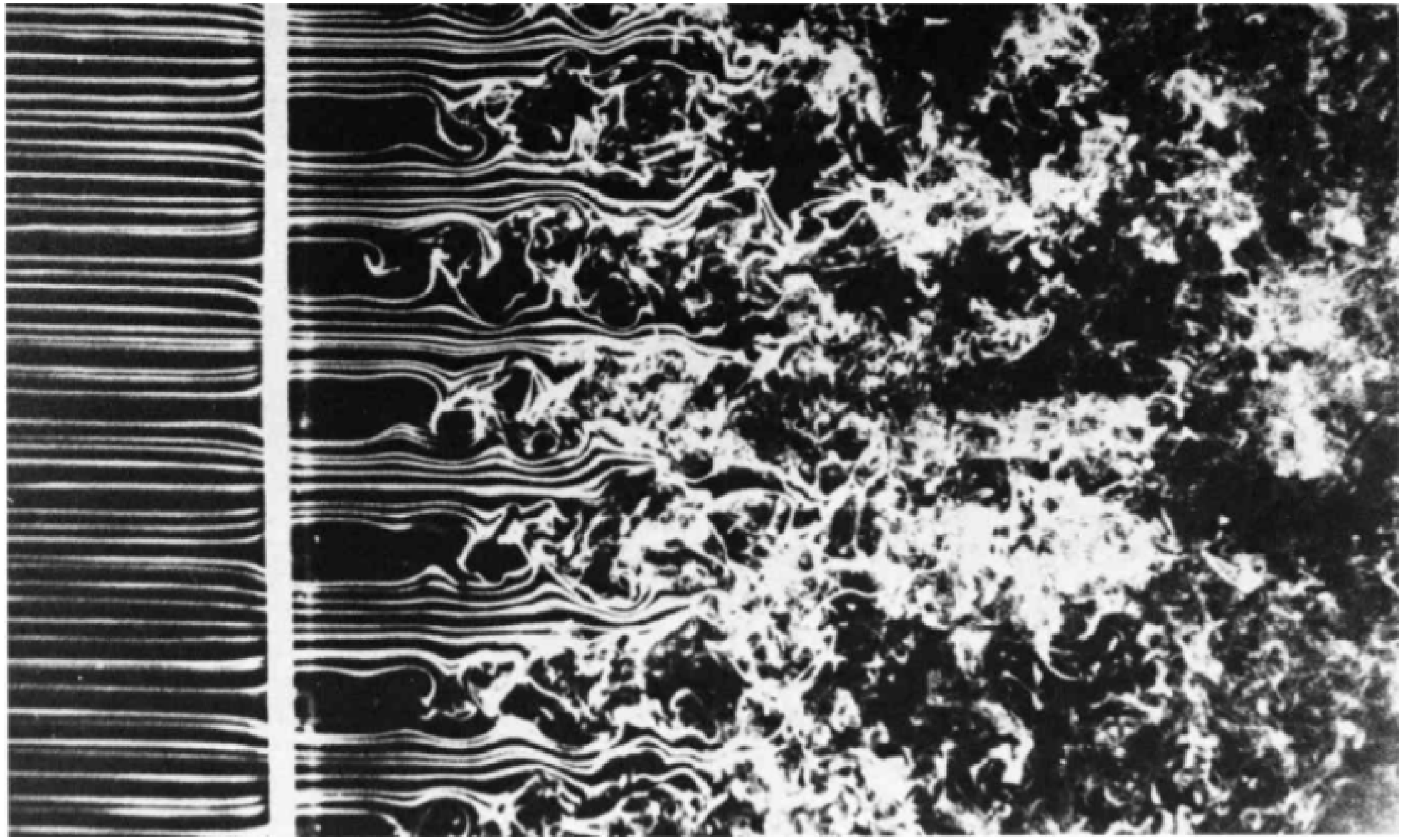




- At very high Reynolds number ($\mathrm{Re}\sim 3\times 10^5$ for a cylinder), a new transition takes place in the flow behind cylinder
- This transition is related to a transition within the **boundary layer**
- More generally, what happens in the flow is linked to what happends in this boundary layer

