MAGNITUDE AND COLOR SYSTEMS

INTRODUCTION

"Magnitudes" are an ancient and arcane, but by now unchangeable, way of characterizing the brightnesses of astronomical sources. They were introduced by the Greek astronomer Hipparchus ca. 130 BC. He arranged the visible stars in order of apparent (naked eye) brightness on a scale that ran from 1 to 6, with stars ranked "1" being the brightest. The ranks were called *magnitudes*. The faintest stars visible to the eye under excellent sky conditions were ranked as sixth magnitude.

Much later it became clear that because of the way the human eye responds to a stimulus, magnitudes were proportional to the <u>logarithm</u> of the EM power entering the eye from the source.

The modern magnitude system has been quantified as follows:

$$m = -2.5 \log_{10} \frac{f(i)}{Q(i)}$$

where f is the mean <u>spectral flux density</u> (see Lec. 2) from a source at the top of the Earth's atmosphere averaged over a defined band and Q is a normalizing constant for that band.

Although this definition looks peculiar, it offers two important practical conveniences: (1) Cosmic sources have an enormous range of brightness, and the magnitude system provides a quick shorthand for expressing these without referring to exponents. (2) The change of magnitude caused by a given (small) fractional change in the flux density of a source is numerically equal to the fractional change. E.g., a 5% error in flux density produces a 0.05 mag change.

Nonetheless, magnitudes are the source of considerable confusion among professional astronomers because there is not one magnitude system but instead several. For historical reasons within subfields, the definitions differ in two ways: **(1)** The spectral flux density can be expressed <u>either</u> as $f_\nu(\nu)$ or $f_\lambda(\lambda)$. **(2)** The normalizing constant $Q(i)$ differs among the systems; and even within a given system, it can differ with <u>waveband</u>.

The most widely used magnitude system through the year 2000 was based on a set of normalizing constants derived from the spectrum of the bright star Vega. We are now slowly moving to "absolute" systems based on calibrations in terms of physical flux units.

I. MONOCHROMATIC MAGNITUDE SYSTEMS

In Lecture 2 we introduced a "monochromatic" magnitude system, which is defined as:

$$m_\lambda(\lambda) \equiv -2.5 \log_{10} F_\lambda(\lambda) - 21.1$$

where $F_\lambda(\lambda)$ is the spectral flux density <u>per unit wavelength</u> of a source at the top of the Earth's atmosphere in units of $\mathrm{erg\ s^{-1}\ cm^{-2}\ \mathring{A}^{-1}}$. This is also known as the **"STMAG"** system because it is standard for the Hubble Space Telescope. For more details, see the *Synphot User's Guide* at **STScI**.

The corresponding system based on flux per unit <u>frequency</u> is

$$m_\nu(\lambda) \equiv -2.5 \log_{10} F_\nu(\lambda) - 48.6$$

where $F_\nu(\lambda)$ is in units of $\mathrm{erg\ s^{-1}\ cm^{-2}\ Hz^{-1}}$. **This is also known as the "AB" or "AB$_\nu$" system. This system has been adopted by the Sloan Digital Sky Survey and GALEX. (The resulting magnitudes are therefore very different from the STMAG system in the UV, for example.)**

These are the most intuitively obvious of the various magnitude scales used by astronomers since the normalizing constants are the same at all

wavelengths, and magnitudes are easily convertible to physical flux units regardless of the wavelength observed.

The zero points are defined to coincide with the zero point of the widely used "visual" or standard "broad-band" V magnitude system: i.e.

$$m_\lambda(5500\,\text{Å}) = m_\nu(5500\,\text{Å}) = V$$

However, the infinitesimal wavelength range inherent in the f_λ or f_ν definitions cannot actually be measured in practice. The monochromatic magnitudes are estimates based on observations made in wider bands, and the conversion to flux can only be as good as broader-band observations can be calibrated. In fact, these systems are the stepchildren of more basic broad-band systems in use by astronomers for over 100 years—but which are more awkward in definition and usage.

II. FILTER PHOTOMETRY SYSTEMS

Broad-band magnitude systems were introduced into astronomy by photography, which offered multi-wavelength response not confined to the acceptance band of the human eye (the "visual" band). As photoelectric detectors were implemented, systems defined using <u>filters</u> of varying widths proliferated. (Consult the bibliography for details.)

It is extremely difficult to calibrate astronomical photometric equipment the way one would calibrate laboratory equipment. Observations are made under field conditions, and there are no easily deployable laboratory flux standards. Each piece of photometric equipment, wavelength isolator, and telescope has different properties, and the throughput of the Earth's atmosphere changes nightly.

Thus, astronomical magnitude systems are defined in terms of the brightness of sources in specific wavebands <u>relative</u> to a <u>set of "standard stars"</u> selected by agreement. These relative brightnesses are <u>better determined</u> than any "absolute fluxes" which might derived from them. (Therefore, much astrophysical analysis is of the magnitudes rather than the fluxes.)