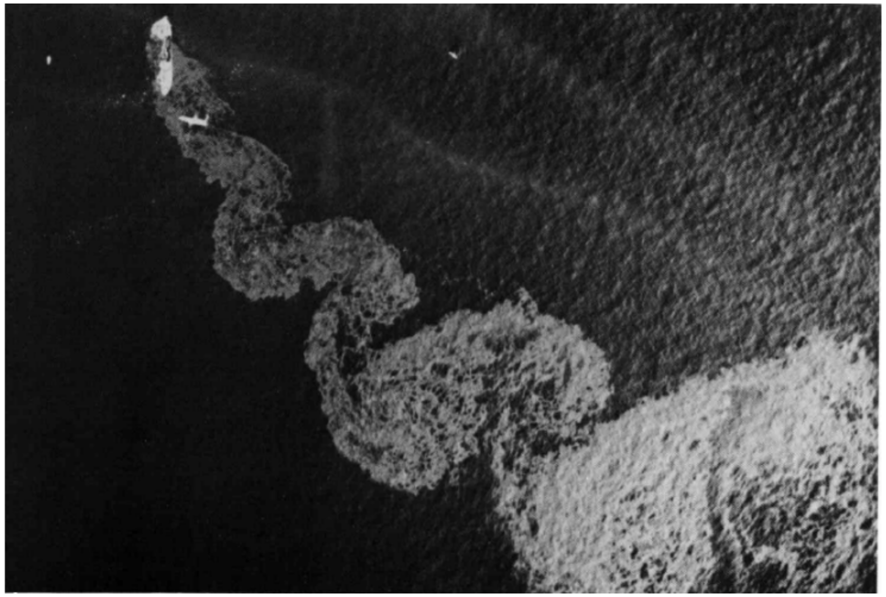
Turbulent flows

Transition to turbulence behind a grid



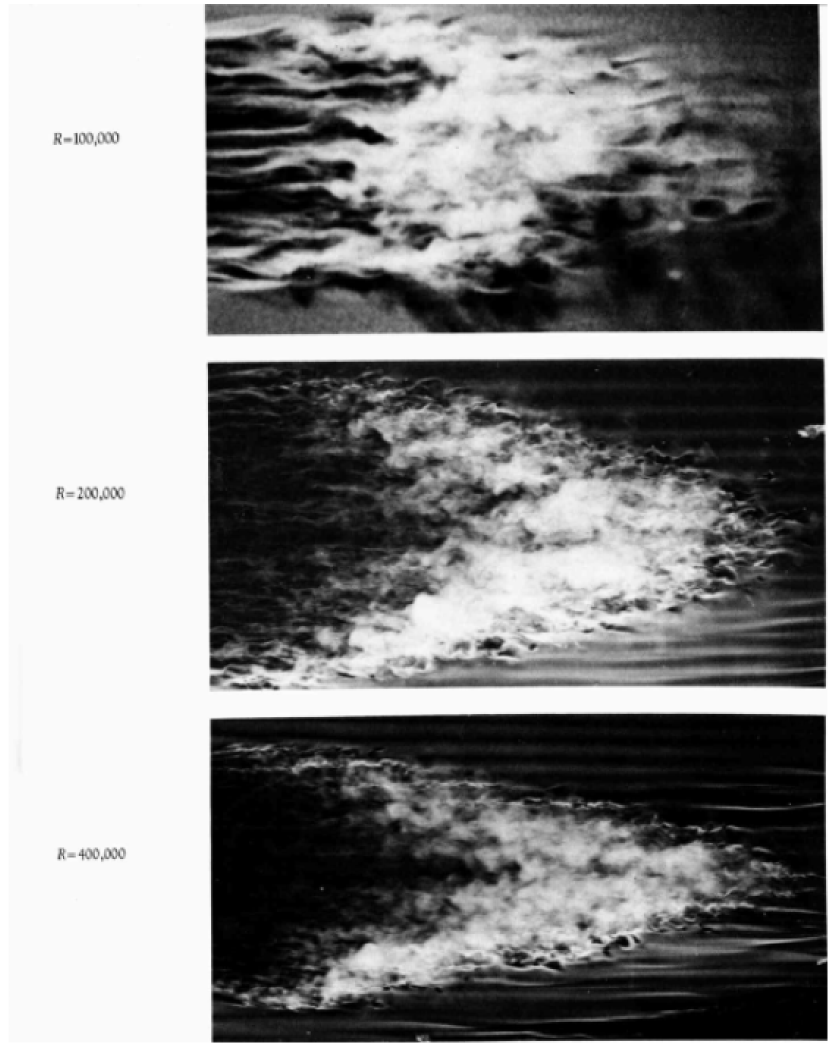
- ▷ Transition to turbulence by shear layer instability
- ▷ Re= 1500 (based on mesh size)

Turbulent wake behind an inclined obstacle



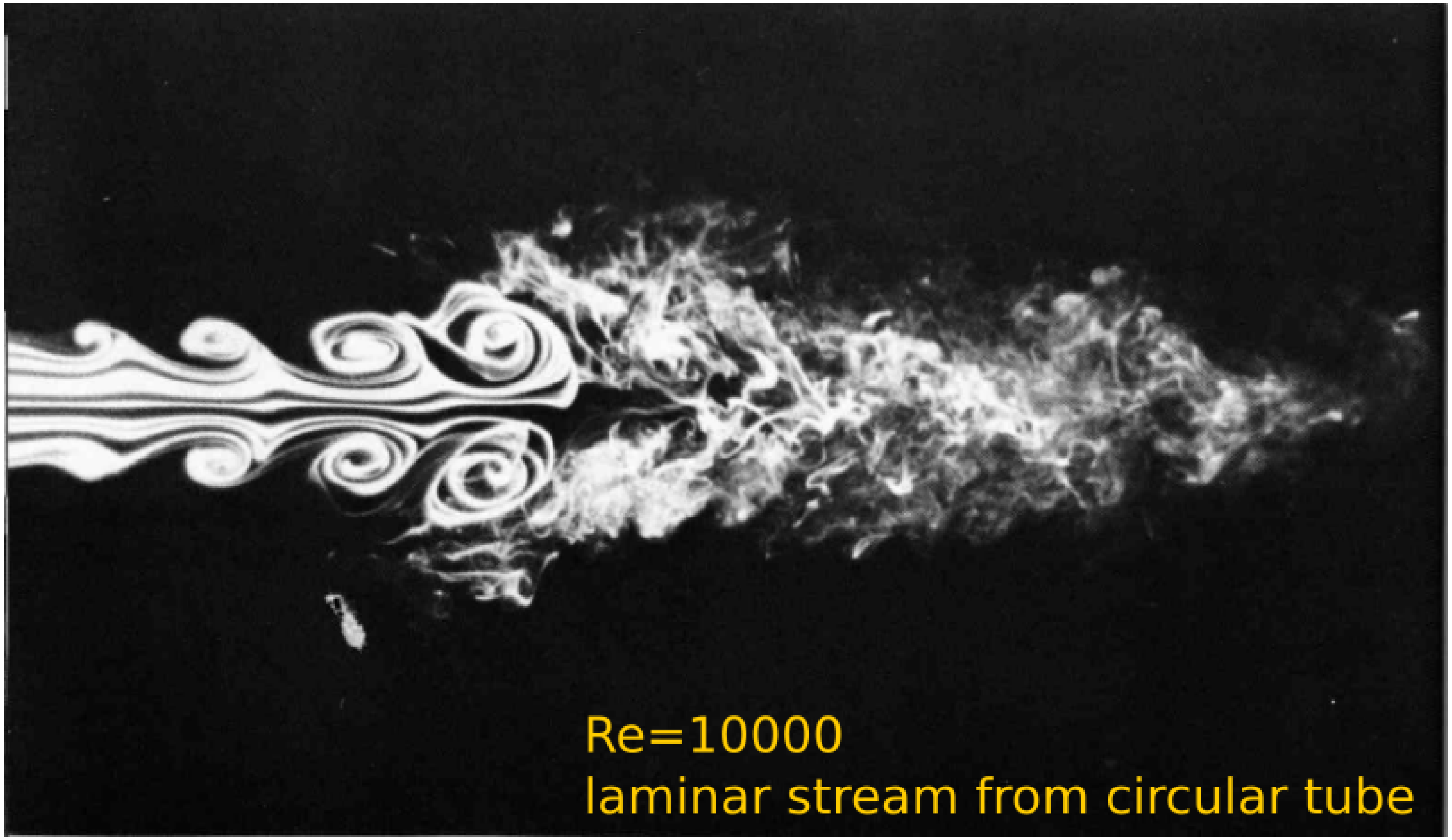
- ▷ Left: experimental setup of an inclined (45°) obstacle at rest in a flow with Re=4300.
- ▷ Right: Wake of a grounded tankship; comparison with left image indicates that the boat was inclined by $\approx$ 45°with respect to the current; $\mathrm{Re}\sim 10^7$;