Apart from the difficulty of flux calibration, broad-band systems suffer from the fact that they are indeed <u>broad</u>, with typical optical bandwidths in the
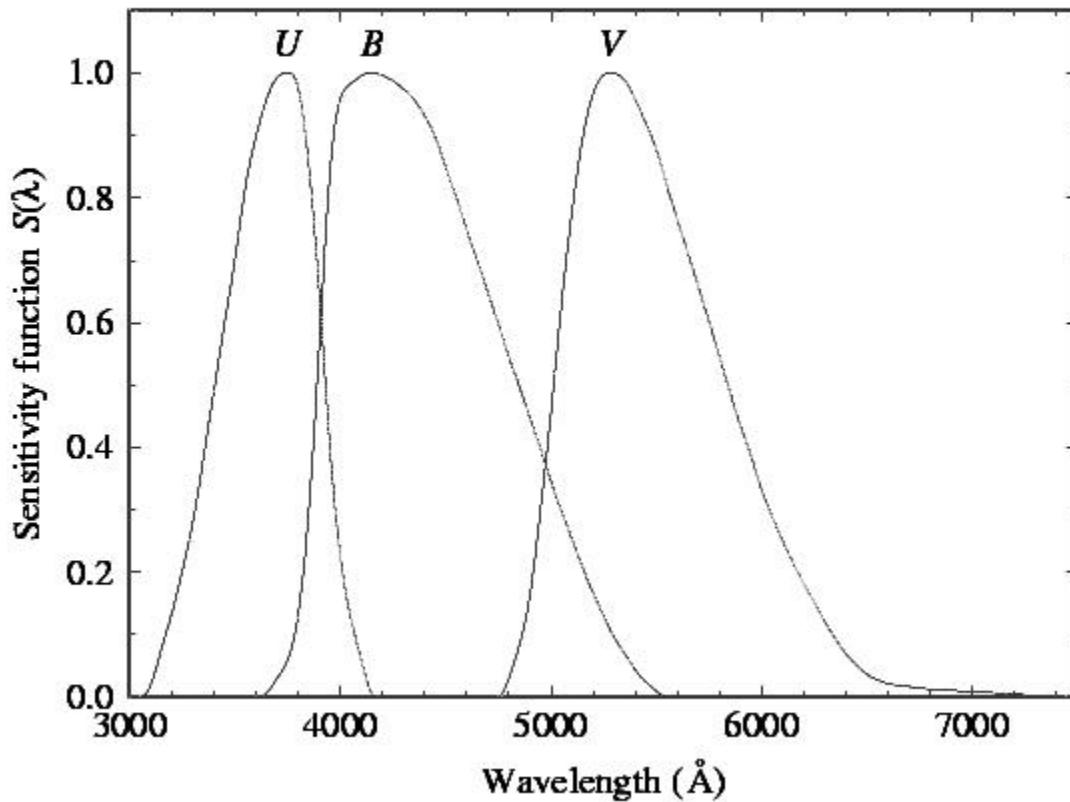
range $\sim$500–2000 Å. The reason for this is, of course, to enable study of faint astronomical sources. But wide bands introduce a host of difficulties arising from the fact that cosmic SED's and instrumental responses can change significantly <u>within</u> the bands.

A brief history:

- The original visual system was, of course, based on the response of the human eye. In the 19th century, various clever designs emerged which allowed simultaneous comparative "photometry" of the brightnesses of two stars or of a star and a calibration source. But visual measures were never very precise.

- The photographic plate gave rise to the "photovisual" system (Pv), using filters which approximated the eye's response, and the "photographic" system (Pg), which used a bluer filter to take advantage of the enhanced photographic response there. Aperture photometers and densitometers allowed quantitative extraction of information stored on a plate (subject, though, to significant uncertainties owing to the nonlinear and nonuniform responses of emulsions).

- The first photoelectric filter photometry system to be used for large numbers of measures of stars and galaxies was the "6-color" system of Stebbins and Whitford (1935-1960).

- The 1P21 photomultiplier tube (1945), with dramatically improved performance in both sensitivity and noise rejection (especially if operated in <u>pulse-counting</u> mode), ushered in a new era in photoelectric photometry. It had a CsSb ("S-4") photocathode, with excellent blue (4000 Å) sensitivity but a red cutoff at about 6300 Å. A number of later PMT designs (e.g. S-1, S-20) extended sensitivity to 8000-10000 Å.

- The prototype broad-band system for those in wide use today was the "UBV" system of Johnson & Morgan (ApJ, 117, 486, 1953). This was carefully defined in terms of specific choices of filter glass matched to the 1P21 PMT <u>and</u> the energy distributions of stars (see plot below). It was normalized to the SED of Vega.

- The UBV system was so successful in applications to stellar evolution, stellar populations, interstellar dust, and extragalactic astronomy that it was energetically extended to the red and infrared (R,I,J,H,K). Unfortunately, this work was undertaken by numerous workers using different filters and detectors, resulting in a confusing and incompatible

set of definitions (especially in R and I). Only in the last 20 years have these systems been consolidated (see bibliographic entries for Bessell).
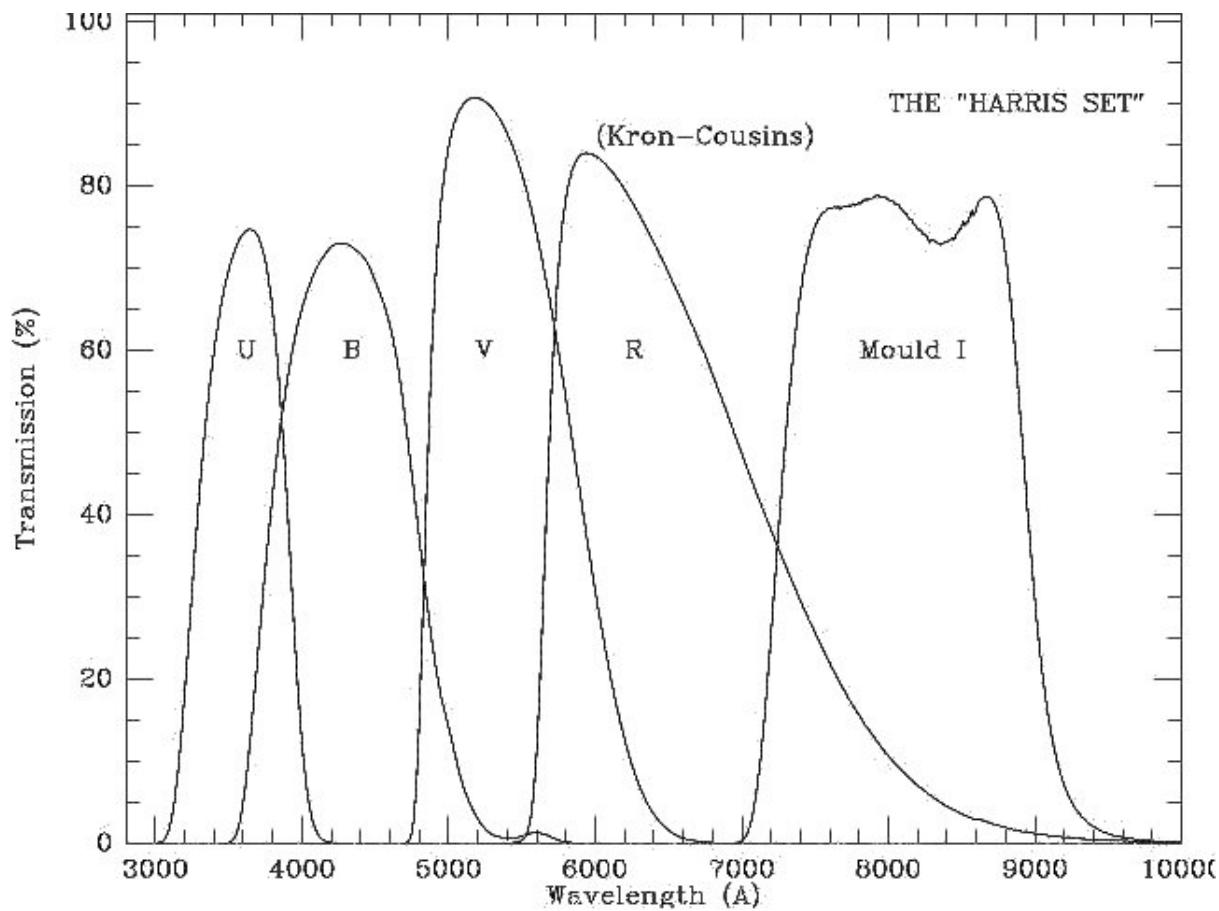
- The basic broad-band system in use today covers the optical range (3300-10000 Å) with UBVRI, the near-infrared (1.0-2.4μ) with JHK, and part of the mid-infrared (3.0-10μ) with LMN. In the future, the most widely used bands in the LMN region will be the four defined by the IRAC camera on the Spitzer Space Telescope.

- The Strömgren system was the first intermediate-band system to be widely used. Its filters (180-300 Å width) were tailored to measure abundance, temperature, and surface gravity effects in AF stars. Various other systems (e.g. DDO) extend this approach to later type stars and galaxies.

- Other useful broad-band systems include the Washington system (intended for faint GK stars) and the Thuan-Gunn system. The latter is like UBVRI but with filters optimized for faint galaxies by rejecting night sky lines; it is used for the Sloan Digital Sky Survey.