Small scales

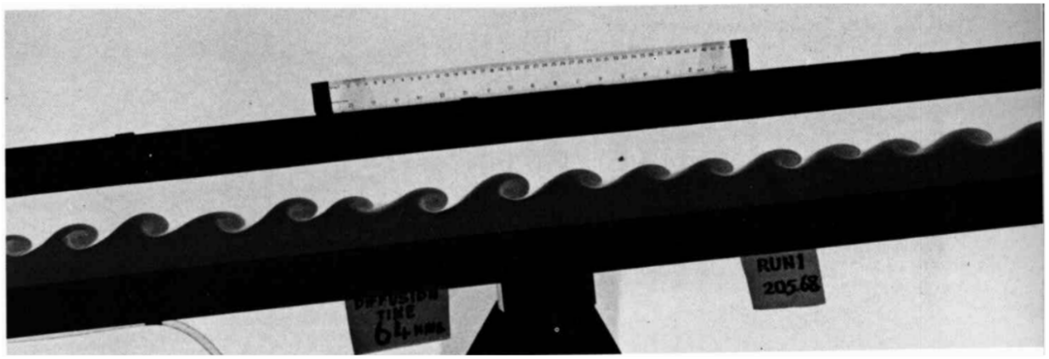


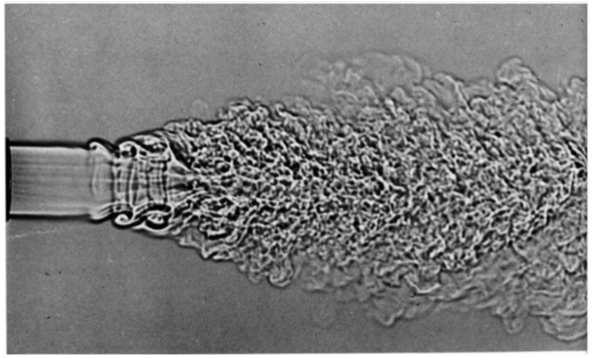
Instabilities

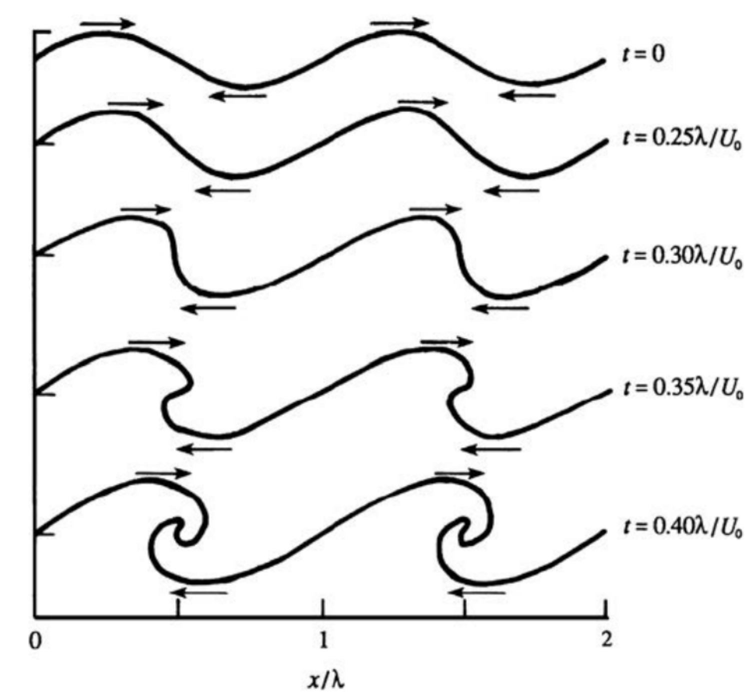
- These are the fundamental mechanism leading to structures in flows
- Instabilities are involved in one way or another in the transition from laminar to turbulent flow: laminar flows are unstable against some classes of perturbations (infinitesimal or finite amplitude);
- Instabilities are found in a huge array of situations in nature
- They play a fundamental role in the formation of stars and planets
- Fantastic examples are observed in the atmosphere of planets (Earth, Jupiter, etc), or in supernova remnants