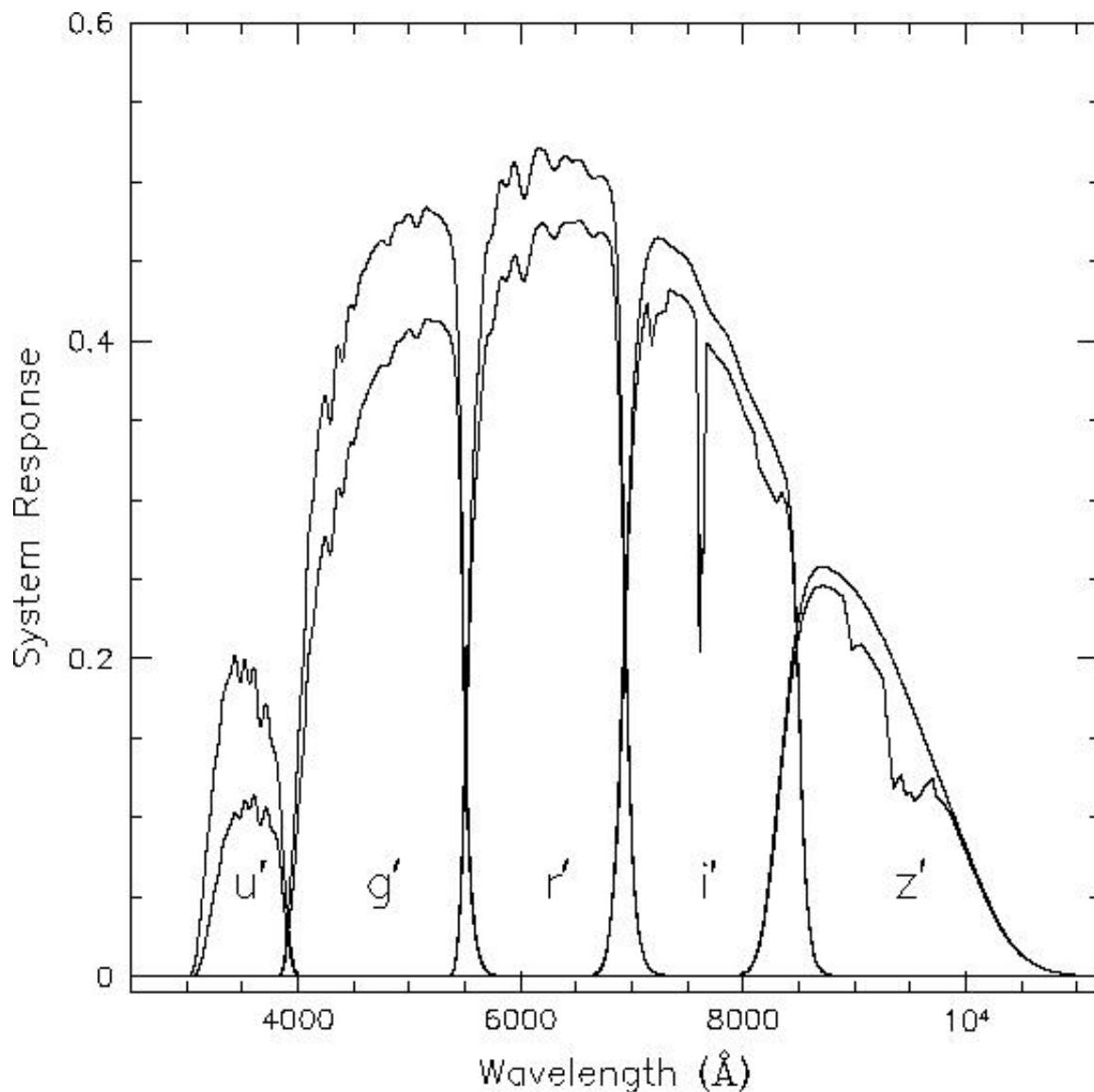- You can find transmission data for all the widely used filter systems at: http://voservices.net/filter/.

The Johnson-Morgan UBV filter system.

Approximate central wavelengths and bandwidths are:

Band	$<\lambda>$ (Å)	$\Delta\lambda$ (Å)
U	3600	560
B	4400	990
V	5500	880

Standard UBVRI broad-band filter response curves (KPNO).

Sloan Digital Sky Survey broad-band filter responses. Filters are on the Thuan-Gunn system. Compared to standard UBVRI, these have more sharply defined band limits and avoid stronger night sky emission lines. The g' band takes the place of standard B and V and z' extends the system to the red limit of standard CCD response. Curves here include net throughput of telescope and detectors. Lower curves show effects of atmospheric absorption at 1.2 airmasses.

III. THE VEGA MAGNITUDE SYSTEM

Following Johnson & Morgan, the set of calibrator stars for most filter photometry systems is defined by one "primary standard," the bright star <u>Vega</u> (α Lyrae), which in turn has been coupled through an elaborate bootstrap technique to a large set of "secondary" standards. The bootstrap involved extensive observational filter photometry and spectrophotometry as well as theoretical modeling of stellar atmospheres.

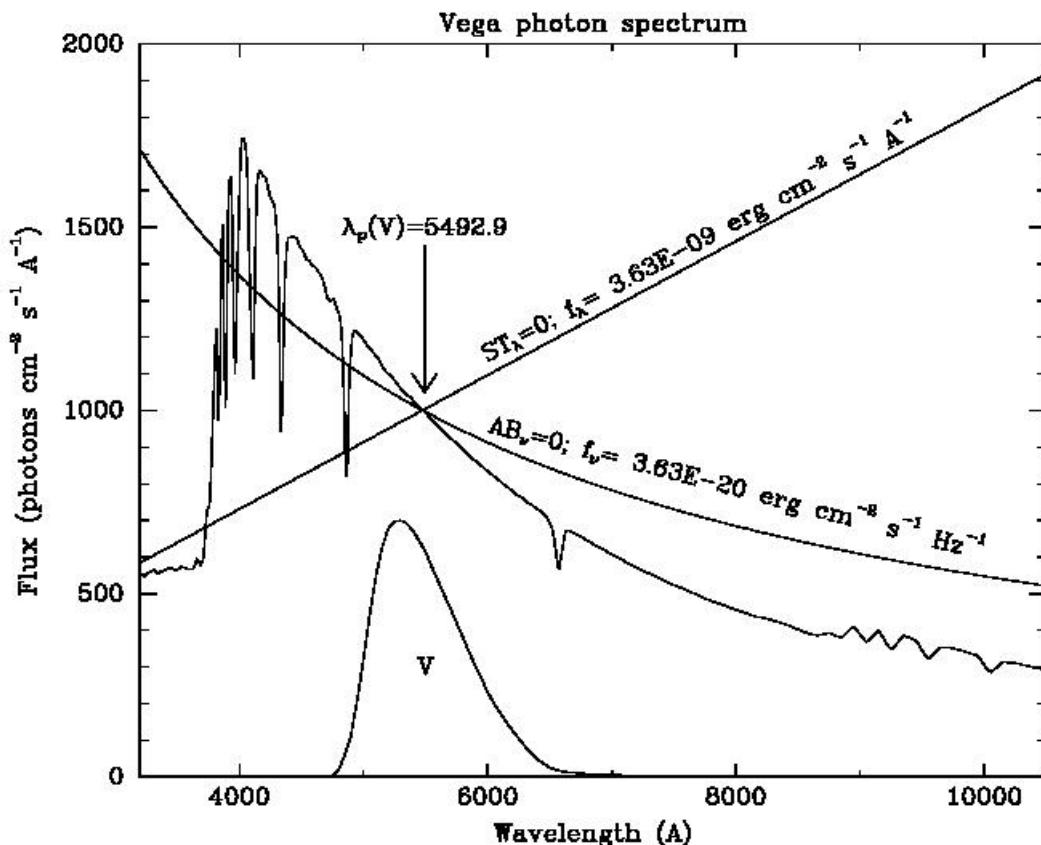**Ideally, the Vega magnitude system (or "vegamag") is defined as follows:**

- **o Let $R_i(\lambda)$ be a transmission function for a given band i. R is usually determined primarily by a filter.**

- **o Then for a source whose spectral flux density at the top of the Earth's atmosphere is $F_\lambda(\lambda)$, the broad-band magnitude m_i is**

$$m_i = -2.5 \, \log_{10} \frac{\int R_i(\lambda)\lambda F_\lambda(\lambda)d\lambda}{\int R_i(\lambda)\lambda F_\lambda^{\mathrm{VEGA}}(\lambda)d\lambda} + 0.03$$

  where 0.03 is the V magnitude of Vega. The system is based on spectral flux density per unit wavelength.

- **o Here we have assumed a photon-counting detector, so that the system response is proportional to the <u>photon rate</u>. This is the origin of the additional λ term in the expression above (the photon rate is $[\lambda/hc]F_\lambda$).**

- **o Vega was chosen as the primary standard because it is easily observable in the northern hemisphere for more than 6 months of the year and because it has a relatively smooth spectral energy distribution compared to later type stars. Bright stars with the same spectral type are relatively common.**

- **o The SED of Vega is shown on the next page, and an IDL save file containing a digital version (theoretical) of the SED is linked to the Lectures web page.**

- **o Vital statistics for Vega (from Kurucz): Spectral type A0 V. Effective temperature $T_e = 9550$ K. Surface gravity $\log g = 3.95$. Metal abundance $\log Z/Z_\odot = -0.5$.**

Figure 2.1: Standard photometric systems generally use the spectrum of Vega to define magnitude zero. The spectrophotometric magnitudes AB_ν and ST_λ refer instead to spectra of constant f_ν and f_λ respectively. Magnitude zero in both systems is defined to be the mean flux density of Vega in the Johnson V passband. Thus all three of the spectra shown here produce the same count rate in the Johnson V passband. The pivot wavelength of Johnson V is defined to be the crossing point of the $AB_\nu = 0$ and $ST_\lambda = 0$ spectra.

Vega photon spectrum

**Plot of zeropoint spectra in three different magnitude systems.
(Note: ordinate is __photon__ flux, not energy flux.)**

Although Vega has a reasonably smooth energy distribution, as compared, for example, to an M star, there are obviously large changes in the vegamag normalizing constants with wavelength.

Implications of this definition:

o In each band, the system <u>weights</u> the photon SED of a source by the defined response function $R_i(\lambda)$. This is usually <u>not</u> a "top-hat" function with a flat top and vertical sides.

     The R_i's for the basic bands are described. for example, in the articles by Bessell in the bibliography.

     Even under the best circumstances, any particular equipment will differ slightly in R_i from the standard system. This means that <u>transformations</u> to the standard system must be part of any photometric reduction.

     Calibration is more difficult the broader is the band. This is because of changes in the source SED and the weighting function within the band. Cf. equation (2) of Lecture 12.

o For Vega, or any other A0 V type star,

$$m_i = m_j = V \text{ for all } i, j$$

o The zero point of the system is defined by the SED of an A0 V star, which means that the <u>zero point differs from band to band.</u>

     Even if $m_i = m_j$, the corresponding mean fluxes, $< F_\lambda >_i$ and $< F_\lambda >_j$ are generally <u>not</u> equal. From the plot of Vega's SED, one can see that the flux zero point at 1μ, for instance, is significantly lower than at 5500 Å.

     This is a major departure from the monochromatic systems, for which equal magnitudes imply equal flux densities.

o With modern highly sensitive detectors, Vega is too bright to observe directly for calibration. Instead, one must observe (every night) a selection of secondary or tertiary standards.

o The actual zero points of the system are defined by the <u>full set</u> of secondary standard stars. Small residual "closure" errors mean that the measured values in practice will depend on the particular set of standards observed on a given night.

IV. FLUX CALIBRATION OF THE VEGA SYSTEM

The zero point of the Vega system is based on high quality data from PMT-based spectrophotometers obtained by Oke, Schild, Hayes, and Latham ca. 1967-1975 (see bibliography). These spectra were calibrated by direct observations of nearby laboratory light sources (e.g. platinum furnaces), though this introduced many complications (e.g. horizontal extinction across the mountain tops).

These data sets have been melded with increasingly high fidelity synthetic stellar spectra (theoretical) to extend and consolidate the system across the various bands. The best current calibration for the UBVRIJHK system is by Bessell, Castelli, & Plez (1998, see bibliography and next page). You were already introduced to the zero point of the V-band system in Lecture 2:

Fluxes for a $V = 0$ star of spectral type A0 V at 5450Å:

$$f_\lambda^0 = 3.63 \times 10^{-9} \, \mathrm{erg \; s^{-1} \; cm^{-2} \; \mathring{A}^{-1}}, \text{ or}$$

$$f_\nu^0 = 3.63 \times 10^{-20} \, \mathrm{erg \; s^{-1} \; cm^{-2} \; Hz^{-1}}, \text{ or}$$

$$\phi_\lambda^0 = f_\lambda^0/h\nu = 1005 \text{ photons } \mathrm{cm^{-2} \, s^{-1} \, \mathring{A}^{-1}}$$

Note that the effective wavelength of the filter shifts with the SED of the source and is closer to 5500 Å for G-K stars.