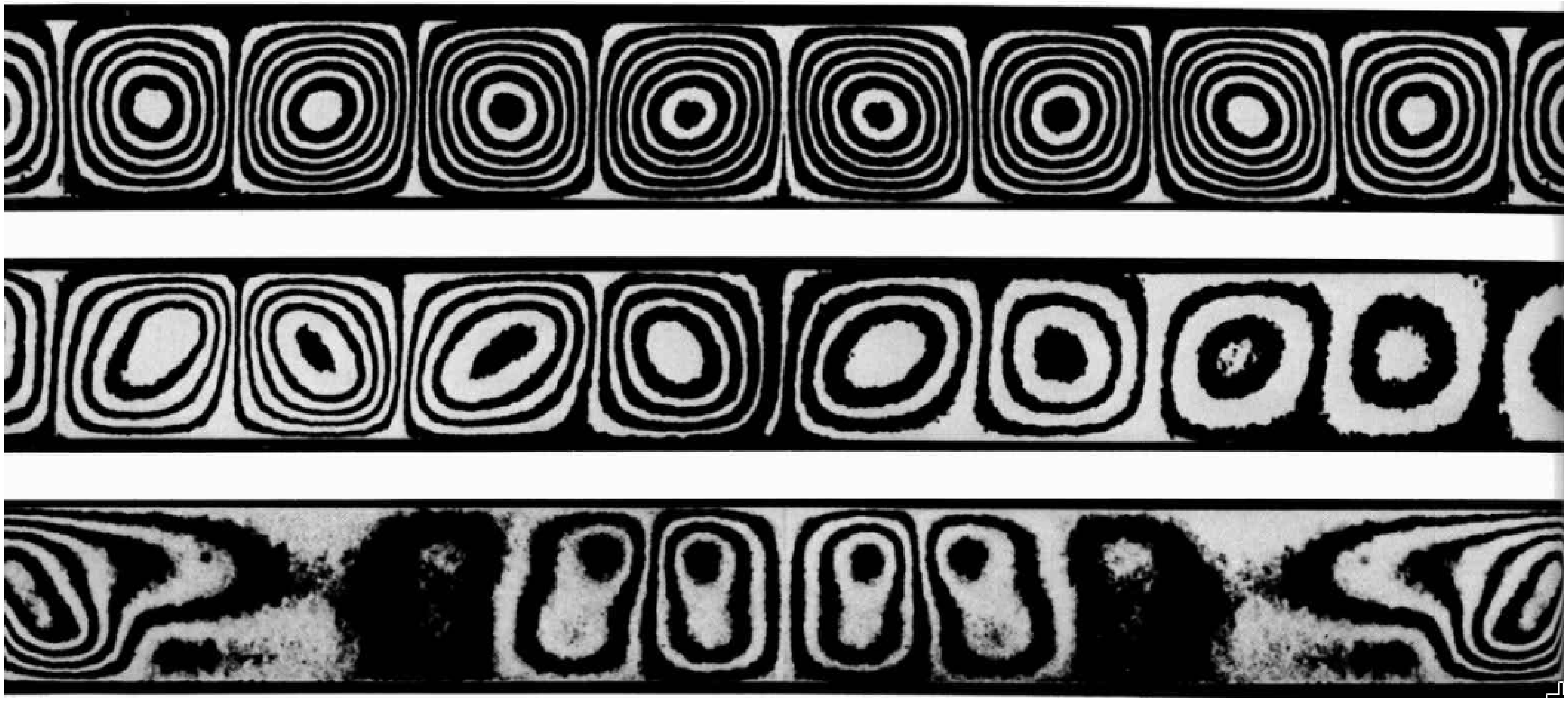

Re=10000
laminar stream from circular tube

Kelvin-Helmholtz instability









Basic definitions and assumptions

- ▷ See your notes