The flux zero point in filters other than **V** is then defined by the spectral energy distribution of an A0 V star. The relationship between the magnitude in a given filter and the mean flux in that filter is then given by:

$$m_i = -2.5 \log_{10} \left(\frac{< F_\lambda >_i}{F_{0,i}} \right)$$

where $F_{0,i}$ is the zero point for band i as given in the Bessell listing (units: $\mathrm{erg \; s^{-1} \; cm^{-2} \; \mathring{A}^{-1}}$).

BROAD BAND SYSTEM ZEROPOINTS (BESSELL ET AL. 1998)

	U	B	V	R	I	J	H	K	Kp	L	L*
λ_{eff}	0.366	0.438	0.545	0.641	0.798	1.22	1.63	2.19	2.12	3.45	3.80
f_ν	1.790	4.063	3.636	3.064	2.416	1.589	1.021	0.640	0.676	0.285	0.238
f_λ	417.5	632	363.1	217.7	112.6	31.47	11.38	3.961	4.479	0.708	0.489
$zp(f_\lambda)$	0.770	-0.120	0.000	0.186	0.444	0.899	1.379	1.886	1.826	2.765	2.961
$zp(f_\nu)$	-0.152	-0.602	0.000	0.555	1.271	2.655	3.760	4.906	4.780	6.775	7.177

f_ν (10^{-20} ergs cm^{-2} sec^{-1} hz^{-1})
f_λ (10^{-11} ergs cm^{-2} sec^{-1} Å^{-1})
$mag_\lambda = -2.5 \log(f_\lambda) - 21.100 - zp(f_\lambda)$
$mag_\nu = -2.5 \log(f_\nu) - 48.598 - zp(f_\nu)$

Bessell's 1998 values for the effective wavelength of each filter (for an A0 V spectral type) and the corresponding flux zeropoints are given above. These, together with mean colors for various spectral types on the Johnson system (which differs from Bessell), will also be handed out.

Note two important typos in the published table: the fourth row of the table should be labeled $zp(f_\nu)$ and the fifth row should be labeled $zp(f_\lambda)$.

V. COLORS

A "color" is simply a difference between magnitudes for a given source in two different bands:

$$C_{ij} \equiv m_i - m_j = -2.5 \, \log_{10} \left(\frac{< F_\lambda >_i}{< F_\lambda >_j} \right) + const_{ij}$$

where the constant is a function of the zeropoints of the two bands.

Colors measure the <u>slope</u> of the spectral energy distribution between bands i and j.

Note that the definition is such that a more positive color implies a larger flux in the <u>second</u> (j) band.

It is usual, though not universal, that the two bands are entered in order of increasing wavelength.

Colors are used in all of the magnitude systems defined above. However, the historical precedent of the classic Johnson-Morgan system is such that labels such as **UBVRIJK** are understood to refer to the Vega system unless otherwise explicitly stated.

The V band is often a convenient reference, so colors like B-V, V-I, V-K are widely used. The standard "UBV" system employs U-B and B-V. Cool sources which emit little light below 8000 Å are often characterized by I-K, J-K, H-K, etc.

VI. ATMOSPHERIC EXTINCTION

The effect of atmospheric extinction on photometry (cf. Lec 4) is usually expressed as:

$$m_{obs} = m_{true} + k(\lambda)\sec Z$$

Here, m_{true} is the magnitude of the source outside the Earth's atmosphere, and m_{obs} is the magnitude observed.

The "air mass," $\sec Z$, where Z is the angular zenith distance, is given by the following expression (for plane-parallel geometry):

$$\sec Z = [\sin\phi\,\sin\delta + \cos\phi\,\cos\delta\,\cos h]^{-1}$$

where ϕ is the latitude of the observatory, h is the hour angle of the source, and δ is the declination of the source.

$\sec Z$ is the total atmospheric pathlength toward the source in units of the vertical pathlength. For best photometry, keep $\sec Z \lesssim 2$.

$k(\lambda)$ is the "extinction coefficient."

Several different physical effects contribute to continuous extinction. These include Rayleigh scattering ($\sim \lambda^{-4}$), ozone or H_2O molecular absorption, and aerosol scattering. Each of these is characterized by a different effective scale height, so that their mixture will <u>change</u> with altitude.

Extinction coefficients have been carefully measured for a number of observatories. A table for Palomar (Hayes & Latham 1975) is included on the next page.

Ignoring its multi-component nature, approximate values of extinction for a given site can be obtained from those for another (assuming a hydrostatic, isothermal atmosphere) as follows:

$$k(X_1) = k(X_0)\exp\left(-\left[\frac{(X_1 - X_0)}{H_0}\right]\right)$$

where X_1 and X_0 are the altitudes of the two observatories and H_0 is the scale height of the atmosphere (7950 meters for STP).
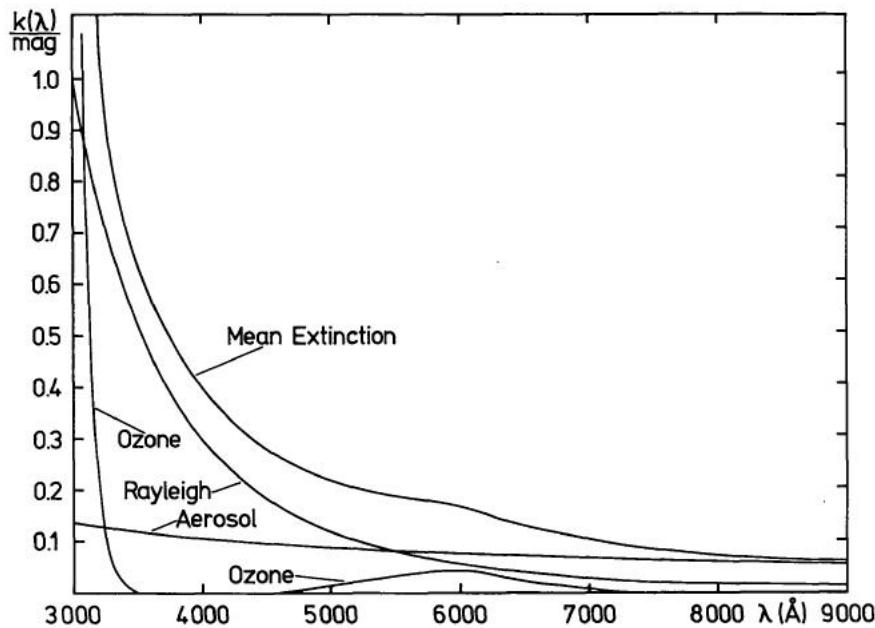
Fig. 1. Mean vertical extinction at Flagstaff, Arizona, in May-June 1976. The assumed ozone and Rayleigh contributions are shown separately

TABLE 5

MEAN EXTINCTION COEFFICIENTS FOR PALOMAR MOUNTAIN
FABRICATED WITH OUR PROCEDURE

Wavelength (Å)	$A(\lambda)$ [mag (air mass)$^{-1}$]	Wavelength (Å)	$A(\lambda)$ [mag (air mass)$^{-1}$]
3200......	1.058	5263......	0.164
3250......	0.911	5556......	0.151
3300......	0.826	5840......	0.140
3350......	0.757	6055......	0.133
3390......	0.719	6435......	0.104
3448......	0.663	6790......	0.084
3509......	0.617	7100......	0.071
3571......	0.575	7550......	0.061
3636......	0.537	7780......	0.055
3704......	0.500	8090......	0.051
3862......	0.428	8370......	0.048
4036......	0.364	8708......	0.044
4167......	0.325	9832......	0.036
4255......	0.302	10255......	0.034
4464......	0.256	10610......	0.032
4566......	0.238	10795......	0.032
4785......	0.206	10870......	0.031
5000......	0.183		

ATMOSPHERIC EXTINCTION (cont)

Apart from the continuous absorption just described, molecules in the Earth's atmosphere produce <u>discrete absorption features</u> that can be very troublesome in certain wavelength ranges. The features can be quite strong and/or variable.

Here are some of the most important absorption features in the optical range:
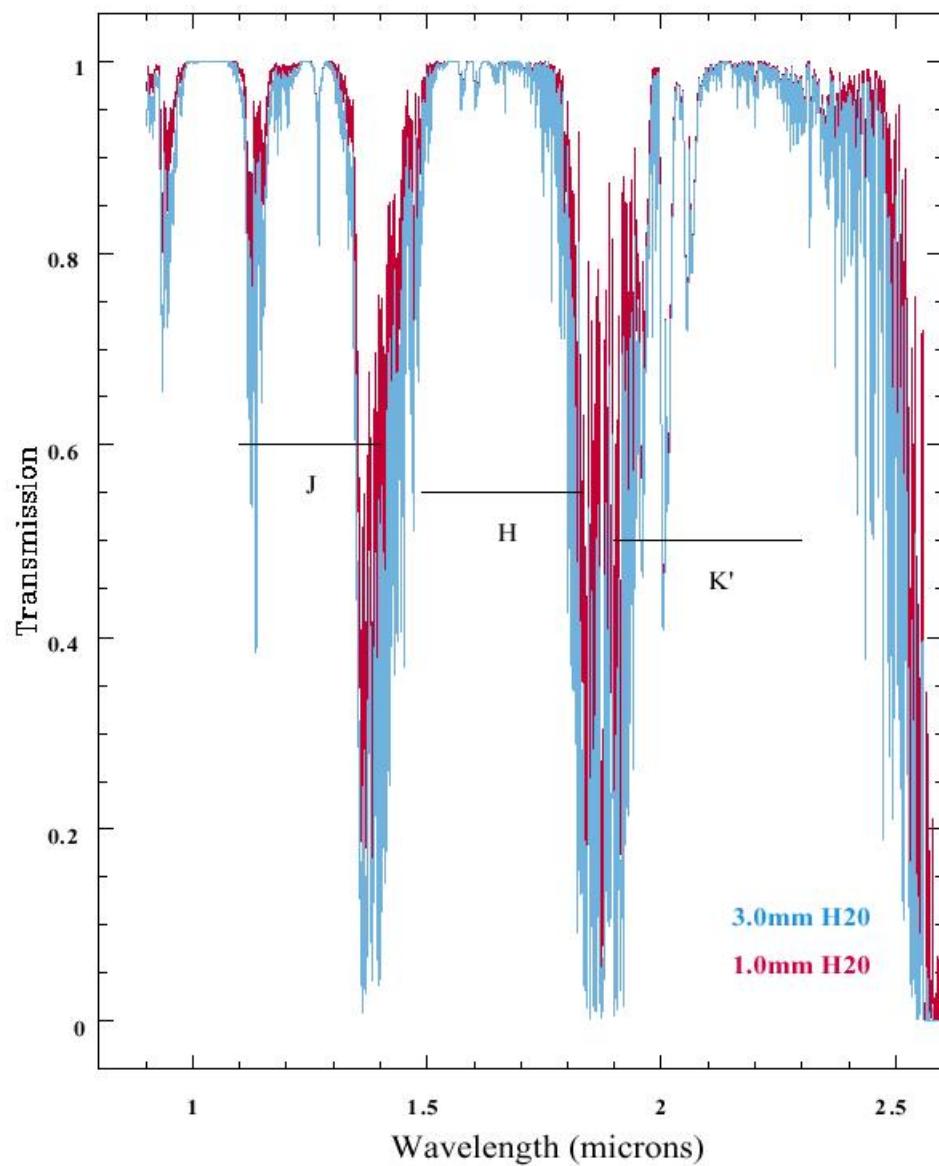
Wavelength	Species
6275-6300 Å	O_2
6860-6950 Å	O_2 "B" band
6900-7100 Å	H_2O
7580-7700 Å	O_2 "A" band
8050-8350 Å	H_2O
9000-9800 Å	H_2O

The very strong H_2O absorption in the 1-3 μ range is shown on the next page.

Because of the large number of transitions within a given molecular band and the potentially rapid change of optical depth with transition, extinction by the bands often does not have the simple $\sec Z$ dependence described above. Some transitions saturate faster than others.

It is best to avoid these regions. Standard broad-band filter systems are designed to avoid the stronger absorption features (e.g. J,H,K', as shown on the next page). However, it is not always possible to do this in narrow-band photometry and spectroscopy. The best way to deal with the features is to find hot reference stars (i.e. with smooth SEDs) near each target and observe them at the same zenith distances.

LINE ABSORPTION IN THE NEAR-IR

Transmission of the Earth's atmosphere in the near-infrared. Absorption is dominated by H₂O. Horizontal lines show the definitions of the J, H, and K' photometric bands, which lie in relatively clean regions.

VII. PHOTOMETRIC CALIBRATION & REDUCTION

Calibration of photometric observations and reduction to the standard broad-band system require observation of standard stars. These are used to determine the effects of atmospheric extinction and the "transformation" between the response of your equipment and that of the standard system.

(i) **Atmospheric transmission varies from night to night. You must make sufficient standard observations each night to calibrate that independently of other nights of your run.**

(ii) **However, you can use combined data from several nights to determine the photometric transformations between your filter set and the standard set.**

(iii) **Calibration can have strong <u>color-dependent</u> terms. This means that your standards must span the color range of your "unknowns." The larger is that range, the larger is the standard set you need to observe.**

(iv) **Standards must be in the range of brightness appropriate for your equipment.**

Recalling equation (2) of Lecture 12, the count rate $\dot{s}$ (detected photons per second in filter i) for a given source is

$$\dot{s}_i \propto \frac{\pi}{4} D_e^2 \int e^{-k(\lambda)\,\sec Z}\, T_i(\lambda)\, \frac{F_\lambda(\lambda)}{h\nu}\, d\lambda$$

A. Narrow-Band Observations

If the bandwidth of the filter is so narrow that all of the terms in the above expression are well approximated by mean values, then it is straightforward to calibrate your data. The basic relation is:

$$m_{ij} = -2.5 \log_{10} \dot{s}_{ijk} - K_i \sec Z_k - a_i$$

Here m_{ij} is the broad-band magnitude in filter i for standard star j; $\dot{s}_{ijk}$ is the count rate for the kth observation of this star at zenith distance Z_k.

K_i is the atmospheric extinction coefficient for band i and a_i is the transformation term between the local and standard photometric systems.

The problem is to determine K_i and a_i. More than two standard star observations overdetermine the problem, but a large set of calibration observations gives valuable information on the scatter from random and systematic errors (e.g. secular variation in atmospheric transparency).

B. Broad-Band Observations

The complexity here arises from the fact that the various terms in the proportion above are not constant across a broad-band filter. The effect of this will depend on the distribution of light of the source within the band, i.e. on the spectral slope of the source. This means there will be <u>color terms</u> in both the effective atmospheric extinction and in the transformation.

The normal approach to this problem is to first define "instrumental colors," based on the relative count rates of two adjacent filters. For example:

$$ C_i \equiv -2.5 \log_{10} \left(\frac{\dot{s}_{i+1}}{\dot{s}_i} \right) $$

Then, rewrite the calibration equation above to include color terms in both the extinction and transformation as follows:

$$ m_{ij} = -2.5 \log_{10} \dot{s}_{ijk} - [K_{0,i} + K_{1,i} C_i] \sec Z_k - a_{0,i} - a_{1,i} C_i $$

The problem now includes 4 unknowns, and solution depends on having a large range of color in the standard star observations. Many more standard observations are needed than in the narrow-band case.

To provide a more robust solution, one normally starts with an estimate of the K terms (based on average atmospheric conditions) and iterates on those.

For more details on extinction corrections and reduction, see the bibliography (the Young articles are very thorough).